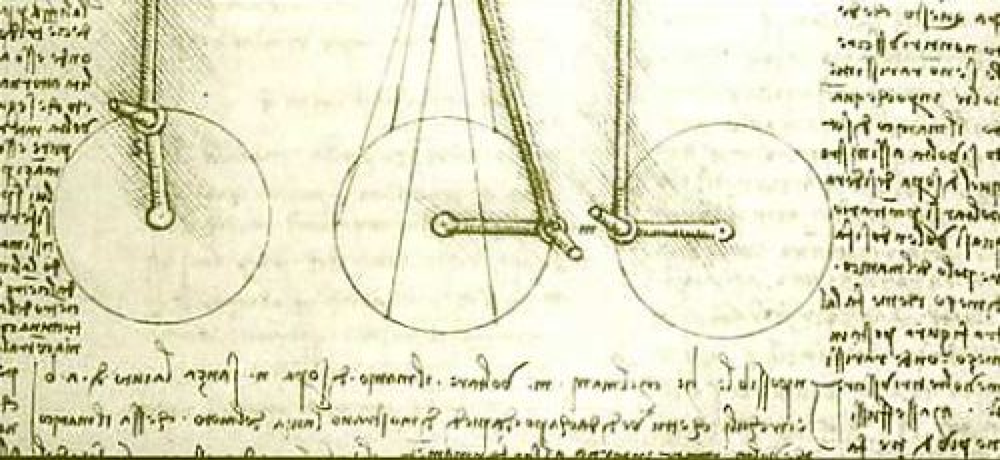
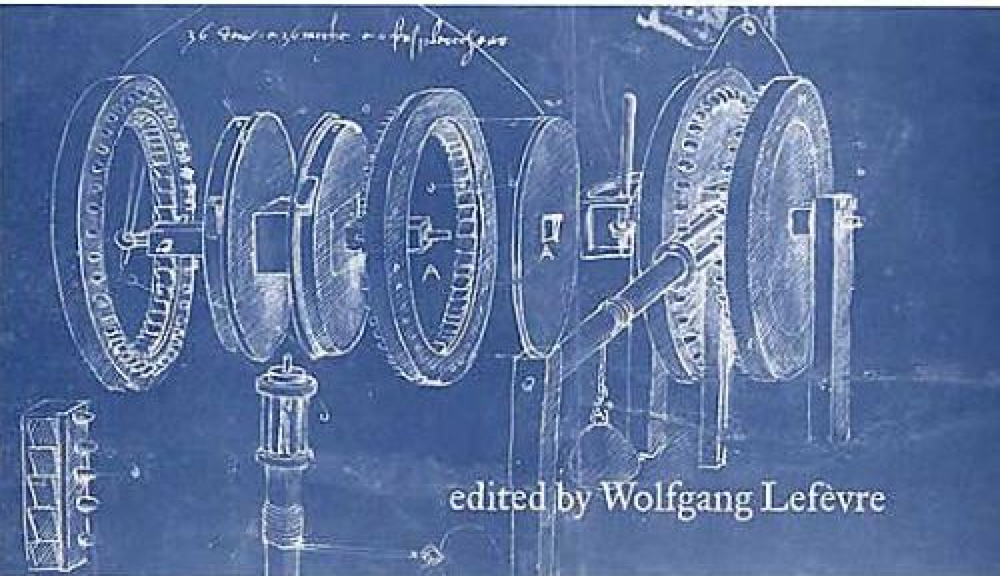


PICTURING MACHINES 1400–1700



edited by Wolfgang Lefèvre

PICTURING MACHINES
1400–1700

TRANSFORMATIONS

STUDIES IN THE HISTORY OF SCIENCE AND TECHNOLOGY

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PICTURING MACHINES
1400–1700

EDITED BY WOLFGANG LEFÈVRE

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INTRODUCTION

WOLFGANG LEFÈVRE

The engineers and architects of the Renaissance are renowned not only for the universality of their genius and the audacity of their creations but also for their drawings. Leonardo da Vinci's famous drawings of technical devices, although unparalleled in many respects, are just one instance of a practice of drawing in the realm of early modern engineering that came into being at the end of the Middle Ages and eventually addressed a broad audience through the *Theatres of Machines* in the last third of the sixteenth century. The new types and methods of graphic representation developed and used by Renaissance engineers have long attracted the attention of historians of art, architecture, science, and technology. Apart from their often fascinating aesthetic qualities, these drawings have been particularly appreciated as historical documents that testify to the development of technology, the spread of perspective, the psychological roots of technological creativity, and the beginnings of modern scientific attitudes.

As an unintentional consequence of this appreciation, however, little attention has focused on the significance that these drawings had, not for present historians, psychologists, and philosophers, but for the historical actors themselves, that is, for the mechanicians, engineers, and architects of the age. Why did they produce drawings? For whom and for what purposes? What were the prerequisites for this drawing practice, what were the contexts, and what were the consequences? In short, how did drawing shape the practice and the notions of early modern engineers? Those were the questions from which the idea of this volume arose.

Among the many shared views of this volume's authors (who also differ with respect to several aspects of its topic), there is the conviction that these questions can be successfully approached only by studies that dispense with large generalizations as regards the cultural, technological, intellectual, and aesthetical significance of early modern engineering drawings—generalizations that hampered rather than promoted an adequate recognition of them in the past. They believe firmly that what is needed instead is studies that actually go into the specific details and properties of these drawings—studies that, in addition, focus thoroughly on different aspects of such properties, regardless of whether or not the pursuit of these different aspects leads to a new large synthesis.

The authors' emphasis thus lay on analysis, fine-grained studies, and close attention to details, and all of them refrained from premature synthesis. The extent to which their studies nevertheless form a connected and consistent whole was surprising even for them. However, the authors being experts in this field of historical investigation, the chapters' connectedness and coherence may be more obvious to them than to a broader readership. This introduction therefore cannot dispense with giving some indications of the context for these studies. The following outline of the histori-

cal setting of these drawings as well as of their crucial aspects constitutes neither shared starting points nor jointly achieved results of the authors, but the attempt of the editor to provide a sketchy topography in which each chapter's choice of issues and perspectives can be located.

TECHNICAL DRAWINGS: SYMPTOM AND INTEGRAL PART OF THE TECHNOLOGICAL TRANSFORMATIONS IN THE EARLY MODERN AGE

In the culture of the West, technical drawings, that is, drawings traced by technicians for professional purposes or those derived from them, appeared only at the end of the Middle Ages and flourished for the first time during the Renaissance. The emergence and development of such a drawing practice is in itself a remarkable fact that indicates profound changes in the social labor process of the West during this age. Traditional production did not employ technical drawings. This holds for agriculture, then and for a long time to come the most important domain of social labor in terms of both the amount of people employed and the wealth produced. For farming, breeding, and growing according to the standards given at the time, and even for manufacturing intricate tools like ploughs, no technical drawings were needed. Surveying, which may come to mind in this context, was not yet a normal part of the agricultural practice. As regards the realm of ordinary crafts, the second important domain of social labor, one encounters by and large the same picture. Almost all of the crafts performed their professional tasks without drawings. An important exception was, naturally, the decorative arts, which passed on established figurative and ornamental patterns through *exempla*, that is, pattern books *(Musterbücher)*. But among the drawings of these *exempla*, only a few could properly be called technical drawings. It was not developments within these established fields of production to which technical drawings owed their birth. Rather, they owed their emergence and development to new sectors of production that transgressed the limits of the traditional labor processes still prevailing, in terms of both the depth of division of labor and the technical procedures applied. And the employment of such drawings is indicative of just these transgressions.

Technical drawings appeared first at the construction sites of Gothic cathedrals. The oldest extant architectural plans date from the thirteenth century. Warfare was the next sector where technical drawings were utilized. Beginning with a few instances in the second half of the fourteenth century and blooming in the fifteenth, drawings of all sorts of assault devices, and increasingly of guns and gun-mountings, heralded the era of early modern machine drawings. Since the middle of the fifteenth century, they were complemented and gradually even outnumbered by drawings of civil devices such as mills of all kinds, cranes and other hoisting devices, and different kinds of pumps and further water-lifting machines for mining and irrigation. Along with this, though apparently following somewhat secluded paths, technical drawings of ships testify to a developmental stage of the ship-building craft where traditional crafts methods, though still indispensable, no longer sufficed.

The emergence and spread of technical drawings thus was part and parcel of new developments in certain exceptional fields in the realm of production that could be called the high-tech sectors of the age. Moreover, these drawings were connected with those very features of all or some of these fields of advanced production by which they distinguished themselves from the traditional ways of production in agriculture and ordinary crafts. The following five features deserve particular attention in this respect, although not all of them occur in each of the sectors of advanced technology.

First, technical drawings were connected with new forms of division of labor characteristic of some of the new high-tech production sectors—forms that developed first at the construction sites of Gothic cathedrals and later, outside architecture, above all along with shipbuilding and mining on a large scale. In these sectors, the flat hierarchy of the typical craft workshop was replaced by complex structures of cooperation, responsibilities, and command. The chief engineer or architect, himself subordinated to clerical or secular commissioners or boards of commissioners, had to instruct and coordinate masters and subcontractors from different crafts who, for their part, directed their teams. The task of coordination often comprised the harmonization of work carried out at different times and different places. Such intricate forms of cooperation among the different parties involved in a project of advanced technology necessitated not only new forms of communication but also new means of communication. Technical drawings are perhaps the most striking of those new means.

Second, technical drawings were connected with new forms of knowledge propagation brought about by the new production sectors of advanced technology. In these sectors, experience with and knowledge gained through new technologies could not be exchanged and circulated effectively by means of the undeveloped and slow communication mechanisms of the medieval crafts. Rather, for these purposes, too, new means of communication were needed, technical drawings included.

Third, and in close relation to this, technical drawings were connected with new forms of learning and instruction that developed along with the new high-tech production. The traditional form in which craftsmen passed on knowledge and skills to the next generation, that is, the system of apprenticeship and journeymen's traveling that rests essentially on learning by doing, proved to be insufficient to acquire all of the capabilities required by the advanced technologies. First forms of schooling technical knowledge developed. In fact, the art of producing and reading technical drawings constituted a central part of the curriculum of the first technical schools, as is indicated in the very name of the first of these schools, namely the *Accademia del disegno* in Florence founded in 1563.

Fourth, technical drawings also were closely related to fundamental changes in the body of practical knowledge induced by the advanced production sectors. This body no longer comprised only the traditional experiences and skills of practitioners, but combined them with elements of knowledge that originated in sciences. The expanding employment of geometrical constructions and theorems in several practi-

cal contexts is particularly characteristic of this development. The broad range of competence required by the new technologies is best represented by a social figure who came into being with the new fields of advanced production, namely the engineer, who was in charge not only of designing and planning ambitious projects but also of their actual realization through the coordinated cooperation of different kinds of crafts. For these engineers, experience in several crafts was no less important than competence in design and planning, which, for its part, included learned knowledge as well as drawing capabilities.

Fifth, technical drawings were connected with the establishment of technology as a matter of public interest. Pictorial representations of machines and devices of all sorts were a chief means by which the protagonists of the new advanced production sectors managed to attract the attention of the educated public for technological issues. With this, technology became, for the first time in the culture of the West, appreciated as a valuable sphere of culture. The technological transformations of the early modern period entailed a cultural transformation in which drawings played a significant role.

TECHNICAL DRAWINGS IN EARLY MODERN ENGINEERING

The practical role of technical drawings in early modern fields of advanced production was not fixed once and for all. Naturally, for which tasks technical drawings could be employed depended partly on the needs of the engineering process in its concrete social embedment and partly on the capacity of drawings to meet such needs. However, both those needs and that capacity underwent changes—changes that often were intertwined. By using drawings and, above all, by developing particular pictorial languages, engineers discovered the possibilities given by the medium of drawings and broadened the spectrum of tasks for which drawings proved to be useful. Thus, an investigation of the actual functions of engineering drawings requires attention to many different aspects.

Mediating Parties

The most obvious function of technical drawings was a social one, that is, their function as a means of communication between different individuals or, more to the point, different kinds of individuals. They served as a means of communication between practitioners, either those of equal rank or those in a hierarchical relation; between contracting parties, commissioners, and responsible practitioners, as well as the latter and subcontracting practitioners; and between practitioners and a broader public interested in new technology. Mere contemplation of the different constellations of interacting parties makes immediately obvious that “communication” is much too vague a term to characterize the variety of roles technical drawings played in such interactions. In one case, they may have conveyed proposals; in another, they may have documented an agreement; in yet others, they may have fixed decisions, given

instructions, served as a basis for consultations and negotiations, exchanged experiences, imposed and secured control, advertised inventions, services, and projects, taught a community of readers, illustrated arguments, and so on. Each of these different cases of social interaction in which technical drawings are employed comprises not only a specific constellation of differing interests but also a specific constellation of differing experiences, competence, and knowledge. Thus, technical drawings could only function successfully as a means of communication and mediation if they allowed for the different and differently informed views of the parties involved. Drawings that served understanding between practitioners of equal rank jointly engaged in a certain project highlighted, or for that matter omitted, other aspects and details of a technical object than drawings that tried to convey an idea of the object to a commissioner. Each case required a different emphasis on completeness, precision, and neatness, and different distinctness as regards information about, for instance, the function of the device in question or its construction.

Studying technical drawings thus can reveal an entire world of social relations between different groups of individuals in which technological projects were embedded. And, conversely failing to take the concrete mediating role of a certain technical drawing into account may seriously impair the understanding of it. Giving testimony as much to the social form of early modern high-tech production as to the technology employed, the technical drawings of the age must be read with social-historical expertise as well as with technological competence. However, the ways in which the extant drawings were passed on often destroyed their original relations to other materials and thus important hints on which a reconstruction of this social context could have rested. As some of this volume's chapters show, it often is impossible to determine the original purpose and the circumstances of a technical drawing. In some cases, however, an analysis of the drawing style may prove helpful.

Between Pictures and Plans

The variety of ways in which technical drawings were employed as means of communication in the social world around the most advanced technologies of the age involuntarily draws attention to the astonishingly flexible capability of drawings as a medium. How did this medium succeed in serving so many different communication tasks, each of which had different conditions, demands, and purposes? A partial answer to this question may be found in the fact that, on closer inspection, the medium of drawings proves to comprise a whole of graphic languages—that of pictures, of diagrams, of plans, and so on. From a semiotic perspective, each of these different graphic languages follows particular rules and grammars. Accordingly, each of them demands particular expertise for rendering objects in the framework of its rules and also for reading and understanding such renderings. Architectural plans of some complexity, for instance, are only partly understandable to nonexperts. Obviously, different graphical languages presuppose different knowledge and are therefore involved in the discriminations that govern the social distribution of knowledge in a

culture. Furthermore, each of these graphic languages has its specific advantages and disadvantages. The graphic language that rules the construction of orthographic plans, for instance, may be unequaled as regards precise information about angles, distances, proportions, and so on. But it cannot compete with the language of perspective rendering when the purpose is to convey an impression of the object as a solid body situated in a surrounding space. Thus there is not only a social aspect of the employment of a specific graphic language but also a material one, that is, the aspect of the possibilities and limits of rendering characteristic of each of these languages. These differences between the material capacities of the various graphic languages by their very nature suggest which specific language is suitable in a certain case. However, the practitioner must additionally consider which of these languages is best understood by the audience in question. Compromises between these two concerns may be unavoidable, and such compromises are, indeed, a characteristic feature of the technical drawings of this age.

Architects and engineers of the early modern period did not only make use of multiple different graphical languages—and often in a virtuose manner—in a certain sense, they also must be regarded as the inventors of these languages. True, from the literary estate of Classic Antiquity, they inherited some clues to the graphic languages employed by their ancient predecessors; and equally true, they could build on some drawing conventions and geometrical techniques used by medieval architects and technicians. They did not have to start from scratch. Yet, as regards the different projection techniques developed in Antiquity—orthographic projection, perspective, geographic projection—each of them was reinvented rather than rediscovered in the Renaissance, and subsequently further developed and refined in an absolutely autonomous manner. Furthermore, against the background of the spread of perspective rendering in the fine arts, the schematic style characteristic of representing machines in the Middle Ages was replaced by a specific style of rendering machines on a single sheet, which was furthermore supplemented by the elaboration of an arsenal of artificial views such as cutaways, exploded views, and so on.

Surveying the styles and techniques of picturing developed from the modest beginnings in the late Middle Ages up to the famous drawings of Leonardo and to the splendid *Theatres of Machines* at the end of the sixteenth century, one encounters an admirably rich world of pictorial languages that was engendered along with the advanced technologies of the age. The range of these languages—from sketches, to perspective views from a deliberately chosen fictitious viewpoint, and to thoroughly constructed projections—testifies to the wealth of graphic abilities the community of engineers and architects commanded. At the same time it mirrors the broad spectrum of competences that had become characteristic of this community—ranging from practical skill and artisanal experience to learned knowledge.

Shaping Engineering

Among the many striking features of early modern engineering drawings, there is one that deserves particular attention. In contrast to architecture, scaled orthographic plans—ground plans, elevations, sections—were the exception rather than the rule in the realm of machine engineering. In this realm, pictorial representations in a quasi-perspective style prevailed. This finding suggests that, in this age, drawings were not as indispensable a means for designing and manufacturing machines as plans and blueprints are today. In the inceptive stage of the designing process, sketches may have been employed by Renaissance engineers in by and large the same manner as by contemporary engineers—I will come back to this in a moment. But, as far as the extant engineer drawings of the age show, the subsequent manufacturing process was not guided by exact plans as it is today. Even the few known instances of employment of orthographic plans in the manufacturing process suggest that these plans served as a means of orientation rather than as a blueprint. The Renaissance engineers could apparently confine themselves to telling the craftsmen in charge of execution some decisive details and leaving the concrete shaping of the machine parts to them. However, this reliance on craftsmen was not reliance on personal experience and tacit knowledge alone. Rather, they were relying on craftsmen who were well equipped with a rich arsenal of geometrical aids developed over centuries—several drawing instruments, templates of all sorts, practitioners' techniques of creating nontrivial geometrical shapes and developing one geometric figure from another. If one takes this arsenal of practical geometrical aids into account, the machine drawings of Renaissance engineers may appear less inappropriate for the manufacturing process than at first glance. Taken together with the geometrical means at the construction site, early modern machine drawings become recognizable as means of the real construction process.

The employment of drawings in design processes deserves further attention. In these processes, drawings are not simply visualizations of ideas. Rather they function as material means that shape ideas. Their role is that of models that simultaneously provide a fictitious and a real opportunity to test possible arrangements of machine parts, try out new combinations and alternative shapes of these parts, and so on. As material creations in space, drawings are subjected to the laws of space and thus represent real conditions as regards possible spatial relations of rendered objects. (The famous drawings by M. C. Escher, which seemingly transgress these laws, confirm this impressively.) The advantages of such flat models on paper over solid ones made of wood or clay are obvious. They can be created and changed almost instantly. Presupposing some drawing skill on the part of the engineer, their restraint to two dimensions can be compensated for almost completely. They are incomparably flexible and allow unfettered experimentation. They furthermore allow unparalleled concentration on issues of interest thanks to the possibility of omitting all interfering or distracting parts of the device in question and reducing it to its essentials. However, the limits of these models on paper are obvious as well. Being two-dimensional creations, their representational potency ends when the focus is no longer on shapes of machine

parts, their spatial relations, and the kinematic significance of such relations—when physical dimensions come to the fore, encompassing the mass and force of the designed object. Nevertheless, within their limits, drawings became an indispensable means of the design process and in this way shaped the very element of the engineering practice on which its fame as an outstandingly innovative activity essentially rests.

THE VOLUME'S FOCUS AND ARRANGEMENT

As stated above, this volume focuses on the functions and significance technical drawings actually had for the professional practice of early modern engineers and architects. It will not address other interesting aspects such as the aesthetic quality of these drawings, the manifold levels of meanings that these drawings had beyond the context of engineering, their place in the visual culture of the Renaissance, and so on. But, as may be obvious after this short introductory outline, despite this concentration on the engineering context, the subject matter is so rich in aspects, dimensions, and structures that one single book cannot aspire to cover it all. Furthermore, as also stated at the beginning, the authors of this volume do not try to present a synthetic view of early modern engineering drawings in the practical context of engineering. Instead they confine themselves to detailed investigations of such drawings under a variety of viewpoints that pertain to this context. If they nevertheless claim to offer more than just a collection of articles that deal with some aspects of this theme, this confidence rests on the conviction that they have concentrated their efforts on such aspects of the topic that are essential for an adequate understanding of technical drawings as means of early modern engineering. The five parts into which the book is divided represent these aspects.

Without anticipating the short introductions that precede each of these parts, their respective focus can be indicated as follows. Part I, entitled *Why Pictures of Machines?* and containing chapter 1 by Marcus Popplow, is about basic categories for ordering the huge and extraordinarily diverse store of extant technical drawings of the age with respect to their origin, purposes, functions, and contexts, thereby providing a first survey of this material. Part II, with the title *Pictorial Languages and Social Characters*, which contains chapters 2 and 3 by David McGee and Rainer Leng, respectively, is occupied with the development of the specific style of early modern machine rendering and the question of whether and how this style can be considered a response to the different social functions of these drawings. Part III, *Seeing and Knowing*, with chapter 4 by Pamela O. Long and chapter 5 by Mary Henninger-Voss, addresses knowledge linked with technical drawings, be it the knowledge presupposed and/or conveyed by them, or knowledge that is presupposed but cannot be conveyed. Part IV, *Producing Shapes*, with chapters 6 through 8 by Filippo Camerota, Wolfgang Lefèvre, and Jeanne Peiffer, respectively, focuses on the development of drawing techniques that required, either from the beginning or in the course of their evolution, familiarity with learned knowledge such as geometry or

geometric optics. Part V, finally, *Practice Meets Theory*, with chapter 9 by Michael S. Mahoney, is dedicated to technical drawings at the interface between practical and theoretical mechanics.

To a certain extent, particularly as regards the fifteenth century, the arrangement of the volume also reflects main stages in the historical development of early modern engineering drawings. After chapter 1, which provides an analytical survey of the technical drawings of this age, chapters 2 and 3 discuss the emergence and the first stage of the specific early modern type of machine rendering in fifteenth-century Italy (Taccola) and Germany *(Büchsenmeistertraktate)*, respectively. Whereas chapter 7 contains an outline of the development of architectural plans up to the first decades of the sixteenth century, chapter 4 presents the mature stage that machine drawings achieved with Giorgio Martini and Leonardo da Vinci in the second half of the fifteenth century, a maturity that was not surpassed by the succeeding development of early modern engineering drawing. This further development is not traced coherently in this volume, with the conspicuous consequence that the famous *Theatres of Machines* from the last decades of the sixteenth and the beginning of the seventeenth centuries are not discussed as a special issue. This omission resulted mainly from the volume's focus on the functions and significance technical drawings had in the engineering practice. Although partly deriving from those of immediate practical context, and although constituting in no way a separate or particular genus of engineering drawings, the drawings of the *Theatres* served the propagation rather than the real practice of engineering. The same holds for the rightly renowned woodcuts in Georgius Agricola's *De re metallica* from 1556. The fact that this treatise and those *Theatres*, the majority of which is accessible through exemplary modern editions, have enjoyed thorough scholarly attention in the last decades made the decision easier to refrain from addressing them in favor of much less investigated issues such as decision making by means of plans (chapter 5), drawing techniques of a learned character (chapters 6 through 8), and the relation between engineering drawings and theoretical mechanics (chapter 9).

ACKNOWLEDGMENTS

The idea of this volume arose after a small conference on engineering drawings of the Renaissance, which was organized by the editor together with David McGee and Marcus Popplow and held at the Max Planck Institute for the History of Science, Berlin, in summer 2001. The authors who eventually joined in this book project circulated drafts of their chapters several times and met in winter 2002, again at the MPI in Berlin, for a thorough discussion of each contribution as well as of the focus and arrangement of the volume as a whole. My first thanks go to them. Seriously engaged in the volume's topic and cooperating in an unusual spirit of good fellowship, they never failed in promptly and patiently reacting to all of the greater and smaller demands such a book entails. Our joint work was an exciting and rewarding experience!

Warm thanks also go to James Bennett, Raz Chen, Judith V. Field, Bert S. Hall, Volker Hoffmann, Alex G. Keller, Antoinette Roesler–Friedenthal, and Thomas B. Settle, who contributed to the 2001 conference and discussed earlier versions of a number of the volume's chapters both critically and constructively.

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Furthermore I would like to thank Jed Z. Buchwald for the decision to include the book in his Transformations series. Dealing with a topic that relates to the history of both science and technology, the volume could not have found better company.

Like machines, to actually come into being, books need to be not only designed (written) but also manufactured. My thanks go therefore to Angelika Irmscher, Heinz Reddner, and Susan Richter, who assisted in the editorial work and helped to bring the manuscript with all the figures to its final camera-ready form. Finally, I want to thank Sara Meirowitz, responsible for the science, technology, and society books at the MIT Press, who discreetly and reliably provided every support needed.

PART I
WHY PICTURES OF MACHINES?

INTRODUCTION TO PART I

In the pictorial world of the Middle Ages and the Renaissance, depictions of technical objects occur everywhere, in paintings and frescoes, on stained-glass windows, in reliefs carved in wood, chiselled in stone, cast in bronze, in illuminated Bibles, prayer-books, books of hours, on single-sheet woodcuts and engravings, in manuscripts and printed treatises with illustrations, on sketches, in notebooks, on plans—everywhere. Only a small fraction of them are technical drawings and thus subjects for this book. Presupposing a rather pragmatic definition, the authors of this volume consider those drawings to be technical that were either traced (or commissioned to be traced) and used by technicians in the pursuit of their professional life or derived from such practitioners' drawings. Of these technical drawings, only machine drawings and a few architectural drawings are dealt with in this volume. However, despite this concentration, the number of drawings addressed here is still huge. According to some experts' estimation, for the period 1400–1700 alone, one has to reckon with five to ten thousand drawings of machines and machine parts.

The unsatisfactory vagueness of such guesses results from the fact that nobody knows how many such drawings might be buried in such locations as the archives of states, cities, dioceses, monasteries, and princely families, and in the manuscript departments of libraries and museums. The expectation that hitherto unknown materials will surface there in the future rests primarily on a suspicious feature of the known material. The bulk of this material consists in presentational drawings that were published in booklets and books—manuscript books *(Bilderhandschriften)* as well as printed ones—in the early modern period. Only a small part consists of workshop drawings pertaining either to the documents of commissioners of machinery and buildings or to the private store of engineers and architects themselves. This fact can certainly be explained by the assumption that drawings of the design and construction process of technical artifacts were not kept but thrown away after a certain span of time. However, the unbalanced ratio of the presentational and the workshop shares of the known machine drawings of this period may also result, at least to some extent, from the storage and display policy of archives and libraries in past eras, when the cultural divide between the realm of technology and the realm of fine arts and literature was still prevalent.

For a first orientation, table I.1 showing the most important sources of early modern engineering drawings, to which the chapters of this volume frequently refer, may be convenient.

The prevalence of the presentational over the workshop material among the extant engineering drawings poses a serious problem to our enterprise. For a picture of the actual role technical drawings played in the practice of engineers and architects, workshop drawings are naturally of far more significance than the presentational ones. The latter may be telling in this respect as well. But only on the basis of thorough investigations of the drawings actually used in the design and construction processes will one be able to determine to what extent and through which of their features certain presentational drawings, too, give testimony to the use of drawings in early modern engineering.

Table I.1. Prominent Sources of Early Modern Machine Drawings

Time	Workshop Drawings	Presentational Manuscripts or Books
1250	Villard de Honnecourt (A, B)	
1300		
1350		Vigevano (C2)
1400		Kyeser (C2)
		Master Gun-makers' Booklets (C1)
1450	Taccola (B)	Taccola (C2)
		Valturio (1472) (D1)
		Anonymous of the Hussite Wars (C1)
		Giorgio Martini (D1)
1500	Leonardo da Vinci (B)	
	A. da Sangallo (B)	Vitruvius (1521) (D3)
		Tartaglia (1537) (D2)
1550		Agricola (1556) (D1)
		Ceredi (1567) (D1)
		Besson (1569) (C3)
		Monte (1577) (D2)
	Holzschuher (A, B)	Errard (1584) (C3)
	Schickhardt (A, B)	Ramelli (1588) (C3)
		Pappus (1588) (D3)
		Heron (1589) (D3)
		Lorini (1597) (D1)
1600		Zonca (1607) (C3)
		Zeising (1607) (C3)
		Caus (1615) (C3)
		Strada (1617) C3)
		Branca (1629) (C3)

A: Design and construction drawings
B: Sketch-books and notebooks
C: Collections of drawings with or without explanatory text
  C1: Practitioner booklets
  C2: Representational manuscripts
  C3: Theatres of machines
D: Drawings in treatises and editions
  D1: In technological treatises
  D2: In treatises on mechanics
  D3: In editions of classical sources

Likely the most crucial problem for an interpretation of technical drawings of the early modern period is the determination of the probable purpose, or purposes, a certain drawing served. Even in the case of drawings in books with an explicit introduction, this purpose can be dubious when, for instance, experts and nonexperts are equally addressed by the book. On the other hand, as regards workshop drawings, it is

sometimes impossible to determine the purpose. “Why pictures of machines?”—this seemingly simple question proves to be an intricate one. Familiarity with a considerable amount of the extant material is presupposed to outline some characteristic features of these engineering drawings and to propose convincing and useful categories according to which this material may be ordered. Presenting such an outline and proposing those categories, chapter 1 by Marcus Popplow not only provides an analytic survey of the subject of this volume but also prepares the ground for the subsequent chapters.

1

WHY DRAW PICTURES OF MACHINES? THE SOCIAL CONTEXTS OF EARLY MODERN MACHINE DRAWINGS

MARCUS POPPLOW

INTRODUCTION

Early modern machine drawings long have been studied with the purpose of reconstructing details of the machine technology employed in their age of origin.[1] In this context, two distinct groups of sources traditionally have received broad attention and by now, for the most part, have been edited: the numerous manuscripts by Leonardo da Vinci and the representational machine books. From the late Middle Ages the latter served to present spectacular engineering designs to a broader public, first in manuscript form and then in print. Regarding the reconstruction of early modern machine technology, the investigation of both Leonardo's machine drawings and the designs of the machine books always has been confronted with one central problem: It is often difficult to determine clearly the realizability of the designs presented. Thus, research on these sources long has focused on efforts to differentiate more clearly which of their designs represented machines actually in use in the early modern period and which of them should rather be regarded as products of the contemporaries' imagination.

Which role was assigned to the medium of drawing by early modern engineers themselves? And what effects did the employment of drawings have on the communication of existing knowledge and the production of new knowledge on contemporary machine technology? Such questions about the practical as well as cognitive functions of the means of representation used by contemporary engineers have been focused on more closely only recently. This is true for the drawings considered here as well as for three-dimensional models of machines.[2] This delay corresponds to the fact that even today, sixteenth-century engineering drawings with more practical functions, preserved as single sheets or personal sketch-books, are to a great extent unpublished and thus less accessible.[3] This chapter places special emphasis on such

1 The topic of this chapter has been presented to a workshop at the Max Planck Institute for the History of Science (Berlin) and a *Journée des Études* at the Centre Koyré (Paris). I am grateful to the participants for their commentaries and suggestions, in particular for the extensive discussion by Pamela O. Long. An earlier version of this chapter has been published as "Maschinenzeichnungen der 'Ingenieure der Renaissance'" in *Frühneuzeit-Info* 13(2002), 1–21.

2 See Ferguson 1992, Hall 1996, Lefèvre 2003. For the related topic of the visual representation of the trajectories of projectiles, see Büttner et al. 2003. For the early modern employment of scaled-down models of machines, see Popplow (in press).

3 In the pioneering study by Ferguson (1992) on visual thinking in the history of engineering, Leonardo's manuscripts and presentational machine books are the only sources from the fifteenth and sixteenth centuries.

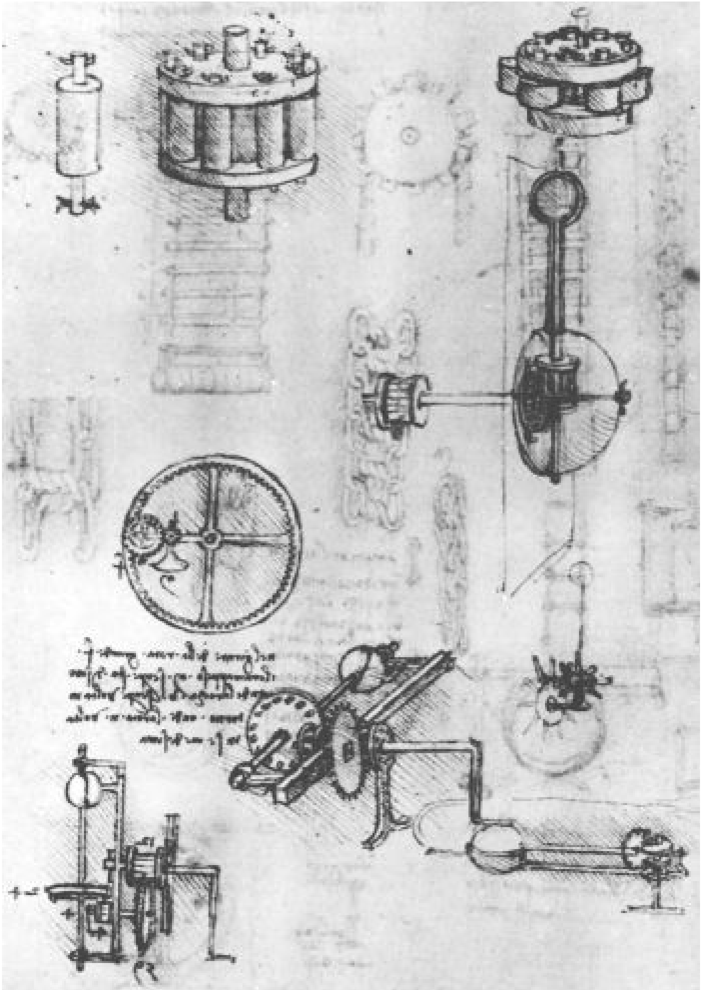

Figure 1.1. Studies of machine elements. It remains unclear whether the drawing documents thought experiments or objects actually assembled in the workshop. Drawing by Leonardo da Vinci. (Photo: Biblioteca Nacional Madrid, Codex Madrid, fol. 10^{v}.)

less formal drawings not addressed to a broader public. It is such sources that document how the medium of "drawing" was indispensable for planning, realizing, and maintaining large-scale technological projects in the early modern period.

The aim of this chapter is to work out a classification of the contexts in which early modern machine drawings were employed.[4] This task confronts a number of difficulties regarding the interpretation of corresponding source material. For Leonardo's machine drawings, it long has proved difficult to determine the purposes they originally served (figure 1.1). Some have been identified as proposals for innovations of specific mechanical devices, for example, his series of drawings on textile machines (figure 1.2). Others have been interpreted as didactic means of conveying his tremendous knowledge on the behavior of machine elements to others, and some obviously served theoretical functions.[5] It furthermore has been suggested that Leonardo used drawings for recording trials with three-dimensional objects made in his workshop.[6] As regards the wealth of machine drawings preserved from early modern authors other than Leonardo, it has been argued convincingly that these must be differentiated according to their functions of documentation, communication, or design.[7] However, confronted with the source material considered below, which early modern engineers employed in the process of realizing mechanical devices, it has proved extremely difficult to assign precisely these functions to such drawings. With regard to these difficulties of interpretation, a different approach is taken here. As a first step, sixteenth-century machine drawings are differentiated according to four main contexts of employment: First, they served to present devices to a broader public; second, they could take on a role in the concrete manufacturing process; and third, they could constitute part of an engineer's personal archives. Fourth, and this final group to some extent amounts to a special case, engineering drawings could merge into or be connected with considerations of a more theoretical nature. Of course, such a classification does not exclude

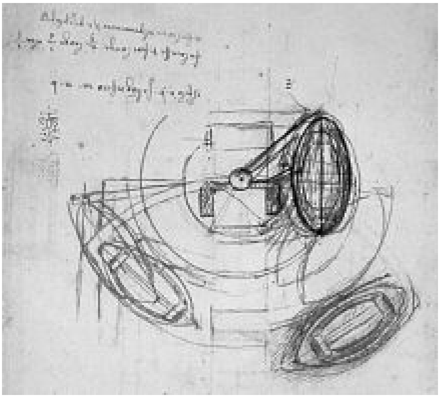

Figure 1.2. Detail of machine for weaving braids. Drawing by Leonardo da Vinci. (Codex Atlanticus, fol. 884r.)

4 For a classification of medieval technical drawings, see Knobloch 1997.
5 See Truesdell 1982, Maschat 1989, and Long (this volume).
6 See Pedretti 1982 and Long (this volume).
7 See Lefèvre 2003.

the possibility that one and the same drawing could be employed in more than one of these four contexts over the course of time.

The following remarks are limited to describing the situation in the sixteenth century without investigating the question of the origins of the employment of machine drawings for more practical purposes in the Middle Ages. With the exception of the numerous illustrated gunners' manuals,[8] early engineering drawings from the fourteenth and fifteenth centuries have been preserved almost exclusively in the context of the production of presentational manuscripts. However, it is hard to imagine that the numerous fifteenth-century manuscript machine books could have been produced without any foundation in some less formalized practice. Scattered textual evidence that still awaits closer investigation indeed testifies to a more informal employment of machine drawings as early as the beginning of the fifteenth century.[9] Furthermore, it must be noted that any attempt to explain when and why drawings came to be employed in mechanical engineering in the Middle Ages must take into consideration the tradition of late medieval architectural drawings.[10] As the different roles of machine builder, architect, and fortification engineer emerged more clearly only in the course of the sixteenth century, it can be assumed that the employment of such a crucial medium as drawing in earlier periods still showed similar characteristics in all three of these fields.[11] As the focus of this chapter lies on the social contexts of employing machine drawings in the sixteenth century, neither will the development and use of different graphic techniques—most prominently, changes induced by the invention of perspective in the fifteenth century—be investigated here.[12]

1. PRESENTING DEVICES TO A BROADER PUBLIC

The above-mentioned machine books in manuscript[13] and in print[14] served to present machines to a broader public, formally continuing a manuscript tradition dating back to Antiquity and the Arab Middle Ages.[15] This public initially consisted of courtly audiences before expanding ever more to learned laymen and fellow technical experts during the sixteenth century. Drawings and later woodcuts and engravings allowed

8 See Leng (this volume).

9 Documents from the *fabbrica* of Milan cathedral mention in passing that proposals for a mechanically driven stone-saw were to be submitted first in the form of a drawing before the most promising designs were required to be presented as scaled-down models. See Dohrn–van Rossum 1990, 204–208.

10 See Lefèvre (this volume).

11 Contexts of employing architectural and fortification drawings in the early modern period have received only scarce attention to date. See Schofield 1991 and Frommel 1994a. Architectural treatises from the fifteenth century onwards often contain explicit discussions of the role of the medium of drawing in the design process. See Thoenes 1993.

12 See Ferguson 1992, Lamberini (in press), and Camerota (this volume). For the density of information conveyed and the broad variety of graphic techniques employed in Leonardo's machine drawings, see Hall 1976b; Heydenreich et al. 1980; and Galluzzi 1982. The argument brought forth by Samuel Edgerton, according to which geometrically constructed perspective drawings in sixteenth-century machine books paved the way for the "geometrization of nature" in the "Scientific Revolution" has, by now, been refuted convincingly. See Mahoney 1985 and Hall 1996, 21–28.

13 See Hall 1982a, Hall 1982b, Galluzzi 1993, Galluzzi 1996a, Friedrich 1996, Leng 2002, Long 2001, 102–142, and Leng (this volume).

14 See Keller 1978, Knoespel 1992, and Dolza and Vérin 2001.

15 See Lefèvre 2002 and Hill 1996.

these audiences to study siege engines, mills, water-lifting devices, and other examples of early modern machine technology. In the fourteenth and fifteenth centuries, these manuscripts contained mostly military devices. Well-known examples are the manuscripts assembled by Guido da Vigevano (c. 1335), Konrad Kyeser (1405), Mariano Taccola (1449), Roberto Valturio (1455, printed in 1472), and the author known as "Anonymous of the Hussite Wars" (c. 1470/1480). The first pioneering manuscripts showing engines for civil purposes were composed in Italy, again, by Mariano Taccola (c. 1430/1440) and Francesco di Giorgio Martini (c. 1470/1480). While large devices for military and civil purposes had existed only rather vaguely in the visual memory of medieval contemporaries, this situation now changed, at least for those among whom these manuscripts circulated. With the regard to the works of Konrad Kyeser, Mariano Taccola, and Francesco di Giorgio Martini, some dozens or even hundreds of copies of the original manuscripts have been discovered. They still await closer investigation concerning the questions of who commissioned them and who was responsible for the artistic process of producing the manuscript copies.[16] Towards the end of the sixteenth century, with the printed machine books of Jacques Besson (1578), Jean Errard (1584), Agostino Ramelli (1588), Vittorio Zonca (1607), Heinrich Zeising (1612ff), Salomon de Caus (1615), Jacopo Strada (1618), and Giovanni Branca (1629), machines for civil purposes became a subject of learned knowledge as well. In addition to the printed *Theatres of Machines,* a number of manuscripts have been preserved, which very likely document a preparatory stage of publication. One example is a manuscript version of Jacques Besson's "Theatrum instrumentorum et machinarum" (1571/72), which was later published posthumously.[17] The intention of publication also can be presumed in the case of a manuscript of the Florentine scholar Cosimo Bartoli (c. 1560/70),[18] which already shows typical traits of machine books: complete views of devices as well as additional detailed views, carefully ordered text sections and labels of reference (figure 1.3).

Late medieval and early modern machine books all have a comparable structure: Full-page images of technical devices are each accompanied by a more or less detailed text explaining their general features. While these explanations often consist only of a few lines in the early manuscripts—in some cases, there are no textual explanations at all—in sixteenth-century works, the length of the explanatory texts grew considerably. This is especially true for the printed machine books. Yet to be investigated is the question as to whether this growth of textual information corresponded to a shift in the contexts of employment of these works. It is well possible that some authors of the earlier manuscripts assumed that the inspection of their manuscripts would be accompanied by oral explanations. Authors of the printed sixteenth-century works, in contrast, from the start had to presuppose a "silent reader" who had to be provided with more detailed explanations of the functioning of the devices presented.

16 See Leng 2002 and Scaglia, 1992. For the manuscripts of Konrad Kyeser, see Friedrich 1996.
17 See Keller 1976.
18 See Galluzzi 1991, 223.

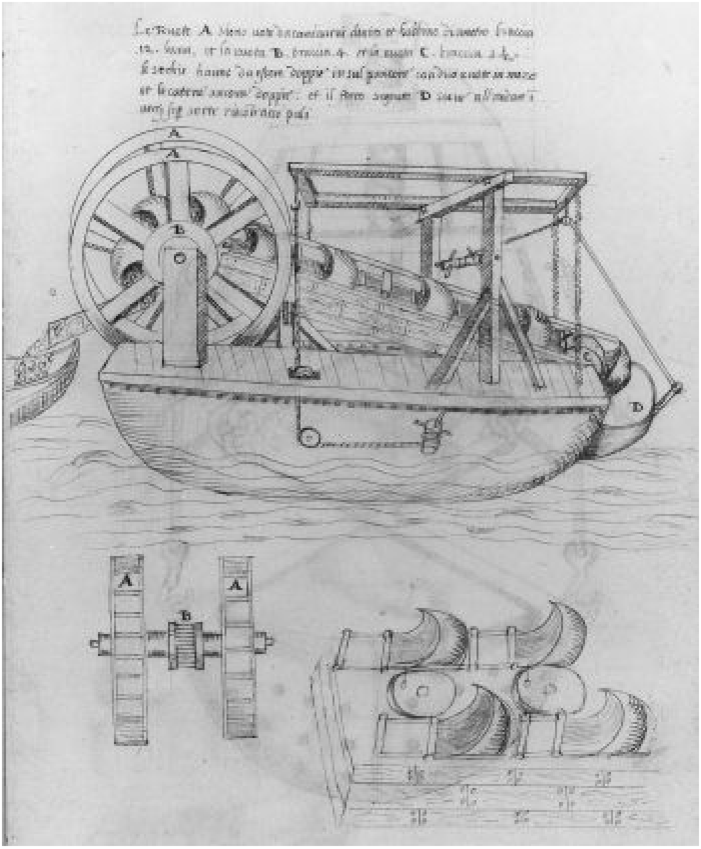

Figure 1.3. High-quality drawing of a dredger with didactic implications. Note the details of the mechanism drawn separately below. Drawing by Cosimo Bartoli, c. 1565. (Florence, Biblioteca Nazionale Centrale, Palatino E.B. 16.5 (II), fol. 60^{r}, courtesy Ministero per i Beni e le Attività Culturali, all rights reserved.)

Regarding techniques of graphic representation, the perspective illustrations in printed books on machines did not differ much from fifteenth-century presentational manuscripts: An elevated viewpoint enabled the spectator to discern machine elements that would remain hidden if the device were represented from the front on

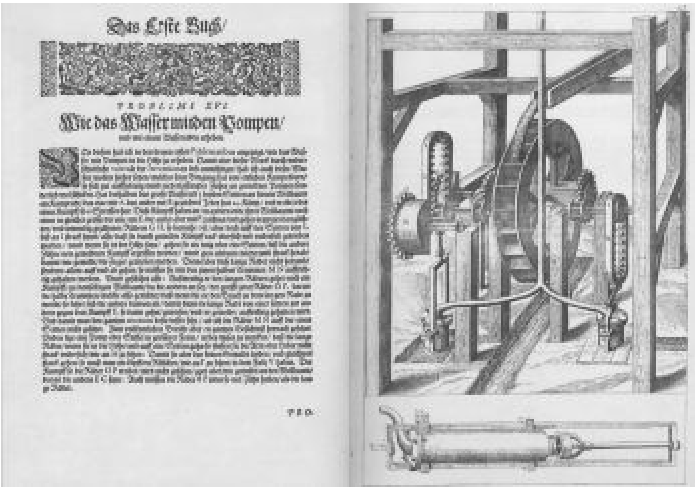
Das Erste Buch/
PROBLEMA XVI
Wie das Wasser mitden Pompen/

Figure 1.4. Presentation of pumps in a printed machine book. Below a separate drawing to emphasize technical details of the arrangement to drive the pumps' pistons. (Caus 1615, fol. 23^{r}.)

ground level. However, the perspective techniques now had become more refined. In continuation of fifteenth-century manuscripts, total views of a device were accompanied by separate drawings of technical details usually also rendered in perspective (figure 1.4). Other graphic techniques like horizontal or vertical sections or ground plans, more difficult to read for the lay spectator, are found only rarely in printed machine books (for a late exception, see figure 1.5).

It has often been stressed that representational manuscripts and printed machine books mirrored technical reality only to a very limited extent. Over time, however, this interpretation has changed considerably. In earlier research it was sometimes argued that the authors of these books did not yet dispose of modern exactness in their technological descriptions. More recently, such "playfulness" has been interpreted as a response to specific expectations placed on technical experts, especially in the context of court culture. From this perspective, early modern machine books appear as a distinctive genre characterized by a carefully selected information: "Unrealistic" designs in general might well be interpreted as expressions of anticipated future achievements. Usually, such designs represented combinations of machine elements, which themselves were already employed in practice. Indeed, authors of the printed machine books often stressed that the designs they presented also were to inspire their colleagues to try out ever new combinations of machine elements to improve traditional machine technology and to extend its fields of application. In this

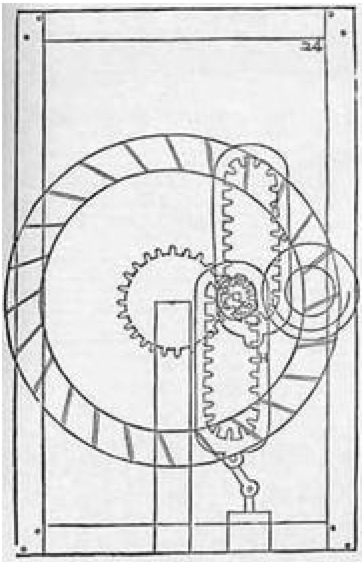

Figure 1.5. Vertical section of machine elements of the water-lifting device shown in figure 4. (Caus 1615, fol. 24^{r}.)

sense, the machine books could be ahead of their time without necessarily losing their relation to technical practice. A number of European territorial powers explicitly promoted the application of mechanical technology by granting privileges for the invention of newly designed mechanical devices.[19] This practice, which spread all throughout Europe in the sixteenth century, provides a very concrete background for the sense of experimentation conveyed by the broad variety of designs in the machine books.

Machine books served the communication between engineers and potential investors or interested members of the republic of letters, rather than that between engineers and artisans or workmen. The functions performed even by the late medieval manuscripts were manifold: They could serve to entertain or to present factual information, or to prove the erudition of princely commissioners and promote the self-advertisement of technical experts. Such implications of the practice of authorship in early modern engineering have only recently began to be investigated more closely.[20] Assembling textual and visual information on machine technology with the aim of presentation to others created a new kind of reflecting knowledge. This new kind of knowledge distinguished the engineer from the ordinary artisan and thus underlined the legitimacy of claims to higher social status of these technical experts.

Even though machine books served as a kind of visual inventory of contemporary technical ideas even among the engineers themselves, they played only a marginal role for engineers' everyday practice. The materiality of technology was often ignored in these presentational treatises. Machines in these books should be understood as a product of the engineer's brain, his *ingenium*; their material realization was not the topic of these books. The organizational activities of the engineer on the building site were mentioned as scarcely as materials, measurements or gear ratios—it was considered self-evident that such factors had to be established at a later point

19 See Popplow 1998b.
20 See Long 2001.

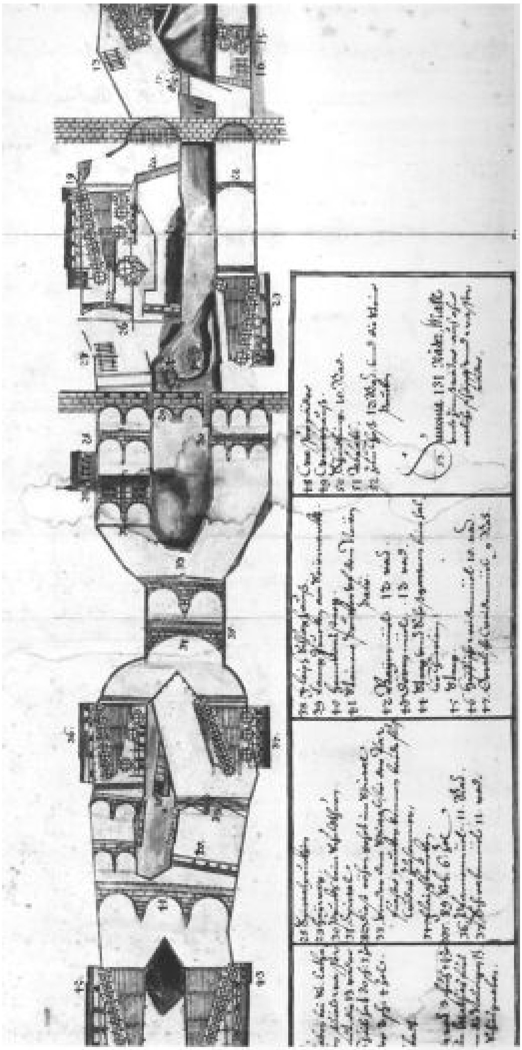

Figure 1.6. Survey plan by the town's master builder Wolf Jacob Stromer of the Nuremberg mills alongside the river Pegnitz (detail), 1601. (Photo: Germanisches Nationalmuseum Nürnberg, HB 3089, Kaps 1055h.)

of time at the site. However, illustrations of machines were often incorporated in lively landscapes or workshop scenarios to suggest the possibility of immediate employment.

Information on the process of creating illustrations for presentational treatises is scarce. For most of the earlier manuscripts, the persons known as “authors,” such as Guido da Vigevano, Konrad Kyeser, or Roberto Valturio, were responsible only for the texts and the composition of the treatises and commissioned the production of the illustrations to artists who remained anonymous. The selection of these artists and how they were instructed about which devices they had to illustrate, and how and with which technical details, remains unclear. This is also true for the woodcuts and engravings in the later printed works. That artists visited each machine at its original site is documented, as an exception, in the case of Georgius Agricola’s preparations for his encyclopaedia on mining, *De re metallica,* published in 1556. His letters show that he had to send different artists to the mining centres of Saxony several times until he found one who produced drawings of the machines employed there in a quality sufficient to serve as templates for the woodcuts.[21] It is difficult to imagine that this was a standard practice, however; artists sometimes might have drawn from three-dimensional models of machines or from some sort of sketch. In any case, the production process of such books on machines presupposes some tradition of more informal machine drawing, of which only faint traces have been preserved from the period preceding Leonardo da Vinci’s notebooks.[22]

In addition to their incorporation in machine books, machine drawings with representative functions have also been preserved as single leaves. A special case is provided by plans kept in communal archives representing, for example, a town’s waterways. From early fifteenth-century Basle, such a plan has been preserved with a coloured scheme of the different waterworks crossing the town. It is assembled of pieces of parchment and is, in total, nearly ten meters long.[23] In other cases, such plans also depicted water-mills alongside the town’s waterways in symbolized form, that is, as mill-wheels turned ninety degrees laterally (figure 1.6).

A different example of a carefully composed machine drawing with representative functions has been preserved among the documents of Württemberg master builder Heinrich Schickhardt (1558–1635). It shows a device that had been built a short time earlier by the carpenter Johannes Kretzmaier, probably under supervision of Schickhardt himself, to provide the castle of Hellenstein near Heidenheim with water (figure 1.7).[24] Between the source and the castle, a height difference of a total of ninety meters had to be overcome. The drawing emphasized only the core element of the transmission machinery: a combination of a lantern and an oval rack. It served to transfer the rotary motion provided by the water-wheel to the reciprocating motion of the horizontal beam driving the piston rods of the pumps. Indispensable construction details of the device, like the guide rails of the rack, are missing. The lower part

21 See Kessler–Slotta 1994.
22 See McGee (this volume).
23 See Schnitter 2000.
24 See Müller 2000.

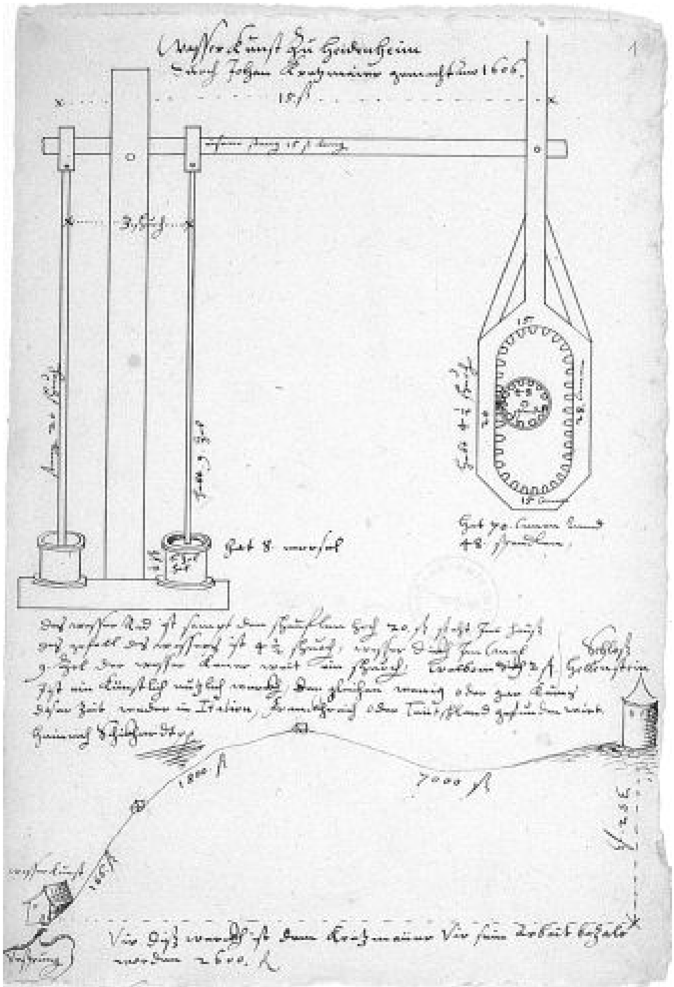

Figure 1.7. Commented presentational drawing of the pumps supplying Hellenstein castle. Drawing by Heinrich Schickhardt, 1606 or after. (Stuttgart, HStA, N220 T149, all rights reserved.)

of the drawing serves to visualize the concrete setting of employment: at the bottom left, the building that housed the device; above right, Hellenstein castle. In addition to specifications of measurements and the performance of the device, a written commentary signed by Schickhardt testifies to the documentary character of the sheet: “This is an artificial and useful device of which only few or even none are to be found neither in Italy, nor in France or Germany in these times.”[25] The exact purpose for which this drawing had been produced nevertheless remains unclear.

2. MACHINE DRAWINGS IN THE PROCESS OF REALIZING MECHANICAL DEVICES

Starting in the late Middle Ages, the separation of the social roles of engineer and artisan became more clearly discernible. While the former was responsible for the design and the organization of a given project, the latter carried out the actual work. This development is discussed as one of the central prerequisites for the growing relevance of drawings to mechanical engineering since the late Middle Ages.[26] In the sixteenth century, in any case, drawings in the process of realizing mechanical devices served the engineer to communicate with the investor on the one hand and (although presumably to a lesser extent) with the artisans carrying out the work on the other.

Communication with the investor especially concerned the preparatory stage of realizing mechanical devices. With regard to the competition among early modern engineering experts, drawings could serve to present engineers’ abilities at a foreign court, even though for such purposes the demonstration of scaled-down models presumably was preferred, because of the more immediate impression it created. Both of these media played a central role in the above-mentioned practice of granting privileges for inventions.[27] Applicants for such a privilege in a certain territory often submitted drawings or models to underline the credibility of their inventions. Such presentations, however, were not necessarily required, as in any case the inventor had to prove the realizability of his invention after the privilege had been granted by constructing a test specimen in full size in the course of the subsequent six to twelve months. In some cases, applicants presented a whole set of inventions by means of illustrated manuscripts, which ultimately strongly resembled manuscript machine books. This was true in the case of the above-mentioned manuscript by Jacques Besson, the designs of which were protected by a privilege in 1569 when the compilation was presented to King Charles IX.[28] A quite similar manuscript was composed in 1606 by the Spanish engineer Jerónimo de Ayanz for King Philipp III.[29] To obtain a privilege for inventions, Ayanz presented drawings of forty-eight of his inventions,

25 “Ist ein künstlich nutzlich werckh, der gleichen wenig oder gar keins dieser Zeit weder in Italien, Franckhreich oder Teütschland gefunden wirt.” Stuttgart, HStA, N220 T149.
26 See McGee (this volume).
27 See Popplow 1998b.
28 See Keller 1976, 76.
29 See Tapia 1991, 53–256.

most of which were presented as sketches in perspective and accompanied by extensive descriptive texts.

A manuscript composed with quite different intentions, under the supervision of Duke Julius the Younger of Brunswick-Wolfenbüttel around 1573, represents an attempt to employ catalogues of designs of mechanical devices for concrete regional innovations.[30] The manuscript assembled illustrations of devices and instruments that were to facilitate and speed up labor in the quarries and on the building sites of the duchy. Moreover, it contained a list of persons active in the duchy's administration and declared that they were obliged to consult the volume accordingly to improve the technical equipment available at the sites for which they were responsible. Some of the illustrations had been copied from earlier manuscripts, others show instruments reportedly already employed elsewhere in the duchy, and a few represent inventions allegedly made by Duke Julius himself (figure 1.8). While a number of formally similar manuscripts mentioned above seem to have been composed rather for reasons of prestige or entertainment at court, this manuscript was thus composed with the aim of practical employment. In this context, the designs of the Wolfenbüttel manuscript clearly refer to local circumstances in the Wolfenbüttel duchy, a trait that is not discernible in other cases. Attempts to turn designs encountered in such manuscripts into practice are, of course, quite conceivable in other cases as well, but they have not yet been documented. With the deliberate intention of realization after his death, the unique sketches of mills left behind by Nuremberg patrician Berthold Holzschuher were similarly meant to serve as a guideline to construction, although in a purely private context.[31]

In the concrete process of realizing mechanical devices, drawings helped the engineer to bridge the different locations of decision processes and the actual realization of a project—the court or the town hall and the building site. These contexts are especially easy to discern with regard to examples from the broad collection of some two to three hundred loose leaves containing drawings of all sorts of mills and water-lifting devices by the above-mentioned Heinrich Schickhardt. Schickhardt, who served the dukes of Württemberg for decades as master builder and engineer,[32] in general does not appear as an ingenious inventor of new devices, but rather as somebody trying to provide the duchy with up-to-date technology that had already proven its efficacy elsewhere. In contrast to machine drawings in Leonardo da Vinci's manuscripts, composed roughly one hundred years earlier, Schickhardt's collection contains no theoretical reflections at all, whereas the relationship of his drawings to actual technical projects is extensively documented. Schickhardt's ability to employ all kinds of graphic techniques for drawing mechanical devices might have been above average. Nevertheless, it seems that his drawings represent an extraordinary case of preservation rather than an extraordinary way of using the medium. Thus they most likely testify to standardized practices in early modern engineering.

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30 See Spies 1992.
31 See Leng (this volume).
32 See Schickhardt 1902, Popplow 1999, Bouvard 2000.

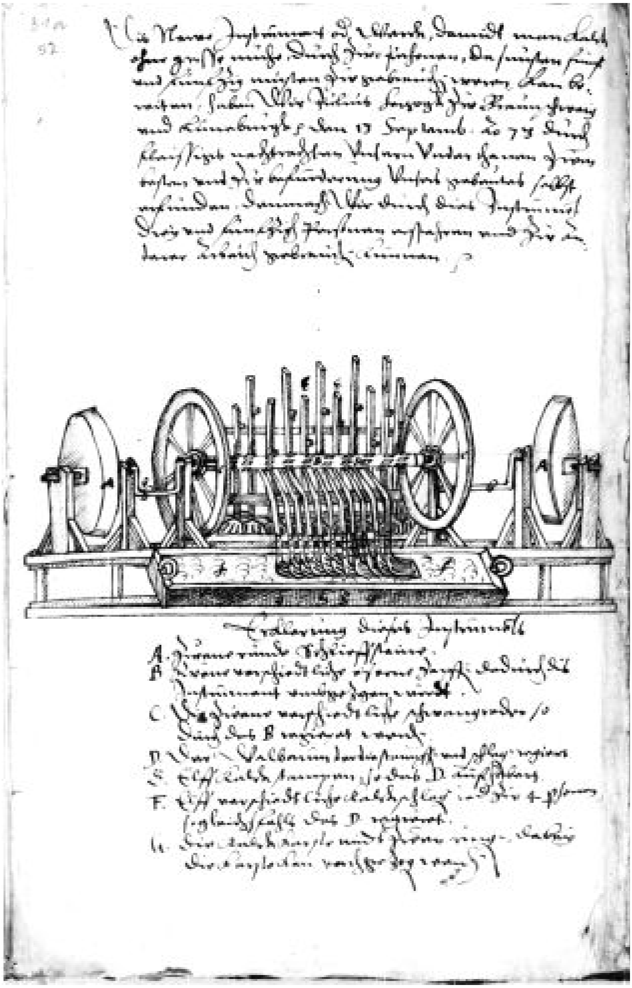

Figure 1.8. Presentational drawing of a device for stamping and mixing lime allegedly invented by Duke Julius of Brunswick-Wolfenbüttel, c. 1573. (Photo: Niedersächsisches Staatsarchiv Wolfenbüttel, Instrumentenbuch I, fol. 31r.)

Around 1600, Heinrich Schickhardt supervised the building of several mills in Montbéliard, a project extensively documented in his papers.[33] One of the devices realized was a paper-mill. A survey drawing of this mill shows, in an idealized way, the most important parts of its inventory—two of the basins where soaked rags were reduced to pulp as the raw material for the production of paper have been carefully omitted to leave space to show such components as the mill's press (figure 1.9). At

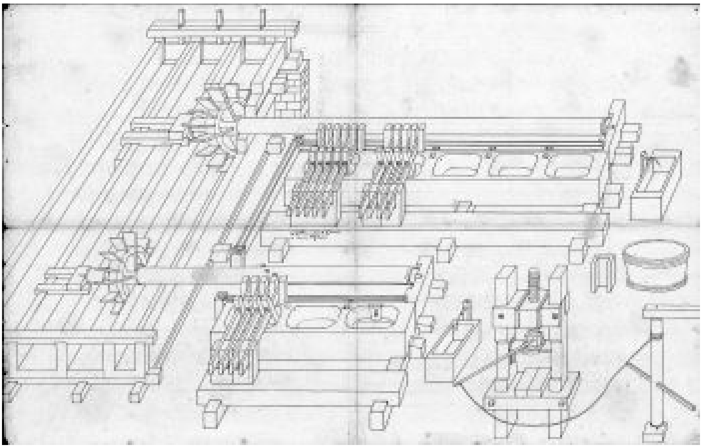

Figure 1.9. Inventory of a paper-mill in Montbéliard. Drawing by Heinrich Schickhardt, c. 1597. (Stuttgart, HStA, N220 T193, all rights reserved.)

which point of time the drawing was made is not definitively clear, but it is very likely that it was composed before the mill was actually built. A tiny comment written below the upper left basin says: "There shall only be four stamps in one hole" (instead of five as shown here). And indeed, the stamp at the extreme left is marked as obsolete by several diagonal lines. Most probably, a drawing like this was presented to the Duke of Württemberg for his formal approval or to keep him informed about such a costly project. Why the changes to details of the design were later documented in the way seen here remains unclear, however. Among several more detailed drawings of parts of this paper-mill is one showing a vertical section of one of the stamps, complete with measurements and, again, disclosing several corrected features (figure 1.10). Others concern the press, for example. Plans of the different storeys of the building have been preserved as well.

33 See Bouvard 2000, 63–77.

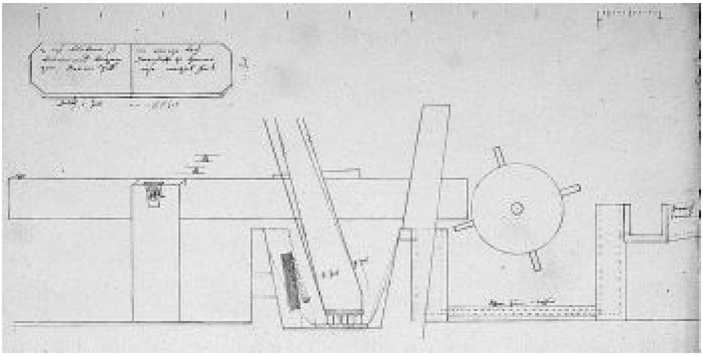

Figure 1.10. Vertical section of the cam-shaft and one of the stamps of a paper-mill in Montbéliard. Drawing by Heinrich Schickhardt, c. 1597. (Stuttgart, HStA, N220 T186, all rights reserved.)

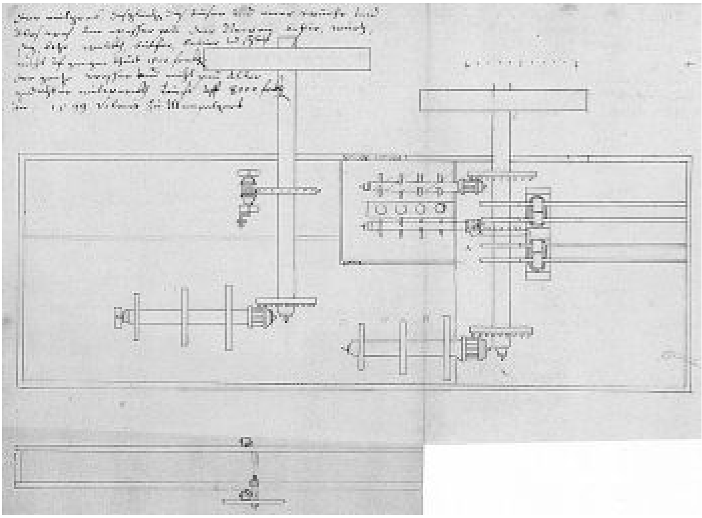

Figure 1.11. Ground plan of a fulling-/stamp-/grinding-/polishing-/drilling-/sawmill in Montbéliard, earlier version. Drawing by Heinrich Schickhardt, c. 1597. (Stuttgart, HStA, N220 T182, all rights reserved.)

Documents pertaining to another of the devices realized by Heinrich Schickhardt in Montbéliard document well the role of drawings in the relationship between engineer and artisan. In the course of constructing a combined fulling-/stamp-/grinding-/polishing-/drilling- and sawmill, Schickhardt again used different kinds of graphic representations, among them two ground plans. The first of these plans is a preliminary study of the disposition of the different mechanisms most probably rendered before the mill was actually built: A closer look reveals one set of stamps crossed out and a small note says that the water-wheels have to be set a greater distance from each other (figure 1.11). The latter addition shows that Schickhardt produced such plans to scale, and a roughly drawn scale is indeed to be found on the plan near the water-wheels. The second plan of the same mill shows that these changes had been carried out (figure 1.12).

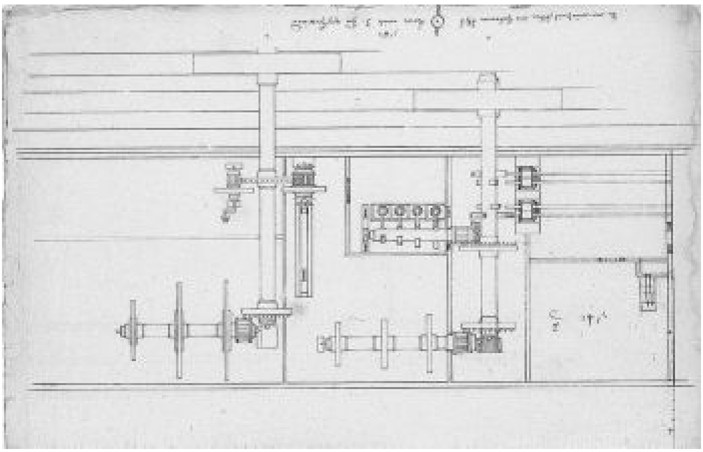

Figure 1.12. Ground plan of a fulling-/stamp-/grinding-/polishing-/drilling-/sawmill in Montbéliard, later version. Drawing by Heinrich Schickhardt, c. 1597. (Stuttgart, HStA, N220 T182, all rights reserved.)

The interesting thing about these two plans is that the second plan, at least, not only served to record what Schickhardt had planned, but was also part of the contract between Schickhardt and the carpenter who actually built the mill—as becomes clear from copies of documents preserved together with these drawings. Schickhardt, like most of his colleagues, was always engaged in several projects in different places at any given time. As he himself once wrote, he was in most cases responsible only for the design of a building or machine. He left plans and other information for the artisans to use, coming back weeks or months later to check on the realization of the project. In the case of the mill discussed here, Schickhardt composed a document on

behalf of the Duke of Württemberg on 24 October, 1597, which specified how the mill was to be built by the carpenter. The text included the remark that “everything concerning the mechanisms and the rooms should be made properly and diligently according to the drawing.”[34] The drawing mentioned is the second plan, which indeed corresponds in detail to Schickhardt’s written description. At a later point in time, Schickhardt again noted the change of one detail, both on the plan and on the margin of the written document: The carpenter had provided the axle of the spice mill with three cams. This, however, resulted in the mill working “too fast.” Two cams, Schickhardt remarked, sufficed in this case. The importance thus placed on this detail is somewhat puzzling, however, because the document for the carpenter had mentioned only the gearing of the mills without specifying such details as the number of teeth on the toothed wheels. This is also true for other aspects of the project. The document laid down only the breadth and the width of the building; none of the other measurements were fixed in written form. This “openness” proves that drawings from the sixteenth century, even when they were used as plans to realize mechanical devices and thus at first glance resemble modern orthographic projections, still are not equivalent to modern blueprints. Furthermore, such drawings did not provide unambiguous instructions on the three-dimensional arrangement of the machine parts.[35] Even though Schickhardt provided the artisans with a wealth of information, a lot of “gaps” concerning the realization of certain machine elements remained to be filled in by oral instructions or through the expertise of the artisans. Finally, this example also documents the proximity of machine drawings to architectural drawings. As the realization of large mechanical devices also comprised the building in which they were housed, it can be assumed that drawings used in that process adhered to standards similar to that of plans used in the construction process of buildings, for example, larger residential houses. Such reciprocal dependencies of machine drawings and architectural drawings remain open to future investigation.

Figure 1.13. Documentation of the size of a leather disc for sealing pistons in a pump cylinder. Drawing by Heinrich Schickhardt, 1603. (Stuttgart, HStA, N220 T150, all rights reserved.)

34 “alles an mülwercken und gemecher dem abriß gemeß sauber und fleißig gemacht.” Stuttgart, HStA, N220 T182.

35 See Lefèvre 2003.

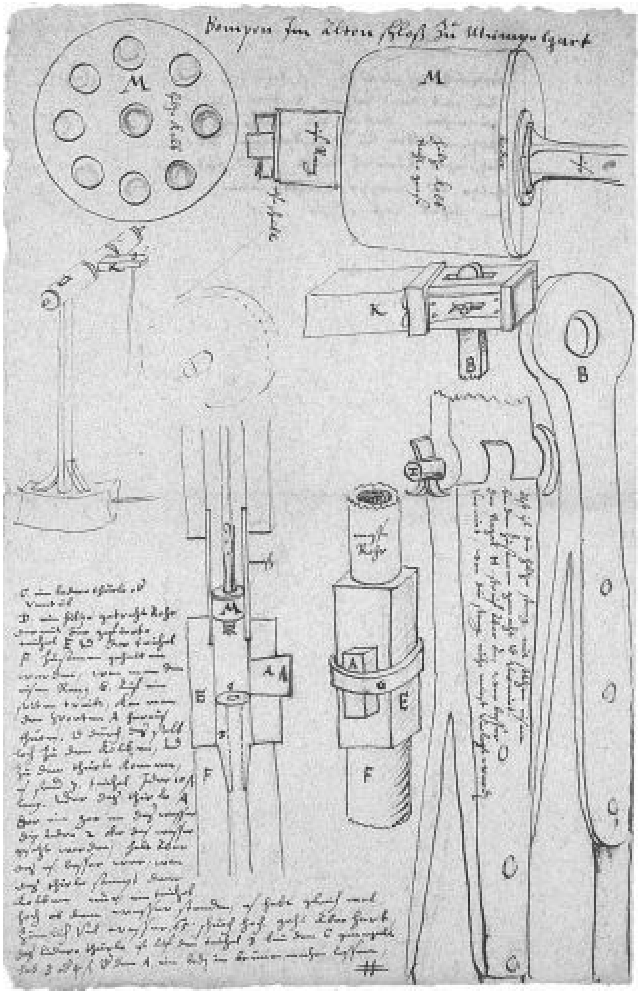

Figure 1.14. Inventory of parts of pumps for Montbéliard castle. Drawing by Heinrich Schickhardt, 1603. (Stuttgart, HStA, N220 T150, all rights reserved.)

A rarely documented and completely different type of drawing could play a minor role in the process of realizing mechanical devices: drawings determining the size of workpieces. Such dimensioning was, of course, a procedure that had long been required in any kind of building project and was solved by different means such as moulds and templates. Given its increasing availability in the sixteenth century, paper could be used for such a purpose as well. Such procedures seem especially likely in the production of the numerous toothed wheels for clockwork and automata. An example of this kind of drawing, again from Schickhardt's legacy, concerns a leather ring that served to seal up pistons moving up and down in pumping cylinders. This drawing with the remark "leather disc for the pumps"[36] (figure 1.13) probably was produced because the wear and tear of these discs frequently made their replacement necessary such that it was advisable to always have new discs at hand. Another drawing makes clearer the context of the employment of this disc: Here Schickhardt was concerned with restoring the pumps for the water supply of Montbéliard castle. The leather disc is to be found on the upper right part of the page, represented by the thin circle fixed to the right of the wooden piston marked "M" (figure 1.14). The function of this visual inventory of the parts of the pump is, again, not discernible.

3. MACHINE DRAWINGS AS ENGINEERS' PRIVATE ARCHIVES

Early modern engineers in many cases assembled personal archives with drawings of their own projects and drawings of devices realized by others. To be sure, the sorts of drawings discussed so far also could find their place in such collections. The following paragraphs, however, after briefly discussing drawings that served to illustrate engineers' own thought experiments and to document their own experiences with machines or machine elements in their workshop, will concentrate on different sets of drawings that helped engineers record the design of mechanical devices they saw during their travels.

As has been remarked above, it is still open to what extent Leonardo da Vinci's drawings of machine elements represented not only thought experiments, but arrangements of objects that had been tested in his workshop (figure 1.15).[37] In other engineers' documents known to date, hardly any drawings with these two functions can be discerned. This makes it extremely difficult to judge the role they might have played in the design practice of the fifteenth and sixteenth centuries in relation to three-dimensional arrangements or scaled-down models of machines. An early example of drawings that might be interpreted as thought experiments are a number of small studies of war ships in a manuscript by Mariano Taccola.[38]

Drawings produced by engineers while traveling are documented to a much greater extent than the thought experiments mentioned in the preceding paragraph. Parallel to artists' and architects' practices of keeping model books for reproducing

36 "lederne scheiblein zu den pompen." Stuttgart, HStA, N220 T150.
37 See Long (this volume).
38 See McGee (this volume).

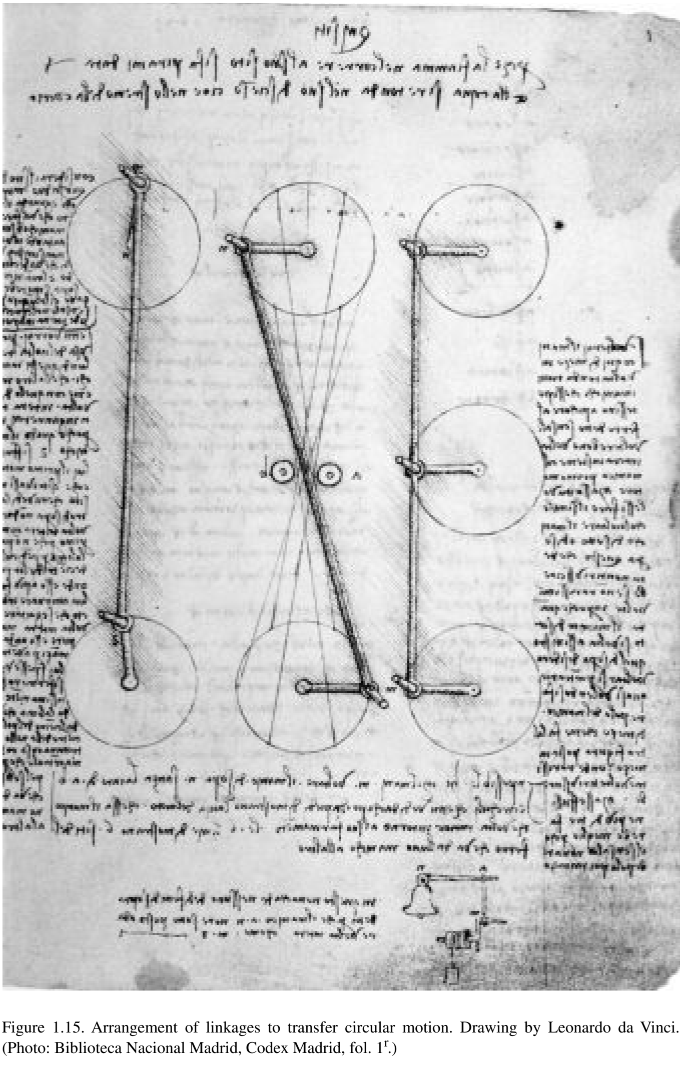

Figure 1.15. Arrangement of linkages to transfer circular motion. Drawing by Leonardo da Vinci. (Photo: Biblioteca Nacional Madrid, Codex Madrid, fol. 1^{r}.)

different kinds of objects at some future point of time, a tradition reaching far back into the Middle Ages, engineers similarly assembled information on the variety of mechanical engines employed in early modern Europe. For the engineer, such drawings were an indispensable means of quickly recording information on devices seen elsewhere. Even if, for example, standard solutions for the design of flour-mills were widespread, there existed a multitude of designs for devices employing more complex gearing, as standardization in early modern mechanical engineering was by no means fostered institutionally. Engineers thus kept records of remarkable devices seen elsewhere, either in notebooks—usually, for practical reasons, of relatively small size—or on loose leaves.[39] In spite of their diversity, both north and south of the Alps such drawings seem to have followed some standard conventions with regard to the numerical and textual information conveyed. Measurements, gear ratios, and commentaries on the device's performance appear regularly, as either personally observed or orally communicated on the site. Especially with regard to this body of information, such drawings obviously adhered to conventions quite different from those which characterized presentational treatises.

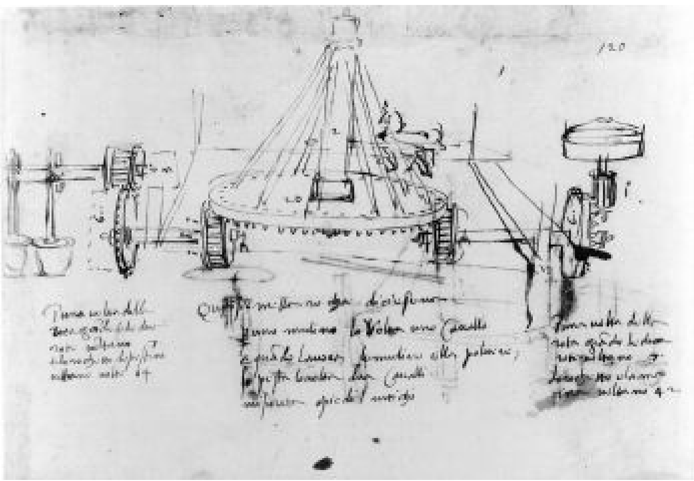

Figure 1.16. Sketch of a combined flour- and stamp-mill in Cesena. Drawing by Antonio da Sangallo the Younger. (Florence, Gabinetto Disegni e Stampe, U1442A^r.)

39 See, for example, the diary of two journeys to Italy by Heinrich Schickhardt, illustrated with numerous drawings of buildings and machines. Schickhardt 1902, 7–301.

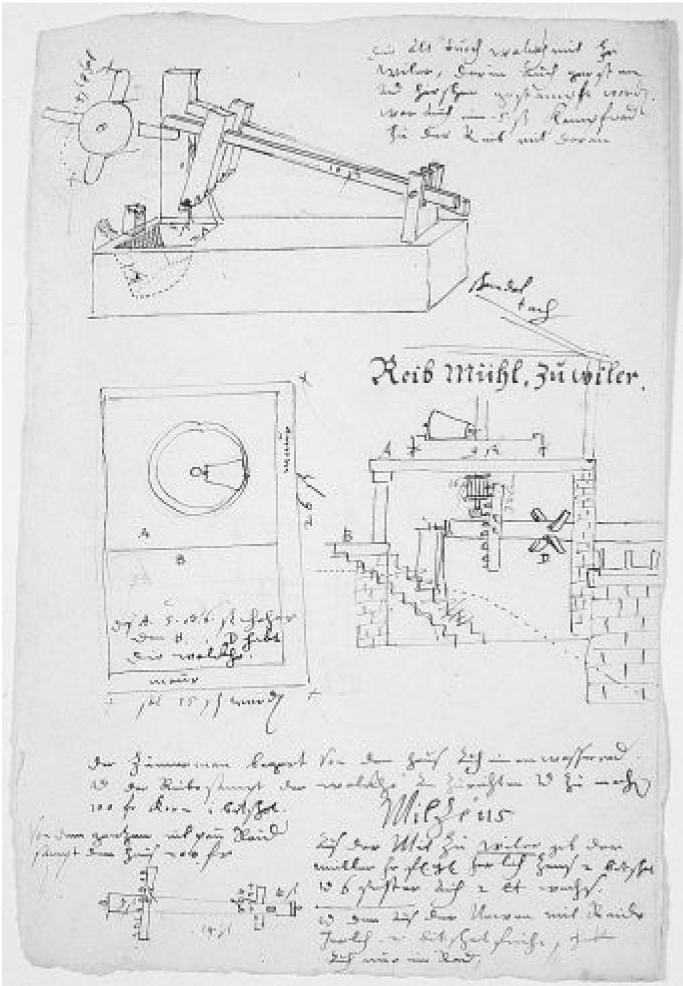

Figure 1.17. Sketches of a combined fulling- and grinding-mill at Wiler near Montbéliard. Drawing by Heinrich Schickhardt, c. 1610. (Stuttgart, HStA, N220 T241, all rights reserved.)

An early example of such drawings recording devices seen elsewhere is provided by Antonio da Sangallo the Younger in the first half of the sixteenth century. It shows a horse-driven combination of a flour- and a stamp-mill, which is said to have been located in Cesena (figure 1.16).[40] As was typical, the leaf includes information on measurements and gear ratios as they had been observed on the site. Similar examples from that period can also be found in the famous sketch-book of the Volpaia family from the 1520s.[41] Another example, one of numerous of such leaves included among the personal records of Heinrich Schickhardt, represents a combined fulling- and grinding-mill near Montbéliard.[42] This drawing exhibits an even greater density of information than the Sangallo example (figure 1.17). It furthermore testifies to the fact that, compared to the presentational treatises, engineers in such cases sometimes used a broader variety of graphic techniques to record what they had seen. Schickhardt, in this case as in many others, used not only perspective representations, but also vertical sections and top views. In each part of the drawing, he noted dimensions of the different parts of the machine and also wrote down the gear ratios of the machinery. The production of such drawings required a considerable sense of abstraction. Firstly, machines were housed in buildings, which, of course, were not transparent, so that it was actually impossible to see the machinery as it was portrayed by the drawing. Secondly, the point of view chosen for the representation of the machine—at a certain distance and slightly above ground level—is practically always constructed virtually, as it was hardly available to contemporary spectators.

How exactly engineers later made use of information recorded in this way is difficult to say. Of course, such drawings not only served individual purposes, but also provided a basis for communication with artisans, colleagues, and potential investors. In the case of Heinrich Schickhardt, the importance of such a collection is proven by the fact that he kept such leaves at home in a desk with drawers, each of which was reserved for one special kind of device.[43] In the sixteenth century, such archives of well-off engineers also usually included a collection of books—not necessarily on technical subjects only. Lists of books owned by engineers are documented, for example, for Leonardo da Vinci; again, Heinrich Schickhardt; and for the Italian engineer Giambattista Aleotti.[44] Towards the end of the sixteenth century, such personal libraries might also comprise printed machine books. At the same time, illustrations from printed books were also copied for private use. This is testified to by a loose leaf from the papers of Heinrich Schickhardt showing copies of machines employed in the German mining regions as they had been depicted in Georgius Agricola's *De re metallica* of 1556 (figure 1.18). The reason for the production of exactly these copies are unclear, as Schickhardt himself owned a copy of Agricola's book.

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40 See Frommel 1994c, 418.
41 For this manuscript in general, see Brusa 1994, 657–658.
42 See Bouvard 2000, 60–63.
43 This can be deduced from a note in a document pertaining to the building of a mill in Pleidelsheim: "Mühlwehr wie das gemacht; ist beü den wasser gebeüen in der oberen Schubladen zuo finden." Stuttgart, HStA, N220 T212.
44 For Leonardo, see Leonardo 1974, II fol. 2^v–3^r and Leonardo 1987, 239–257; for Schickhardt, see Schickhardt 1902, 331–342; for Aleotti, see Fiocca 1995.

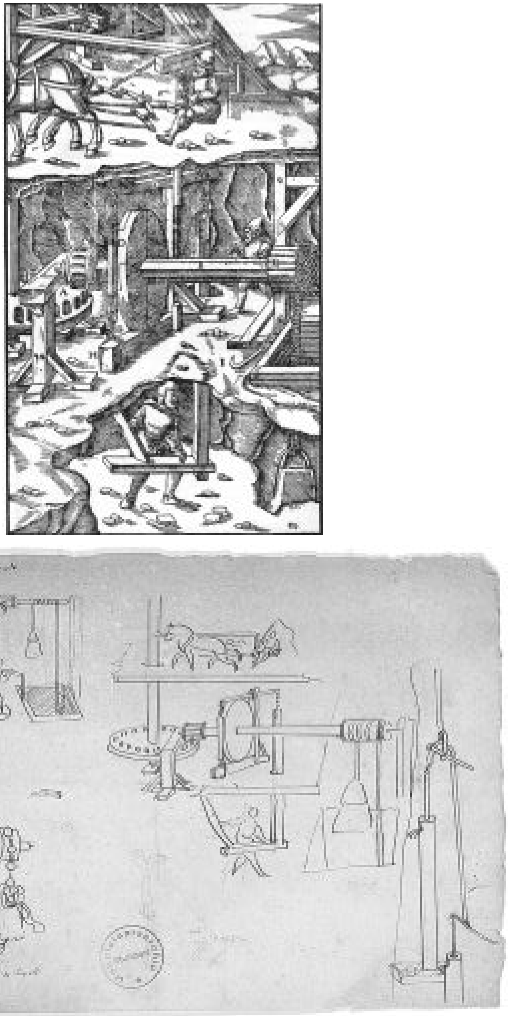

Figure 1.18. Top: Lifting device. Woodcut from Agricola 1556, 167. Bottom: Copies from Agricola's *De re metallica* by Heinrich Schickhardt, c. 1605. (Stuttgart, HStA, N220 T151,)

Finally, another drawing from the legacy of Antonio da Sangallo the Younger might serve to illustrate the difficulties of unambiguously assigning early modern machine drawings to the three contexts of representation, realization, and documentation discussed so far. The leaf shows different views of a pump (figure 1.19).[45] As the brief explications make clear, its peculiarity was that the valves usually employed in pumps had been replaced by metal balls. From this innovative feature it could be assumed that the leaf represented a study that was not connected to a particular project. From the information given on the numbers of teeth of the toothed wheels, it could also be assumed that the drawing shows a device that was actually in use. Even if this was indeed the case, it would still remain unclear whether the device had been designed by Antonio da Sangallo or whether it represented a device made by others, which had been investigated during his travels. Ultimately, it remains open which purpose such a documentation actually served. The analysis of early modern machine drawings is often confronted with such problems of interpretation. To narrow down the possibilities of interpretation, descriptive texts, text fragments on the drawing and textual documents preserved with the drawings again and again prove most useful. Where such additional material is missing—which is often the case due to the frequent separation of pictorial and textual sources practised in a number of European archives some decades ago—interpretation is often confronted with considerable difficulties.

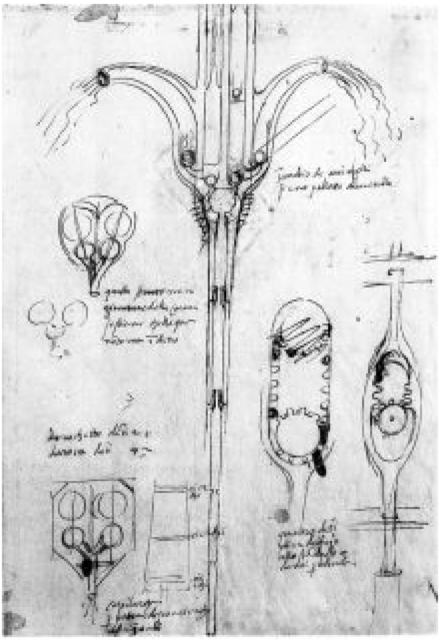

Figure 1.19. Studies of pumps with the usual valves replaced by metal balls. Drawing by Antonio da Sangallo the Younger. (Florence, Gabinetto Disegni e Stampe, U847A^{r}.)

45 See Frommel 1994c, 335.

4. DRAWINGS SERVING THEORETICAL CONSIDERATIONS OF MACHINES

This fourth category of machine drawings represents a special case in the classification proposed here. Up to this point, machine drawings have been classified according to the social context of their employment: presentation to a broader public, realization of concrete projects, storing information for the engineer's own use. The theoretical analysis of machines by means of drawings appears to be orthogonal to these categories, as such an approach might be found in each of these three categories. To be sure, the definition of "theoretical" in the context of early modern engineering drawings is still an open question. In general, the sixteenth-century theory of mechanics is understood as consisting in the analysis of the simple machines based on the lever and the balance. However, there also are engineering drawings that testify to general reasoning on machines without any reference to preclassical mechanics and its visual language of geometrical diagrams. Drawings of standardized types of mills in a treatise by Francesco di Giorgio Martini could be adduced as an early example,[46] a drawing by Antonio da Sangallo that will be discussed below as a later one. Engineers' considerations of working principles of machines in drawings like these might be labelled "theoretical" as well, but the establishment of corresponding definitions lies beyond the scope of the present contribution. The following paragraphs thus mainly concern the appearance of the visual language of preclassical mechanics in sixteenth-century engineering drawings.

In the sixteenth century, mechanics gradually emerged as an independent discipline. This process was inseparably connected to the reception of ancient sources. Pseudo-Aristotle's "Mechanical Problems" were now edited and commented upon as well as the works of Archimedes, Hero of Alexandria and later those of the Alexandrine mathematician Pappus. Additional sources comprised medieval treatises in the tradition of the *scientia de ponderibus,* most prominently those of Jordanus Nemorarius. All of these approaches were founded on the theoretical analysis of the balance with unequal arms and the lever by means of geometrical proofs. This common basis facilitated attempts in the sixteenth century to unify all of these different strains from the Greek and Hellenistic eras and the Arab and European Middle Ages. In this context, special attention was devoted to the classification of the five simple machines (lever, wedge, winch, screw, and pulley) dating back to Hero of Alexandria, which, for example, guided Guidobaldo del Monte in structuring his influential "Mechanicorum liber" (1577). As early as the late fifteenth century, as soon as the work on "rediscovered" texts on mechanics began, engineers strove to use this body of theory to investigate more closely the properties of the sixteenth-century machinery with which they were dealing. As the analysis of the simple machines proceeded by means of geometrical proofs to determine relationships of distance, force, weight and velocity, graphical representations played an important role. The corresponding visual language is documented, for example, in the illustrations of Guidobaldo del Monte's treatise and was presented concisely on the title page of the German translation of

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46 See Long (this volume).

Figure 1.20. Geometrical analysis of the simple machines as the foundation of mechanics, and their practical application. Frontispiece of Daniel Mögling's *Mechanischer Kunst-Kammer Erster Theil* (Frankfurt 1629).

Guidobaldo's work in 1629 (figure 1.20). The frontispiece presented an overview of the simple machines and their geometrical analysis, alluding to their practical application as well.

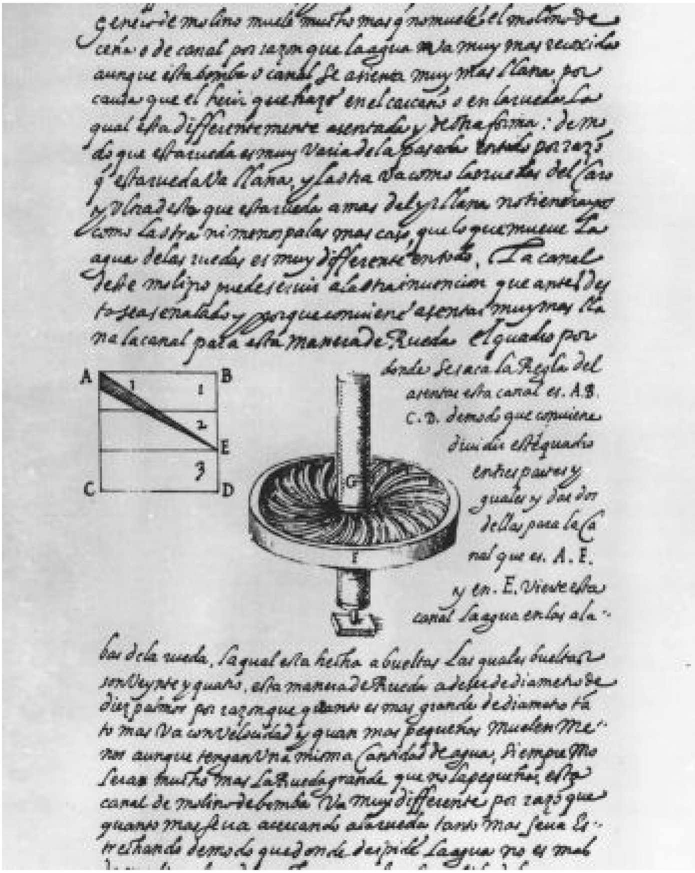

Figure 1.21. Diagram to determine the inclination of a conduit to drive a horizontal water-wheel. (Photo: Biblioteca Nacional Madrid, Los veintiún libros …, Mss. 3372–3376, fol. 290[r].)

The few examples known to date that combined engineering drawings with geometrical analysis in terms of the simple machines are found in engineering treatises, in which they aimed to underline the author's acquaintance with the foundations of contemporary science. The author of the Spanish engineering treatise "Twenty-one books of engineering and machines" in the 1570s thus analysed the inclination of conduits to drive a water-wheel by means of a geometrical diagram (figure 1.21).[47] Giuseppe Ceredi, physician to the Dukes of Parma and Piacenza, in a treatise published in 1567 concerning the application of the Archimedean screw to irrigation, also dealt extensively with the theory of the balance and the lever as the theoretical foundation of the analysis of machines.[48] Arguing for the superiority of the kind of crank he had chosen to drive his Archimedean screws by manpower, Ceredi also incorporated geometrical abstractions in the illustration of his own solution in order to allude to the scientific reasoning underlying his choice (figure 1.22).

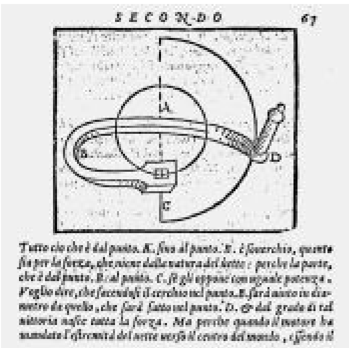

Figure 1.22. Geometrical analysis in terms of the lever of a crank to drive an Archimedean screw. (Ceredi 1567, 67.)

A similar kind of explanation was later given by Simon Stevin in his treatise "De Weeghdaet" with reference to a typical crane operated by a tread-wheel employed on early modern riversides.[49] In private documents of sixteenth-century engineers, the employment of drawings for theoretical reflections is documented more extensively only in Leonardo da Vinci's notebooks. His analysis of such factors as friction and the strength of materials, in particular, still appears to have been singular. In sixteenth-century manuscript material, no comparable theoretical analysis of machine elements is known. In the process of realizing early modern machines, such theoretical analyses seemed hardly to play a role. An exception, which, however, again concerns a preliminary stage of evaluating the design of a machine, was later reported by Galileo Galilei. While Galileo was at the Florentine court, a foreign engineer who remained anonymous presented the Duke of Tuscany with a model of a geared mechanism allegedly suitable for employment in different kinds of mechanical devices.[50] The crucial fact about the engineer's proposal was that his device entailed a pendulum, which, the engineer claimed, greatly increased its performance. In an undated letter sent to the engineer, Galileo Galilei, who had been present at the demonstration, sub-

47 See Turriano 1996, fol. 290r.
48 See Ceredi 1567.
49 See Stevin 1955, 344.
50 See Galilei 1968b.

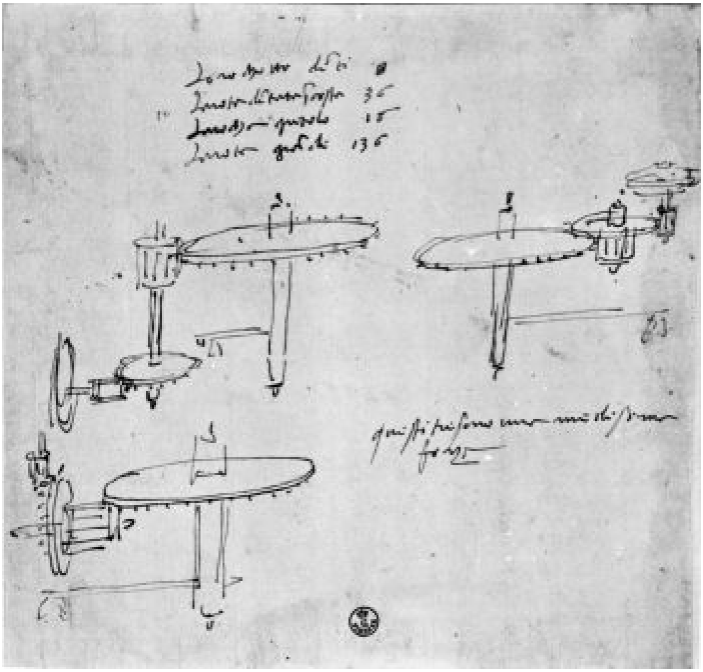

Figure 1.23. Study on different combinations of the same gears for a horse-driven flour-mill. Drawing by Antonio da Sangallo the Younger. (Florence, Gabinetto Disegni e Stampe, U1487A^{r}.)

stantiated to the engineer why he considered this hope to be unfounded. In part of this letter, Galileo reduced the major features of the model presented to a geometric drawing in order to enable its study according to the principles of the balance. The original drawing, however, has not been preserved. This example shows that such theoretical analyses in the context of discussing the design of particular machines can be expected above all in court contexts. In this framework, the “scientific” foundation of personal judgements gained increasing importance over the course of the sixteenth century.

In engineers’ personal accounts, it was primarily measurements and gear ratios that were reported extensively. However, it does not seem that they generally used this information as a starting point for further reasoning of a more general nature. The only drawing known so far that points in such a direction is, again, part of the collec-

tion of Antonio da Sangallo the Younger. It shows ways to combine identical gearing assembled differently in space, stating that they are all "of the same power"[51] (figure 1.23). This comment shows that in early modern engineering practice, concepts like "force" or "velocity" were used by engineers to describe the performance of such devices as a matter of course, without referring to the contemporary scientific definitions of these terms. Even if theoretical reasoning in mechanics did not emerge directly from the use of such prescientific concepts, it seems obvious that the intensified dissemination of preclassical mechanics towards the end of the sixteenth century, at least in Italy, sharpened the perception of the gaps between the scientific and the colloquial use of such terms and thus further stimulated reasoning among figures familiar with both cultures. Such gaps also became obvious with regard to the different visual grammars of engineering drawings and geometrical diagrams of preclassical mechanics, which in the end concerned similar objects, namely basic machine elements. The merging of such different traditions of knowledge raised fruitful challenges for the theoretical investigations in mechanics pursued, for example, by Guidobaldo del Monte and Galileo Galilei.

CONCLUSION

The preceding investigations have shown that early modern machine drawings are not only relevant for the reconstruction of the state of the art of contemporary machine technology. Although an epistemic history of early modern engineering still remains to be written, the analysis of the drawings technical experts produced testifies to the fact that their knowledge far exceeded the *tacit knowledge* of the artisan: Machine drawings turn out to have been the product of a highly differentiated form of knowledge that could take on a number of functions in different contexts of employment. This is especially evident with regard to the drawings discussed here, which were, for the most part, closely related to engineering practice. Such drawings open up new possibilities of contextualizing Leonardo da Vinci's drawings as well as those of the more representative machine books.

51 "[…] sono una medesima forza." Frommel 1994c, 448.

PART II
PICTORIAL LANGUAGES AND SOCIAL CHARACTERS

INTRODUCTION TO PART II

At first glance, almost all of the machine drawings of the fifteenth and sixteenth centuries may appear unprofessional to a modern beholder. They in no way resemble the orthographical plans and schematics that engineers trace and employ today. Usually omitting crucial details, often representing instead superfluous particulars of apparently rhetoric nature, and rarely giving measurements, they may even evoke the suspicion that they are creations of interested laymen with limited competence as regards technological matters. Taking modern blueprints as standard, it is, indeed, hard to imagine that drawings of this kind were of any use for the practice of early modern engineers.

It is particularly the seemingly naiveté of these drawings that is misleading. As the chapters by David McGee and Rainer Leng show, these drawings are anything but naïve. Even the most awkwardly drawn ones do not testify to unskilled attempts of practitioners groping for any manner of depicting intricate technical objects. Rather, such drawings testify to first experimental steps toward depicting them in a specific style of rendering that distinguishes early modern machine drawings from the machine drawings of the Middle Ages as well as those of the modern age.

The few extant machine drawings from the Middle Ages, which David McGee discusses in the first part of his chapter, show a preceding style of technical drawings reminiscent of modern schematics. Focusing exclusively on some of the essential parts of a certain device and their arrangement, no effort is made to represent its appearance. Because of this, they are often as unintelligible for a nonexpert as modern engineering drawings. However, medieval technical drawings share one feature with the seemingly naïve engineering drawings of the early modern period that distinguishes them from modern ones. They, too, omit crucial details, do not give measurements, and leave the question of the device's dimensions unanswered.

Against the background of this preceding style of machine rendering, two points become immediately clear. First, the early modern style of representing machines was developed in a rejection of an earlier professional style of machine rendering and, thus, cannot be regarded as naïve and unprofessional itself. Second, the fact that it shares with the preceding style a practice of neglecting crucial details, measurements, and dimensions, calls for a reconstruction of the engineering practice in these ages that accounts for the apparent usefulness of such incomplete drawings. Both points together warn us not to judge the early modern machine drawings from the viewpoint of present engineering drawings. Rather, one has to try to interpret them in their actual setting, that is, in the actual practice of early modern engineering.

The two chapters of part II do exactly this. They trace the origin and first developmental stages of the specific language of early modern machine drawings. The chapter by McGee explores the emergence of the new style with the Sienese engineer Mariano di Jacopo, called Taccola, whose drawings constituted the starting point for the Italian tradition of early modern engineering drawings, which became influential for the entire West. The chapter by Leng focuses on the less-known pictorial catalogues of German master gun-makers from the fifteenth century. They are of particu-

lar interest with respect to the development of the language of early modern machine drawings for two reasons. First, the bulk of them addresses exclusively fellow experts; their pictorial language thus must have fit exactly the practical needs of these practitioners. Second, in these drawings, one can study how artificial views such as cutaways and exploded views were elaborated step by step.

Investigating carefully and in detail the pictorial means developed and employed, the two chapters inevitably lead to the social conditions that shaped the designing and manufacturing of machines in the fifteenth century. And insights into these conditions lead, in turn, to a better understanding of the peculiarities of the new drawing style, which looked so strange at first glance. This style now appears as a highly artificial compromise between the different ends these drawings had to fulfill at the same time, namely to present the device in question to nonexperts such as (potential) commissioners, on the one hand, and, on the other, to give the masters in charge of the (possible) construction of the device all of the technical information they needed. In reconstructing this double-faced nature of the early modern machine drawings, these chapters restore to these drawings their genuine technical character, which is easily eclipsed by their more obviously representational function for a contemporary beholder.

2

THE ORIGINS OF EARLY MODERN MACHINE DESIGN

DAVID MCGEE

From approximately 1450 to 1750, machine designers made use of a singular kind of graphic representation. Where craftsmen used partial representations or none at all, and architects used plans and elevations to show different views of a building, machine designers stuck to a tradition of one drawing and one view for one machine in both sketches and presentations. Such a long and stable association between machine design and one kind of drawing calls for explanation. This chapter attempts to lay some of the groundwork for such an explanation through an investigation of the origins of the early modern tradition of machine drawing in the period before 1450.

Any attempt to delve into the origins of early modern machine design is beset by methodological problems. One set of these problems arises from the nature of the evidence. There are only a handful of extant manuscripts and they are spread across three succeeding centuries. Such a distribution compels the historian of machine design to develop an account of long-term historical trends, with all the problems that implies, knowing that any such account can only rest on a slender foundation. Such a predicament generates a real need to get all we can out of the available evidence. But doing so is not easy. Early technical drawings are extraordinarily difficult to interpret. Confronted with a drawing from the fifteenth-century notebook of artist-engineer Mariano Taccola (figure 2.1), for example, it is not at all clear what is important, why these particular devices are found together, why they are drawn at different sizes, or even what the drawings are for. Indeed, one rather quickly realizes that we simply do not have the conceptual tools we need to get at this kind of evidence.

A second set of problems is of our own making, beginning with our Platonic approach to design, where design is taken to be a more or less entirely cognitive affair that is concerned exclusively with the *conception* of form.[1] Our tendency is to take drawings as expressions of ideas, then move backwards from the drawings into the minds of the artists, either to describe what they were thinking, or, since knowing what they were thinking is almost impossible, to characterize their mentality, or to opine about cognition. This is to leap from the methodological frying pan into the fire, jumping from evidence we do not understand very well into the even more treacherous realms of psychology—perhaps even to celebrate “tacit knowledge,” or “nonverbal thinking”—long before we have really come to grips with the evidence at hand.[2]

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1 A good example is Ferguson 1992. But the emphasis on conception and psychology is rampant in the design professions. See, for example, Lawson 1980; Akin 1986; and Rowe 1987.

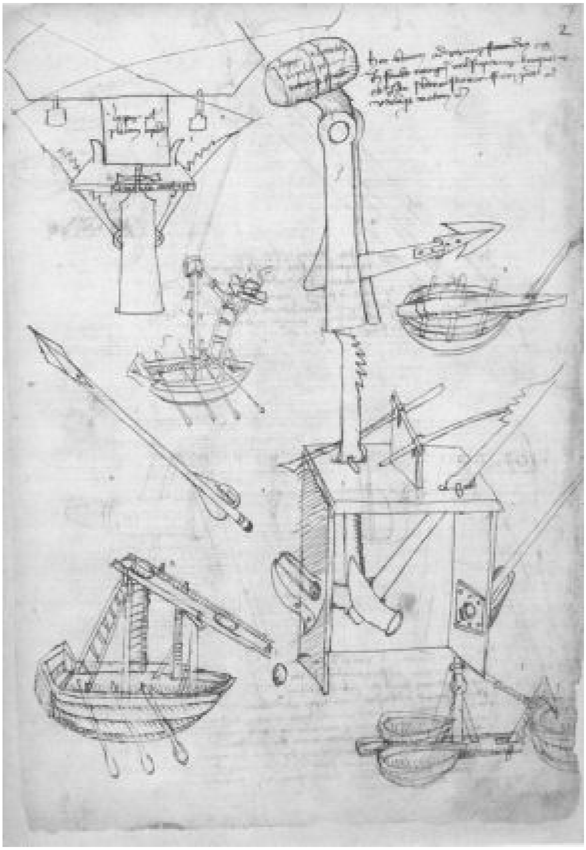

Figure 2.1. A typically difficult to interpret folio from the notebook of an early Renaissance engineer. Drawing by Mariano Taccola. (Munich, Bayerische Staatsbibliothek, Codex Latinus Monacensis 28800, f. 2^{r}; reproduced in Taccola 1971.)

2 My concern here is twofold. On the one hand it seems to me that historians do a tremendous disservice to their discipline by attempting to devolve historical explanation off onto the "truth" of psychological theories (which are, of course, constantly changing), thereby abandoning the effort to build up historical explanation itself. On the other, I very much worry about theories that locate the essential evidence in the mind, which is to base a theory on evidence which (like tacit knowledge) is unknowable by definition.

A related problem stems from modern familiarity with the conventions of linear perspective and our almost unconscious acceptance of two assumptions that go with it. One assumption is that, when people look at things, they really do see something very much like a perspective drawing—and especially that they see things from a distinct, single viewpoint. The second is that making pictures is about rendering the appearance of things as they are seen either in reality or in the mind. These assumptions are problematic in themselves, and extremely problematic when it comes to pre-perspective drawings, which do not render the overall appearance of things accurately, and frequently do not have viewpoints. Nevertheless, on the assumption that the artists had perspective images in their minds, we proceed to interpret their drawings in terms of perspective, using all its language of viewpoints and picture planes. Apart from anachronism, the usual result is an interpretation of the actual drawing as simply a poor rendering of the much clearer mental image, due either to inadequate drawing methods or simple incompetence. We have all read of the childlike "naiveté" of early machine drawings and of the "mistakes" their artists are supposed to have made.[3] This line of reasoning leads to an account of the development of drawing over time as a process of closer and closer approximation to the more accurately realistic representational methods of linear perspective, which is not only teleological, but takes for granted that people should want such a thing, thereby relieving us of the need to actually offer an explanation. More important, discounting the *actual* properties of early drawings as "error," this approach dismisses the *actual* evidence in favor of what is *assumed* to be in the mind, when it is closer attention to the evidence that we need.

One last set of difficulties may be referred to as the Princes problem. It stems from the observation that none of the authors of early machine drawings were actually machine makers, and that most of the early manuscripts up to 1550 were created for princely patrons. Much fine work has been done by Paolo Galluzzi and Pamela Long using the written content of these manuscripts to stress the importance of courtly humanism on the development of Renaissance machine design, and especially on the emergence of "engineers" as a distinct social group.[4] Yet, an unspoken assumption is that the fact of a princely audience explains something about the contents and properties of the drawings. For example, the more fanciful drawings or impossible machines can be regarded as "dreams," or a form of play, composed by amateurs as entertainment.[5] The point here is that on this basis we tend to push the drawings away as objects of serious attention, worthy of investigation for what they can tell us.

To help combat these problems, this chapter proposes that early machine drawings should be analyzed in terms of design, where design is fundamentally regarded as a form of *doing* rather than thinking, and as *the process by which artifacts get the*

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3 Perhaps the worst example is Edgerton 1991.
4 See particularly Galluzzi 1993 and Long 1997.
5 Thus even Gille 1966 suggested that the authors of early manuscripts were not the "true agents" of technological progress, and that invention was mostly a pastime, a form of amusement, an intellectual game. See also Hall 1979b.

dimensions they actually have. A moment's reflection reveals that many artifacts get their final dimension only when they are actually made, something that was particularly true of early machines. That is to say, the definition of design employed here can include both conception *and* construction, things we habitually regard as separate on the basis of our understanding of modern design techniques. One methodological principle to arise out of this definition is that, unless we have good evidence to think otherwise, it should be assumed that the drawings we see were considered by their authors to be perfectly adequate to the process of completing a design. The second is that, in order to get the most out of premodern machine drawings, it will often be much more useful to consider what came *after* them, rather than what went before them. That is to say, it will be crucial to consider the context of construction—even though we can be reasonably sure that none of the machines seen in this chapter was ever built.

The claim here is that by paying attention to *doing* we can get a better understanding of the properties of early modern drawings, and through a better interpretation of the evidence we can arrive at a better tale of development over time. To support this claim, the chapter examines four episodes in the development of early machine drawings, beginning with the lodge books of Villard de Honnecourt, followed by the manuscripts of Guido da Vigevano and Konrad Kyeser. These brief studies set the stage for an examination of the drawings of Mariano Taccola—a man who may justly be regarded as the father of the style of machine drawing that would remain intact for three hundred years, and continues to dominate even now in the twenty-first century.

A word of warning to the reader. This is a methodological chapter. The normal historical apparatus, particularly the discussion of historical context, has been kept to a minimum. The intention is to outline an argument as clearly as possible, rather than provide a history, so that the approach may be judged, so to speak, by its good works.

1. THE CONTEXT OF CONSTRUCTION

The value of looking at early machine drawings in terms of the context of construction can be illustrated with a famous drawing of a self-powered sawmill by Villard de Honnecourt, the French architect active circa 1250 whose sketch-books provide us with some of the earliest machine drawings in the West (figure 2.2).[6]

Villard's depiction of the sawmill already displays two basic characteristics of machine drawings that would remain stable for the next 500 years. The first is that Villard has adopted what may be called the principle of one machine, one drawing. The second is that there are no measurements, to which we may add that parts of the arrangement are clearly missing. In short, final dimensions have not been determined.

However, perhaps the most striking thing about this drawing is the visual confusion it presents to the modern viewer, a confusion that stems from our expectation, honed by years of experience with perspective drawings, that one drawing of one

6 The standard version of Villard's sketch-book is Hahnloser 1972.

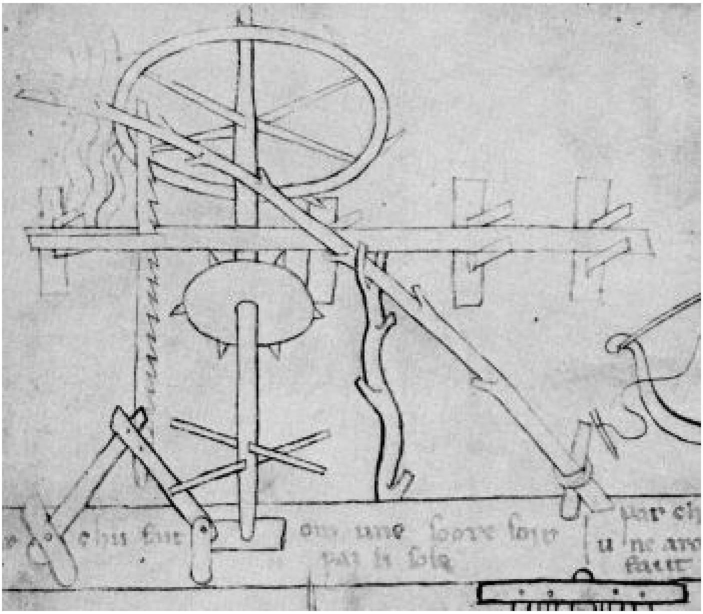

Figure 2.2. Villard de Honnecourt's drawing of a self-powered sawmill, c. 1225. (Detail of Paris, Bibliothèque Nationale de France, MS Fr 19093, f. 44; reproduced in Hahnloser 1972, plate 44.)

machine should also have one viewpoint. To us, Villard appears to have rotated different parts of the sawmill away from their "true" position, relative to any viewpoint and, moreover, has done so without any system. The timber seems to be shown from the top, the water-wheel from a three-quarters view, the saw blade from the side, the cams that bring the saw blade down from a different side, the tree-branch that brings the blade up from the side at one end but the top at the other, while the point at which the saw blade meets the timber seems to be shown from the top, side and three-quarters view all at once. We read the drawing as having multiple viewpoints at the same time, some of them mutually exclusive. Hence the difficulty we have understanding the picture.

Given our confusion, it is essential to point out that Villard can and does draw in different and (at least to us) "better" ways. In his architectural drawings, for example, Villard uses plans and elevations to show two sides of the same building in different drawings. Moreover, it turns out that Villard is also able to draw machines in a "better" way, as can be seen in his depiction of a new arrangement for fixing the hub of a

wagon without having to cut the shaft (figure 2.3). Here it appears that Villard may even have used a compass to construct a drawing that simultaneously shows both the true shape of the wheel and the way the arrangement would actually appear from one side. With its apparent viewpoint, we read this drawing quite easily.[7]

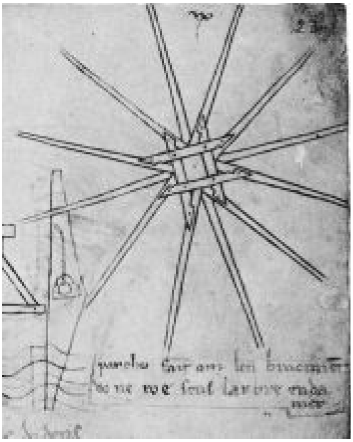

Figure 2.3. Villard de Honnecourt's drawing of a wagon wheel. (Detail of Paris, Bibliothèque Nationale de France, MS Fr 19093, f. 45; reproduced in Hahnloser, 1972, plate 45.)

The main point, however, is not whether one of Villard's drawing styles is better than another, but recognition of the fact that Villard has *different* drawing styles available to him. This immediately rules out interpretations of the style used in the drawing of the sawmill as "naïve" or "childlike," or indicative of some odd medieval way of "seeing" the world (since if styles are equated with world-views, then there are at least three world-views in Villard, which is ridiculous).[8] More important, acknowledging that Villard had different drawing styles available to him leads to the realization that he *could* have drawn his sawmill differently, and thus that he *chose* to depict the sawmill in the way that he did.[9] Evidently, he did so because he thought this style was best for his purpose.[10] What purpose could that be?

This is where consideration of the context of construction comes in, beginning with the fact that a machine like Villard's, like any machine of its kind at the time, would have to be adapted to the site on which it was built. At different locations, the water-wheel would have to be of a different diameter, the drive shafts would have to be longer or shorter, the support

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7 I say "apparent viewpoint" because the fact that the eye seems to be fixed at the center of the wheel may simply have been the result of using a compass, and thus imply nothing about Villard having any notion of viewpoint, even though he was trying to depict the hub device as it would appear.

8 See for example, Booker 1963, and also Edgerton 1991.

9 The importance of the availability of different styles of mechanical drawing for practitioners is also noted by Marcus Popplow (this volume) in his discussion of the technical practice of Heinrich Schickhardt.

10 I forego what could be a lengthy discussion of why Villard, who may be taken to be indicative of premodern technical practice in general, uses different styles of drawings for different kinds of objects: the flat style for machines, plans and elevations and stonecutting diagrams for buildings, etc. I suggest, however, that part of the answer is to be found in the context of construction. There was a much greater need for precision in the construction of a building, whose parts must fit together precisely, than there was in the construction of the wooden machinery of the Middle Ages. The need for precision implies a greater need to control the worker's hands. Plans represent the first step in excercising that control, stonecutting drawings the next. As I suggest below, no such control over the hands of the machine maker is wanted, but rather precisely the opposite.

structures adapted to the river bank, and so on. In other words, the design of the machine would not be completed until it was actually built. A second point is that the necessity of adapting the machine to the site implies the existence of a *person* who can do just that. And not just any person, but an *expert* capable of completing the arrangement and establishing the final dimensions of the machine for himself. The question then becomes—what needs to be conveyed to such a person? Not precise dimensions or complete arrangement. That would be pointless, since each version of the sawmill must be different from every other.[11] Rather, all one really needs to provide is an indication of the main parts and their general relationship to each other. Knowing what the parts are, the expert can *deduce* how they must go together, as well as whatever other parts are needed while building the machine on site. The same point is made by Rainer Leng in his chapter, concerning the graphic representations of fifteenth-century gunmakers.[12]

Given the existence of an expert, it is not necessary for either the overall machine or its parts to be rendered "realistically," as they would actually appear.[13] All that is needed is to represent the characteristics of the parts in such a way that the expert can understand what they are. Mere icons will do, and the iconic nature of Villard's drawing is nicely illustrated by the buckets or paddles attached to his water-wheel on the lower right.[14] They are on backwards. If interpreted realistically, the water-wheel would drive the timber *away* from the saw blade rather than toward it. However, if we eschew the idea that Villard made a "mistake," in favor of the assumption that the buckets did exactly what Villard intended them to do in this drawing, we realize that the buckets are no more than signs that says to the expert "water-wheel," rather than, say, "grist-wheel." On this account it becomes superfluous to assume that Villard had any notion of rotating parts to show them as they would appear from *different viewpoints*. In fact, the evidence does not suggest any idea of a viewpoint whatsoever. Rather, the drawing takes on the character of a logical schema built of icons, intended for the eyes of an expert builder who does not need to know exactly how either the whole or the parts would appear. All he needs to know is the main parts and their general arrangement.[15]

This interpretation of Villard's sawmill rests on the proposition that early machine designers had the context of construction by experts in mind and made their drawings accordingly. Alas, it must be admitted that there is not enough evidence in Villard's

11 In other words, the dimensions are not missing because the drawing represents only an idea, as might be argued. They are not included because they are unnecessary.

12 The same idea is suggested by Hall 1996, 9, who observes that missing or distorted elements in drawings almost automatically suggest the ability to supply the missing details on the part of those who have to use them.

13 As Leng also points out in his chapter.

14 The word "icon" points to one of the typical language difficulties encountered in the discussion of drawings. I use the word to mean a graphic element whose purpose is to show essential or characteristic attributes of a machine part so that it can be *named* in the virtual context of the drawing, not to show how a machine part actually appears so that it could be *recognized* when seen in the real world. In this sense a machine icon is a symbol, like an Egyptian hieroglyph. But like a hieroglyph, the symbol is not an arbitrary choice like a letter. It is an abstraction of the visual appearance of the real thing, making it difficult to say precisely that one drawing is supposed to show a machine as it appears, while another does not.

notebooks to confirm or refute this contention. Striking confirmation, however, can be found in the work of Guido da Vigevano.

2. PARTS, PICTURES, AND PATRONS

Guido da Vigevano was an Italian medical doctor from Pavia who found service with various kings and queens of France in the fourteenth century. He is of interest here for his *Texaurus Regis Francae*, an illustrated manuscript, which he wrote in 1335 to advise Phillip VI of France on the conduct of a proposed crusade to the Holy Land.[16] The first part of the manuscript concerns such things as proper diet and how to avoid being poisoned. The second part consists of 14 folios of text and drawings about the kind of military equipment the king would need to take with him.

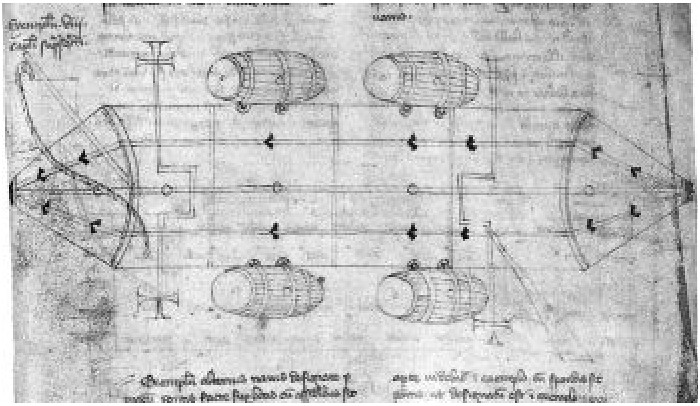

Figure 2.4. Guido da Vigevano's modular paddle boat, c. 1335. (Paris, Bibliothèque Nationale de France, Manuscrit latin 11015, *Texaurus regis Francie aquisitionis terre sancte de ultra mare,* f. 49^{r}; reproduced in Vigevano 1993, plate IX.)

15 The argument that drawings like Villard's were understandable to experts naturally implies the existence of a shared set of visual icons among those using the flat style in any given locale. The existence of such a set raises the question of what would happen if an item had to be indicated for which there was no icon, perhaps because it was entirely new. Without arguing the issue at length, it might be suggested that Villard's spoke and hub represent one attempt to show a new item more realistically, as does the branch in his sawmill. Both, though in very different ways, might be seen as attempts to show unknown items as they would appear—perhaps the only alternative.

16 This expedition never took place. For the original text and marvelous reproductions of the drawings, see Vigevano 1993.

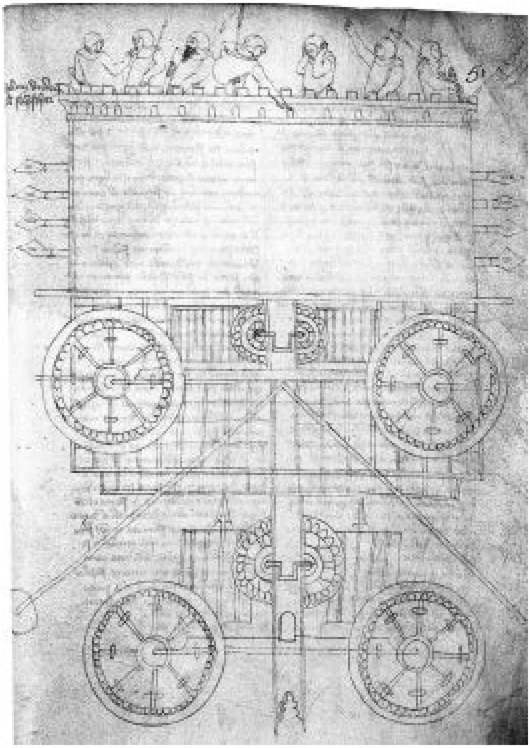

Figure 2.5. Guido da Vigevano's cranked assault wagon. (Paris, Bibliothèque Nationale de France, Manuscrit latin 11015, *Texaurus regis Francie aquisitionis terre sancte de ultra mare,* f. 51^{r}; reproduced in Vigevano 1993, plate XIII.)

Figure 2.4 shows Guido's proposal for a modular crank-and-paddle powered paddle-boat.[17] From it we see that Guido (and/or his illustrator) was working in the same "flat" style as Villard.[18] We have one machine and one drawing. There are no dimensions and little concern for overall appearance.[19] Guido is more rigorous than Villard in epicting all of his parts flat on the paper, so that none are shown in a three-quar-

17 One of Guido's guiding ideas was that very little timber was to be found in the Holy Land so that the equipment needed for the campaign would have to be prefabricated in France in pieces small enough to be carried on horseback. For an English translation, see Rupert Hall 1976a.

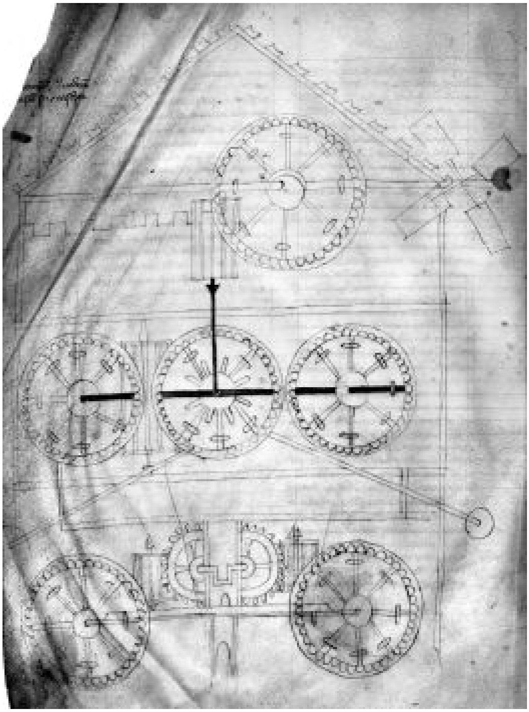

Figure 2.6. Guido da Vigevano's wind-powered assault wagon. (Paris, Bibliothèque Nationale de France, Manuscrit latin 11015, *Texaurus regis Francie aquisitionis terre sancte de ultra mare,* f. 52$^{\text{v}}$; reproduced in Vigevano 1993, plate IV.)

ters view. But the emphasis is again on the parts, which are again shown as icons. The flattened ends of the barrels, for example, are not shown to indicate volume, but to show that these items are casks.

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18 Of the three extant copies of Guido's manuscript, none are the original, so that none of the drawings we have can be said to be by Guido's own hand. It is not known for certain whether Guido himself made the original drawings or had them done by an illustrator, although there is a passage in the text where he suggests that one of his assault devices could be covered with a protective quilt, and that this should be done "as the maker sees fit, for I could not sketch all this in the figure," Hall 1976a, 29. For the sake of convenience, however, I will still refer to the drawings in the manuscript as "Guido's drawings."

19 Although indications of size are given in the accompanying text, they are extremely general.

More important is what Guido says about the text associated with this drawing, which is that “a skilled man will easily understand this because I cannot write it more clearly.”[20] A few pages later, after describing the front wheels of his famous crank-powered assault wagon (figure 2.5), Guido observes that: “the rear wheels should be similarly prepared in this way, and all these things shall be arranged by the master millwright who knows how to match these wheels together.”[21] Similarly, referring to his famous drawing of a wind-powered assault wagon (figure 2.6), Guido comments on his description of the gearing by saying that “all these matters are the concern of the master millwright and especially the master windmill-wright.”[22] These passages clearly show that, while writing his text and using the same style of drawing as Villard, Guido was indeed thinking about the context of construction and particularly about construction by skilled experts. The last two passages make it explicit that Guido believed his drawings would be fully comprehensible to these experts, and that he fully expected them to be able to figure out from the drawings how to complete the design, deducing the necessary details from the arrangement of well known icons of lantern gears, cranks, and so on.[23]

It is worth stressing how different this design relationship is from that of today, where the designer is expected to provide complete drawings and complete specifications of dimensions, parts, and arrangement, and the worker is expected to make the parts (and thus the machine) exactly as shown. But Guido not only expects the participation of expert machine builders, he *needs* them to make positive, creative contributions. In fact, his text is full of comments to the effect that “whoever is to do this will provide for the best,” and that various parts will be “made as those doing the job see fit.”[24] In other words we see that medieval machine design was *not* conceived to be a matter of individual conception, of making a drawing and that was that. It was conceived as a process in which at least two people were needed to give a machine the final dimensions it would actually have when it was actually built. As in Villard, it was not necessary, nor of any use, to provide dimensions or a realistic depiction of the overall machine to the experts who would build the machines and were quite capable of determining final dimensions and arrangements for themselves. Flat, iconic representations would do.

However, in the very passages where Guido says his drawings will be perfectly adequate for experts, he reveals that the flat, iconic style of the machine builders was *not* understandable to everyone—and particularly not to the ideal reader to whom the

20 Hall 1976a, 26.
21 *Ibid*., 27.
22 *Ibid*., 29.
23 Having confirmed the idea of a style of drawings intended for expert makers, the question arises as to the existence of a tradition of technical drawing in this style, stretching back in time. Lefèvre 2002 has shown that whether or not one regards the illustrations as in any way original or authentic, the drawings found in military manuscript treatises on machines and mechanics prove that a flat style tradition of machine drawing existed in both the Byzantine and Arabic worlds from at least the early Middle Ages onward. His examples, as well as those in this chapter, show that there was considerable flexibility in the application of this style, over and above its shared conventions.
24 To give a typical example, Guido writes that the platform of one of his assault towers is to be supported by iron struts: “as the makers shall see fit, for it is not possible to set out every detail. But when someone undertakes this work he will himself make provision for this platform.” Hall 1976a, 22.

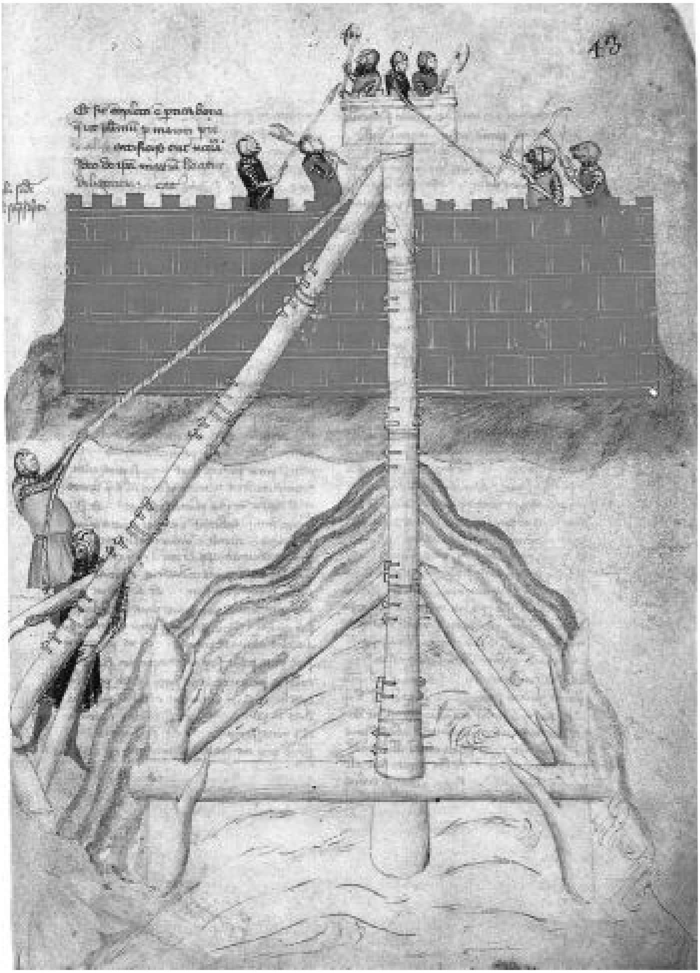

Figure 2.7. One of Guido da Vigevano modular assault towers, where shading is used to show three dimensions. (Paris, Bibliothèque Nationale de France, Manuscrit latin 11015, *Texaurus regis Francie aquisitionis terre sancte de ultra mare,* f. 43^{r}; reproduced in Vigevano 1993, plate IV.

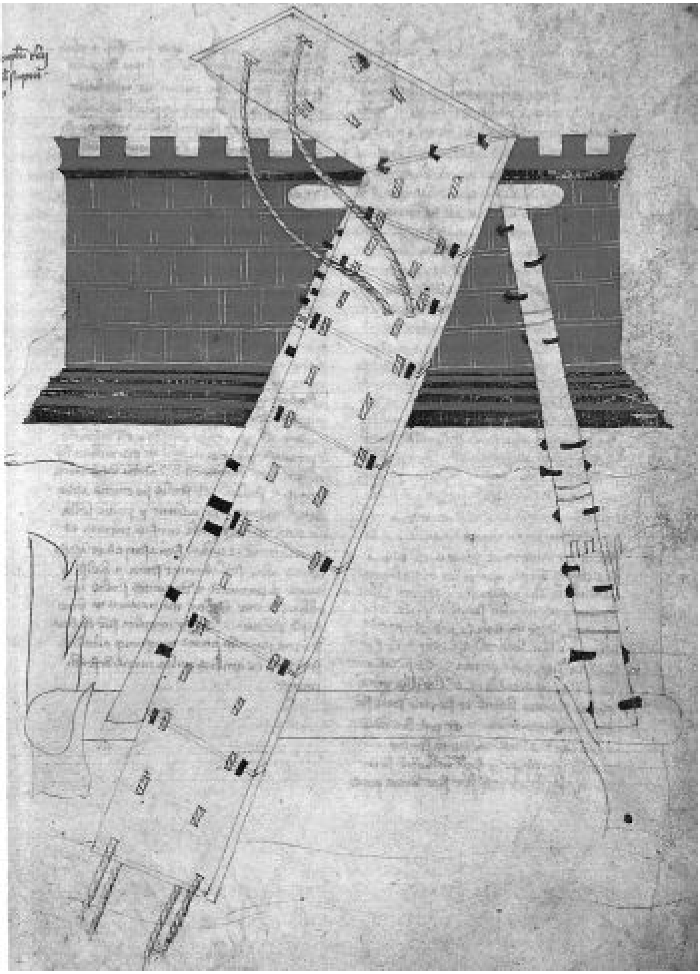

Figure 2.8. Another of Guido da Vigevano modular assault towers, where lines are used to show the three-dimensional volume of the timbers. (Paris, Bibliothèque Nationale de France, Manuscrit latin 11015, *Texaurus regis Francie aquisitionis terre sancte de ultra mare,* f. 44[r]; reproduced in Vigevano 1993, plate IV.)

manuscript is dedicated, namely, his patron the King. This is significant because it points to Guido's recognition of the participation of a *third* person in the design process: namely, the patron who may not determine any dimensions himself, but whose role is nevertheless crucial, since without his approval, no money will be spent and no finalization of the dimensions will ever take place.

In light of Guido's awareness of the fact that the flat style created problems of communication with patrons, two of his drawings may be singled out as particularly important. The first shows an assault tower resting on three big legs, composed of short wooden modules held together by iron pins (figure 2.7).

The poles in this drawing are not represented by mere lines as the flat style would demand. Instead, shading has been added to give the poles three-dimensional volume, and thereby indicate that they are round. The second drawing (figure 2.8) shows a modular assault ramp, up which men are supposed to run until they reach the topmost section, which they flip over the ramparts of the enemy fortress. Here, the ramp has been drawn to show two sides of the modules at the same time, and lines have been drawn in to indicate the volume of the modules without the use of shadowing.[25] In both drawings we also notice that the machines have been set in scenes, with the fortress being assaulted in the rear.

There would appear to be two possible explanations for this combination of setting and appearance, depending on the audience. It may be argued, for example, that Guido's goal was to show the purpose of his machines to the patron and that he set them in their context of use in order to show what they were for. Since it would have been pointless to set the normal flat icons in such a setting, he therefore added the third dimension to the parts in order to show everything as it would appear. Alternately, it may be argued that Guido's modular ramps were quite new. Hence, there were no icons for them in the established repertoire that could be used to communicate with the makers. Since there was no other choice, it was necessary to attempt to show the parts as they would appear.[26] The setting was added to help makers to identify the parts by deduction from the purpose.

It seems most likely that both factors were in play, and that Guido was attempting to find a means of communicating to both audiences at once. If so, these drawings may be taken to indicate the horns of the early machine designer's dilemma. In trying to show the appearance of his assault towers, Guido may have succeeded in giving a better idea of their purpose, but his ability to show the parts to the expert/maker has suffered. Without referring to the text, for example, it is extremely difficult to recognize the spikes or the modules for what they are.[27]

How to devise a style of representation that could answer the needs of both machine makers and patrons at once? This problem may be taken as one of the major difficulties facing machine designers in the fourteenth and fifteenth centuries. A

25 Several more of Guido's drawings also show three dimensions without shadowing.
26 As noted above, the same may be seen in Villard's drawing of a sawmill, where the tree branch used as the return spring seems to be shown as it would appear, there being no standard mechanical icon for such a thing.

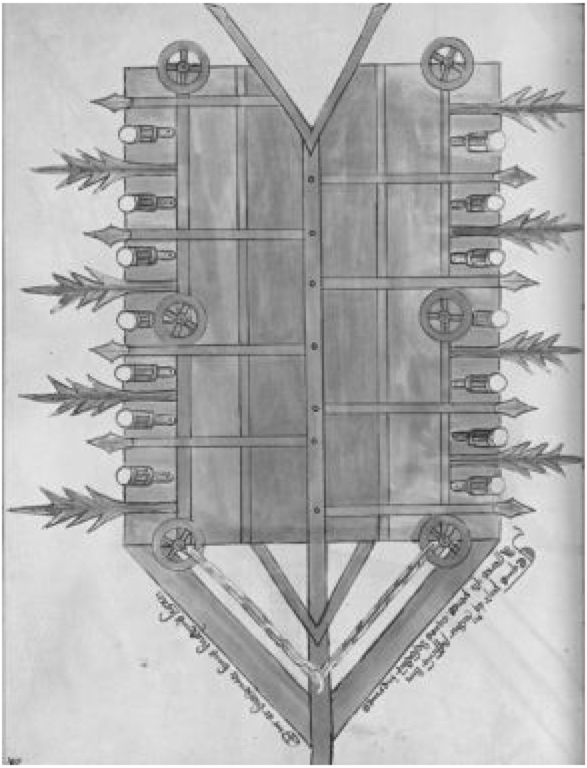

Figure 2.9. A protected assault chariot in the flat style from Konrad Kyeser's *Bellifortis*. (Göttingen, Niedersächsischen Staats- und Universitätsbibliothek, Cod. Ms. philos. 63, fol. 24^{r}; reproduced in Kyeser 1967.)

sequence of drawings in Konrad Kyeser's *Bellifortis* provides fascinating evidence about the nature of the solution to the problem.

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27 There is a wonderful confirmation of the difficulty of interpreting the parts correctly in the Yale manuscript version of this drawing, where the illustrator has not only changed the spikes into dowels, but spread them out along the length of the poles so that they have nothing to do with pinning the modular sections of the poles together. Rather, to judge by what the Yale illustrator does with the pegs in other drawings, he seems to think that they were supposed to be steps, by which one could climb up the poles to the assault platforms. See the images of the New York manuscript included in Vigevano 1993.

3. FLAT TO FAT

Konrad Kyeser was born in Eichstadt, Bavaria, in 1366. He was trained as a medical doctor and spent his later years in the service of the Holy Roman Emperor Wenceslaus of Prague. Despite being a medical man, or perhaps because of it, Kyeser seems to have gained considerable military experience. In any case, we know that he was present at the embarrassing defeat of Sigismund of Hungary by the Ottoman Turks at Nicopolis in 1396. Four years later, Sigismund imprisoned his half-brother Wenceslaus and Kyeser was banished to Eichstadt. There he composed his *Bellifortis*, or Strong in War, dedicating the manuscript to the weak Emperor Ruprecht, who succeeded Wenceslaus in 1400, and who was in turn ousted by Sigismund in 1410.[28]

Bellifortis was a popular work among the princes of its time, judging by the many copies in existence. Much of its popularity must have been due to its lavish drawings, which were produced by illuminators from the Prague scriptorium who had also been banished by Sigismund, and apparently passed through Eichstadt on their way home. How the final drawings in *Bellifortis* relate to any original drawings of Kyeser is not known, but the fact that they were made by various hands means that Kyeser's work, like those of Villard and Vigevano, once again reflects the availability of many different drawing styles.

One of the styles found in many of Kyeser's drawings is the "flat" style, of which a particularly lavish version is found in figure 2.9. It shows a covered assault wagon, armed with small guns. Here we see the continuation of the principle of one machine, one drawing, and no dimensions. Every part of the device is shown flat on the paper in order to reveal its characteristic or iconic shape. Realistic appearance is not a concern, as is very clearly shown by the chain at the front of the wagon, which passes between the spokes of the wagon wheels and would have stopped the wheels from turning. Apparently, this did not bother the illustrator, who knew the wheels would in reality be turned 90 degrees from the position shown and thus that the chain would not really pass through the spokes at all. As this picture shows, we find in Kyeser the kind of logical schematic used to communicate parts and arrangement to an expert machine maker, capable completing the design for himself. That is, makers were also a part of Kyeser's audience. Like Villard and Guido, he needed them to complete his designs.

Kyeser, however, may also be assumed to have faced Guido's problem. The "flat" style might be fine for communicating with wheelwrights, but an unwarlike emperor could not be expected to readily understand them. Thus, in Kyeser we would expect to find a further development of techniques for drawing machines as they would appear. One sequence of drawings is particularly telling about the nature of this development.

The sequence consists of several protective shields, which were to be rolled forward in battle (figure 2.10). All of them appear to be drawn by the same hand. The

28 For biographical information on Kyeser's life, see Quarg in Kyeser 1967 and Long 1997.

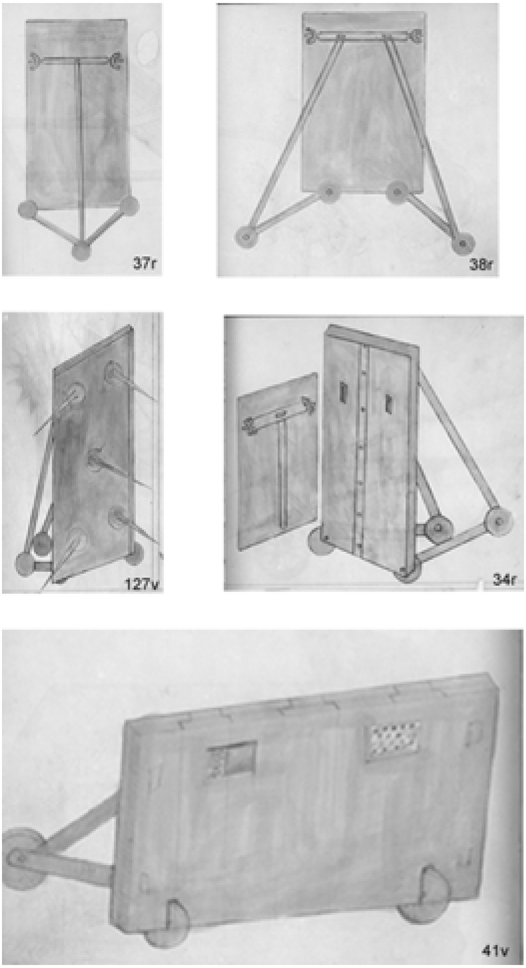

Figure 2.10. Evolving technique in a sequence of rolling shields from Konrad Kyeser's *Bellifortis*. (Details from Göttingen, Niedersächsischen Staats- und Universitätsbibliothek, Cod. Ms. philos. 63, fol. 37^{r}, 38^{r}, 127^{v}, 34^{r}, and 41^{v}; reproduced in Kyeser 1967.)

first shield is from folio 37^{r}. It is drawn in the "flat" style and shows the shield as a rectangle, and its three wheels as iconic circles. The next drawing is from folio 38^{r} and also shows a rectangle for the shield and circles for the wheels. Here, the artist seems to have realized that the triangular frame for the three wheels of his first shield would have got in the way of anyone trying to shelter behind it, and so his second shield rolls on four wheels, each pair on a separate assembly. These assemblies are splayed out, the closest wheels being wider apart than the farther wheels, which are also higher on the page. This may indicate that the artist was already beginning to think about issues of appearance, but a definite turn towards appearance is seen in folio 127^{v}. Here we have the same four-wheeled shield, but the front wheels are partially obscured by the shield as they should be, while the shield itself is shown as a rhomboid rather than a square, as if turned in space. Turned this way, the artist seems to have realized that three sides of the shield could be seen at once. Accordingly, he has drawn in lines to show the thickness of the shield. Unfortunately, the attempt to show the appearance of the shield seems to have presented many problems to our artist. Over and above the odd treatment of the spikes on the front, he has shown the thickness on the top and right of the shield when it should have been indicated on the top and left. He has also continued to draw the wheels and their assemblies in the flat style. The closest wheel strut is drawn at right angles to the shield, when it should tilt up. The farther wheel assembly should have been completely obscured, but is still partly visible.

Given these difficulties, the shield drawn in folio 34^{r} represents a true breakthrough. The artist has now indicated the thickness on the proper sides of his rhomboid shield. He has tilted the front wheel assembly upwards so that it recedes into the picture space. He has once again shown the front wheels as partially obscured. These are major developments despite the fact that he has still drawn the wheels as circles and, the advantages of the "flat" iconic dying hard, despite the fact that he has still shown the back wheel assembly when it should be obscured.

It is fascinating to see that these last two difficulties are the very problems attacked in folio 41^{v}. Here we sense the growing confidence of the artist in rendering the thickness of the shield, which is now more or less correct in appearance and even indicates the jointing of the timbers. The rearmost wheel assembly has rightly disappeared. The front-most wheel assembly is tilted up as it should be. For the first time, the artist attempts to render the thickness of the wheel assembly struts. For the first time, he tries to show the thickness of the wheels, and even how the wheels go through the shield, although he has been let down by his continuing practice of starting his wheels as circles when they should be oblongs.

Our artist, however, seems to be quite aware of this, as can be seen in the final drawing of our sequence, which is the colored image found on folio 73^{v} (figure 2.11). This drawing now shows men behind a shield, which they are rolling along the ground, and such is our artist's confidence that he has even tried to show the thickness of a V-shaped shield. The back wheel assembly is properly obscured. The thickness of the front wheel assembly is shown. He also appears to have recognized that, if

shown from the side, the wheels would really appear as circles and he would not have to deal with the problem of drawing ellipses. Unfortunately, it seems as if he was again unable to resist the temptation to provide further information, and so has drawn in the thickness of the wheels as they would appear from below.

Figure 2.11. A perfected drawing of a rolling shield from Konrad Kyeser's *Bellifortis*. (Details of Göttingen, Niedersächsischen Staats- und Universitätsbibliothek, Cod. Ms. philos. 63, fol. 73^v; reproduced in Kyeser 1967.)

What is truly fascinating about this sequence is that it moves from the "flat" style to what we might call a "fat" style. But the shift clearly did not take place all at once. It did not take place as a result of the application of some established drawing scheme, or involve any rigorous sense of visual geometry (even though the artist uses a compass and straightedge). Rather, it appears the artist adopted the idea that his task was to show the *parts* as they would appear and then proceeded piecewise, replacing the original icons one at a time, perfecting the appearance of individual parts one after another. That is to say, he did not start with an idea of overall appearance or with a set of top-down rules that only needed to be applied. On the contrary, the overall appearance of the shield in the final drawing is built *up*, one part at a time. On this basis, it may be suggested that the goal was not only to adapt the flat style to the needs of patrons, but to *preserve* its original ability to convey enough information about parts and arrangement to builders that they could finish the design for themselves. The author has succeeded. In the later drawings of the sequence, we can quite clearly tell which parts are which, and yet we easily understand the purpose of the machine.

Following the sequence of shield drawings provides the key to interpreting what is undoubtedly the masterpiece of technical representation in Kyeser's manuscript. This is the magnificent picture of a trebuchet seen in figure 2.12. The background in the original is richly figured in red, the ground a stately gray, the timbers bright gold. Every timber and every part is given a three-dimensional shape. The overall impres-

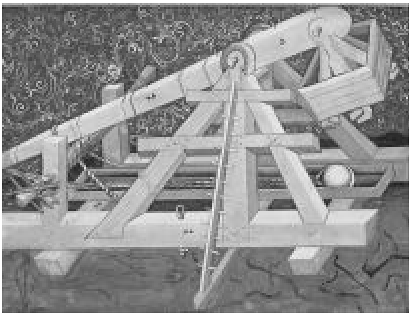

Figure 2.12. A magnificent drawing of a trebuchet from Konrad Kyser's *Bellifortis*. (Göttingen, Niedersächsischen Staats- und Universitätsbibliothek, Cod. Ms. philos. 63, fol. 30^{r}; reproduced in Kyeser 1967.)

sion is of a style so completely different from that of Villard de Honnecourt that it is difficult not to think in terms of some sort of "proto-perspective." This, however, would be to interpret the drawing in terms of something that had not yet happened. It also assumes the artist had a "better" image in his mind, which included a distinct point of view. But we should recall that this was not the case with the final picture of the rolling shields. With the lessons of the sequence of shields in mind, the picture of the trebuchet may be interpreted quite differently.

Passing over the fact that we once again have one drawing, one machine and no detailed measurements, it should be remarked that there is no overall point of view.[29] We seem to see the machine in general from above right. Yet we see the throwing arm and the weight bucket from above left. The right support of the triangular frame in the foreground seems to be pictured from above right, but the left support from below right, and so on. Rather than think the artist *had* a point of view but implemented it badly, we are better off to think that he did the same thing as the author of the shield sequence. The artist drew the machine one part at a time, giving each part its three dimensionality in an effort to depict its appearance. The overall appearance is only an artifact of this piecemeal procedure.

We may acknowledge that the picture contains perfectly adequate information for the expert builder. We may also acknowledge that the result was more than good enough to indicate the use of the machine to the patron. Both goals are accomplished through the depiction of appearance, but it seems clear that even though designers had adopted the idea of depicting the appearance of individual machine parts, they had not yet adopted the idea of a viewpoint as a means of regulating the appearance of machines as wholes.[30]

This, however, is the very step taken in the work of Mariano Taccola.

29 Interestingly, there are four numbers written on the drawings, seeming to indicate the proper length of some parts. They would not, however, relieve the maker of the necessity of determining the final dimensions of each part for himself. There is, for example, no indication of the relative thickness of any timbers.

30 In the case of the trebuchet, we have a drawing of a well-known machine. Leng (this volume) examines the application of this drawing style to gunpowder weapons as a whole new kind of artifact.

4. INVENTION OF THE SKETCH

Mariano di Jacobi detto Taccola was born in Siena in 1381. We know that he was paid for wooden sculptures in the Duomo of Siena in 1408, which suggests he trained as an artist. In 1424, he became secretary of a quasi-religious housing and hospital organization, which suggests he may have trained as a notary. In 1434 Taccola lost his secretary's job, becoming a *stimatore* or quantities estimator. In 1441 he became an inspector of Siena's roads, bridges and, possibly, water supply. He died shortly after 1453. This is almost all we know for certain about Taccola's life as it relates to his actual technical experience.[31]

During his life, Taccola prepared two technical manuscripts. The first is generally referred to as *De Ingeneis*, and was to have consisted of four books on civil and military technology. Books 3 and 4 were finished for presentation to the Emperor Sigismund in 1433. Books 1 and 2 were never completed. Fifteen years later, however, Taccola set to work on a second manuscript, variously known as *De Rebus Militaribus* or *De Machinis*, completed in 1449. What is significant for our purposes is that Taccola worked out many of the ideas he presented in *De Machinis* by filling the unfinished pages of Books 1 and 2 of *De Ingeneis* with hundreds of rough sketches, turning them into a sort of notebook. Examining these sketches and comparing them to the drawings in *De Machinis* we are able to follow a person actually working out technical ideas for the first time in history.[32]

An example of one of Taccola's final drawings from *De Machinis* is shown in figure 2.13. It shows two ships, one with a rotating stone dropper, the other with a sort of swinging-boom "fire dropper." In both examples we note Taccola's adherence to the principle of one machine, one drawing, and the absence of measurements, clearly locating these drawings in the mechanical tradition of design we have been examining. That is to say, Taccola does not expect to make these machines himself. He expects they will be made by experts who can complete the design and determine final dimensions for themselves. To these experts it is necessary only to indicate the parts and their arrangement.

Yet one has to acknowledge that Taccola accomplishes his goal in a strikingly different way compared to what had been done before. Gone is the flat style of the Middle Ages. Looking down from above right, Taccola is able to show us three sides of his ships at once so that they have volume and take up space. He adds to this sense of three-dimensional mass and volume through the use of shadowing. Furthermore, in place of the "flat" style, we finally have the adoption of the idea of a single viewpoint to go with the idea of a single drawing of a single machine, showing the overall appearance of the machine as well as the appearance of the parts at the same time.

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31 For Taccola's life and works see Taccola 1972. For more on the context of the Sienese engineers and further details of Taccola's life, see Galluzzi 1993.

32 The first two books of *De Ingeneis*, also comprising the "notebook," are found in Taccola 1984a. The third book is found in Taccola 1969. Two different manuscript versions of *De Machinis* are found in Taccola 1984b and Taccola 1971.

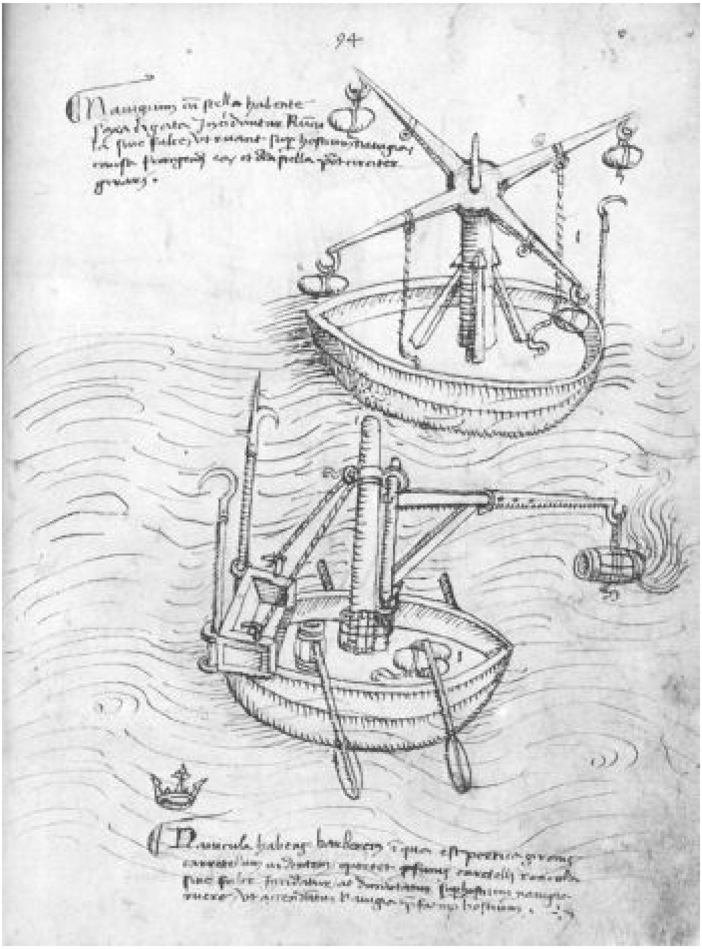

Figure 2.13. A page from Mariano Taccola's *De Machinis* of 1449. (Munich, Bayerische Staatsbibliothek, Codex Latinus Monacensis 28800, fol. 94^r; reproduced in Taccola 1971.)

What brought about this change in style? It has been said that Taccola's *De Machinis* is not really an engineer's book but a prince's book, the implication of this statement being that Taccola's drawings are presentation drawings, intended to entertain as a form of technical fantasy, and that is why they have the properties they do.[33] To a certain extent, this would be in keeping with the trend of development seen in Vigevano and Kyeser, where the attempt to satisfy the needs of the patrons resulted in a shift away from icons to the depiction of machines as they would appear. However, this cannot be the whole story. Figure 2.14 is a page from Taccola's notebook. It contains preliminary sketches for the droppers presented in *De Machinis*. These drawings have the *same* properties as the finished drawings—one machine, one drawing, no measurement, single viewpoint, etc. That is to say, Taccola used the same style to work out his ideas as he did to present them, and hence the story about a presentation style is at best incomplete. Further consideration of Taccola's preliminary drawing style is called for.

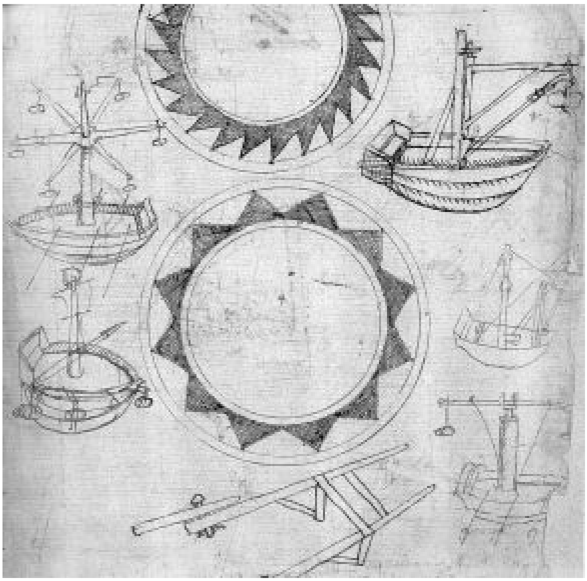

Figure 2.14. Sketches of "droppers" from Taccola's notebook. (Detail of Munich, Bayerische Staatsbibliothek, Codex Latinus Monacensis 197 Part 2, fol. 3^{r}; reproduced in Taccola 1984a.)

33 Long 1997.

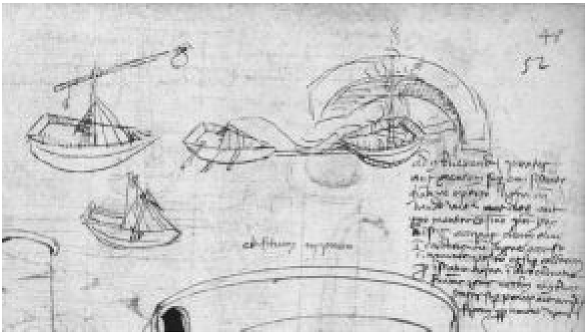

Figure 2.15. Details of folio 52^r from Taccola's notebook, showing several sketches of ships with protective shields, and one ship with a "grappler." (Detail of Munich, Bayerische Staatsbibliothek, Codex Latinus Monacensis 197 Part 2, fol. 52^r; reproduced in Taccola 1984a.)

Assuming the explanation for Taccola's new style must lie in what it allowed him to *do*, we can pursue the issue by turning to another page of the notebook (figure 2.15). Here we see that Taccola has sketched three different kinds of protected attack boats: one with a stone dropper, one with a ram, and one with a large hook or "grappler" on the side. We immediately see that his technique has enabled him to quickly generate three alternatives. Using paper, he is able to store them. Stored, they can be *compared*. In short, Taccola's style provided him with a graphic means of technical exploration.

In this particular case we can attain a closer understanding of the nature of this exploration by turning to another folio (figure 2.16), where the "grappler" boat shows up again in the lower left. This sketch still has the grappler on the side, which is to be dropped by a pulley. In the drawing immediately above it, the grappler is moved to the bow, attached to the boat by a stoutly constructed hinge, and dropped by a weighted, overhead boom. In the sketch above that, Taccola keeps the bow grappler, adds one to the stern, but removes the protective shielding while he considers a system of dropping the grapplers from double pulleys in the mast. The double-pulley mast is then retained in the sketch at the lower right, where one grappler is replaced with an assault ladder and the protection replaced. What we see here is that Taccola's technique does more than allow the rapid generation of alternatives. It allows a systematic holding of some elements steady, while others are explored in successive iterations of the design.[34] More than exploration, Taccola's style provides him with a method of systematic investigation through variation.

34 The importance of doing this is stressed in Jones 1970, 22.

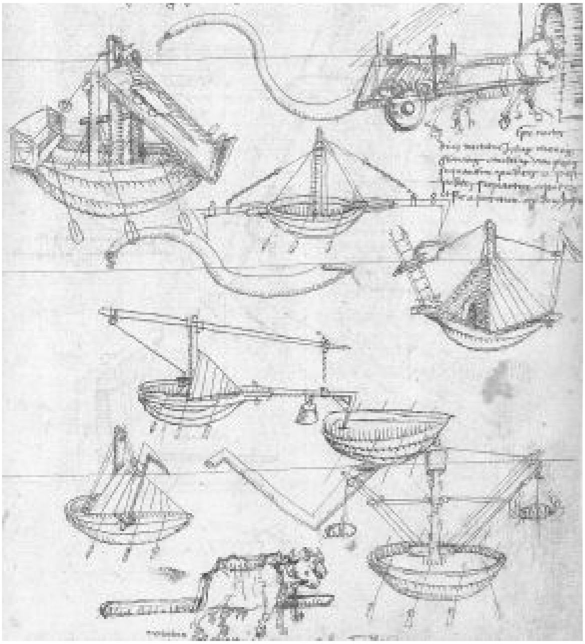

Figure 2.16. Detail of folio 60^{r} from Taccola's notebook, showing several sketches of ships with "grapplers." (Detail of Munich, Bayerische Staatsbibliothek, Codex Latinus Monacensis 197 Part 2, fol. 60^{r}; reproduced in Taccola 1984a.)

But systematic investigation of what? To what considerations are Taccola's variations a response? To answer such questions, it is helpful once again to consider the context of construction—this time the seemingly simple fact that by drawing ships on paper, Taccola does *not* have to go to the trouble of actually making them. Using paper, he does not have to deal with the real costs of labor and materials. Indeed, he does not have to deal with real materials at all, and he can also work in the absence of real dimensions.[35] Thus, we see the masts of his grapplers rise out of the various boats in figure 2.16, but Taccola does not have to be concerned with how big the masts, the boat, or the grapplers should really be. More important, working in the absence of real materials and real dimensions, Taccola is able to *work in the absence of real physics*.

35 Jones 1970, 22 emphasizes the importance of being able to work in the absence of real cost and real material constraints.

In the absence of materials, dimensions, and physics, Taccola is able to iterate freely, in both senses of the word. There is no cost of construction and no cost of failure. He is not constrained as the expert maker would be. However, the absence of physics makes it clear that Taccola's variations in design did not result from the consideration of *physical* problems related to issues in mechanics. The information necessary for any such considerations is just what is missing. For instance, given the absence of dimensions, he could not know either factually or intuitively whether his boats would have the necessary displacement to float themselves, let alone to carry the equipment and protection he proposes, with adequate stability. Indeed, in the absence of dimensions, he could not even calculate the problem of the levers around which all his designs are organized.[36]

Instead, it appears that Taccola's drawing technique allows him to pass *beyond* the context of construction to investigate the future context of *use*. Thus, there is nothing physically impossible about placing the grappler on the side of a boat, as Taccola does in his initial sketches (figure 2.16). The pulley arrangement would work. But it is not very likely to be effective in battle, particularly if the enemy ship approaches from the other side. Seemingly aware of this, in his next sketch Taccola considers dropping the grappler from the bow, which could be more effective, although the stern would be unprotected. Hence, the double-grappler of his next drawing. Similarly, there is nothing physically better about dropping grappling hooks from overhead beams, as opposed to pulleys, or double pulleys. Again, the cause of variation appears to be Taccola's awareness of awkwardness or ineffectiveness for the humans who actually have to use them.[37]

It may be significant that this process of systematic variation, iteration, and exploration of the context of use continues into the final drawings of *De Machinis*. Thus, in figure 2.17 we have what almost looks like the original design, but with a grappler at the front using his perfected hinge arrangement and the pulley dropper. In figure 2.18 we have a sort of grappler and assault ladder combined, but a more sensible form of protection than the original pyramid. In figure 2.19 we have a ladder and grappler combination, with a flat deck for protection, while double pulleys on one mast have been abandoned for double masts with a single pulley each. Such a sequence of drawings flowing from the notebook to *De Machinis* strongly suggests that Taccola's "presentation" drawings are actually an extension of his investigative technique, rather than the reverse. If so, it would be possible to argue that Taccola developed his inno-

36 One might claim that Taccola varied his design in response to his *intuitive* understanding of mechanical problems. My response is not to say that this is impossible, but rather that is an argument for which there can be no proof, because the whole advantage of the only evidence available—the drawings—is that they *eliminate* the need for physical considerations on the part of the artist.

37 It may be argued here that I have violated my own injunction and started to discuss what Taccola was thinking. There is a difference, however. Following the approach presented in this paper, one is able to describe Taccola's thinking as sequence of reasoning, and moreover to point to the evidence for assertions about that reasoning. This is quite different from opining about "cognition," or "tacit" knowledge, particularly in that one can argue about the actual evidence at hand, and not about human nature, whether one particular psychological theory is valid and so on. With more space, I would further argue that, while doing may not always be regarded as thinking, thinking may always be regarded as a form of doing.

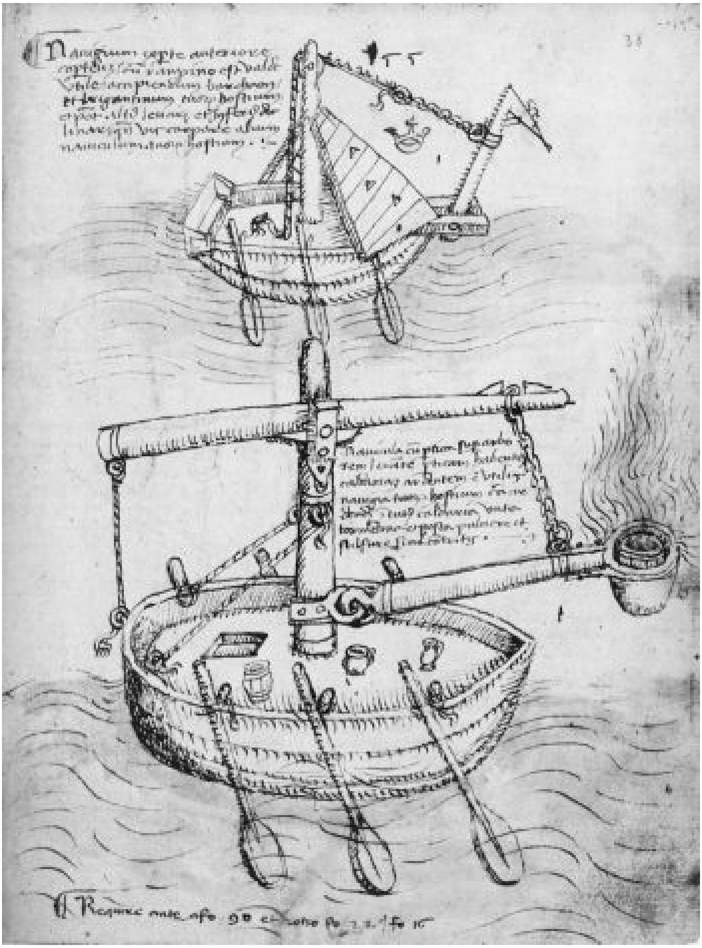

Figure 2.17. A presentation drawing of a "grappler" ship from Taccola's *De Machinis*. (Detail of Munich, Bayerische Staatsbibliothek, Codex Latinus Monacensis 28800, fol. 55^{r}; reproduced in Taccola 1971.)

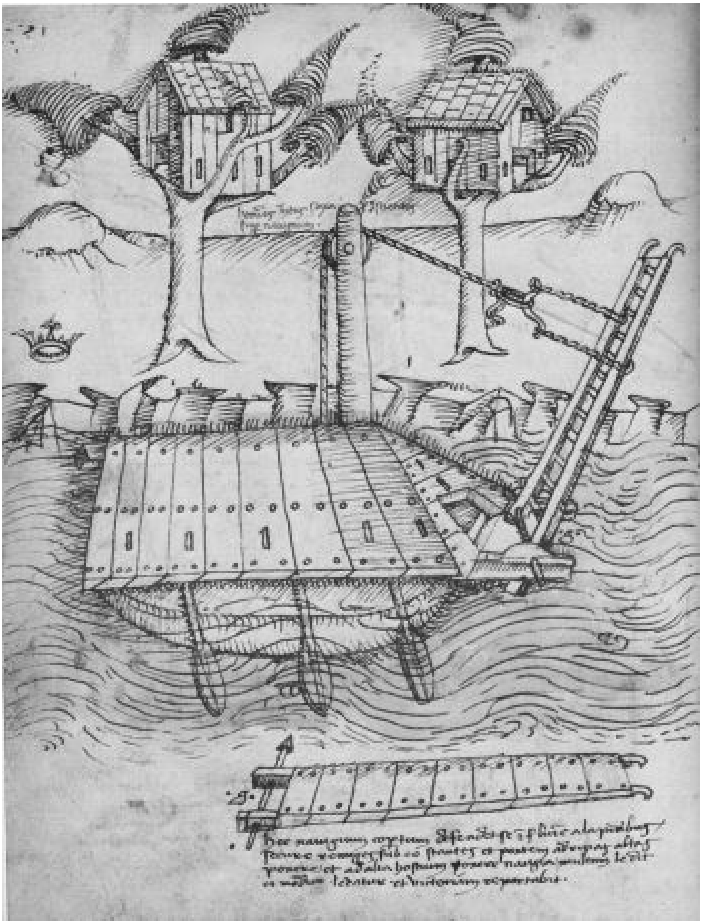

Figure 2.18. A second presentation drawing of a “grappler” ship from Taccola’s *De Machinis*. (Detail from Munich, Bayerische Staatsbibliothek, Codex Latinus Monacensis 28800, fol. 81^{v}; reproduced in Taccola 1971.)

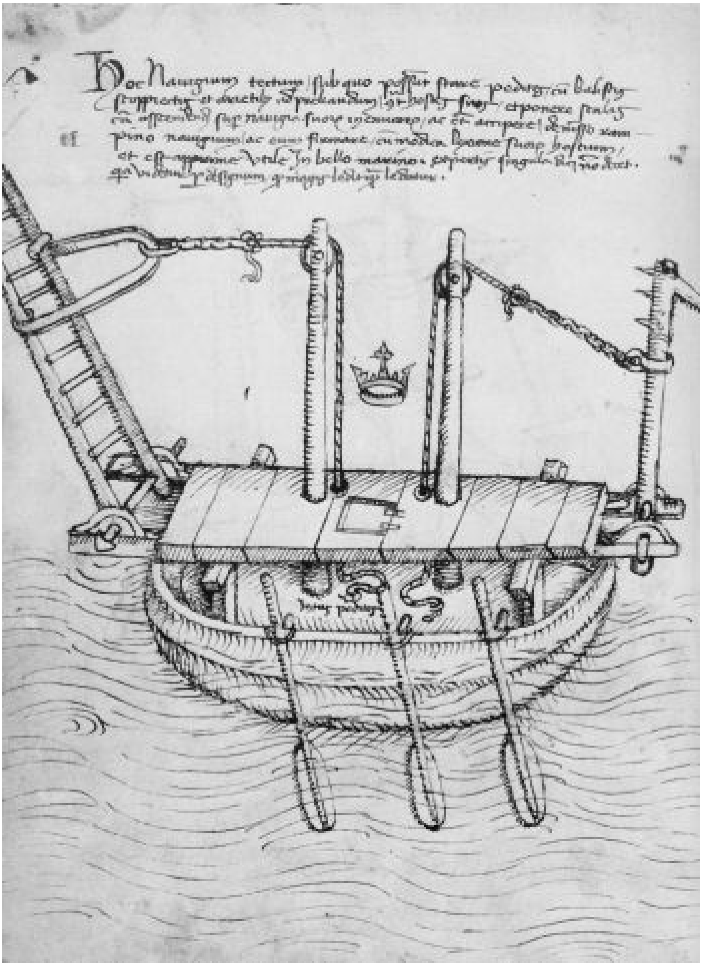

Figure 2.19. A third presentation drawing of a "grappler" ship from Taccola's *De Machinis*. Unfortunately, part of the hook has been cut off the page. (Detail of Munich, Bayerische Staatsbibliothek, Codex Latinus Monacensis 28800, fol. 90^{v}; reproduced in Taccola 1971.)

vative drawing style precisely because of the investigation it involved, rather than for its presentation value. In the larger picture, however, we may at least say that Taccola's search for a means to satisfy the visual needs of his patrons had a momentous, but unexpected result—a graphic technique for investigation and invention that was soon, in the hands of Francesco di Giorgio Martini, Leonardo da Vinci, and others, to become crucial to the development of mechanical technology in the West.

The question arises, of course, as to whether Taccola's technique was really new. Artists of the Middle Ages used representations of appearance to show chairs and buildings, as did the Romans. But we have no evidence to show that they used the technique to work out mechanical ideas. Is this because they did not do so? Or because the evidence is lost? My own opinion is that the technique used by Taccola *is* new in its application to machine design, and the evidence suggests he adopted it because of the kind of mechanical investigations it allowed.[38] Indeed, it may even be the reason for the great interest in Taccola's manuscript by those who consulted and copied it, including Giorgio Martini and Leonardo—not for the machinery depicted, but for the revolutionary manner of depicting the machinery.

5. THE SOCIAL GROUNDS OF CREATIVITY

This chapter began with a question about the stable relationship between machine design and the use of one machine, one view drawings over the period 1400 to 1750. We are now in a position to explain this relationship in terms of the creation of a powerful "paper tool" in the early decades of the fifteenth century, as used by Taccola, if not created by him.[39]

To be sure, the use of one drawing for one machine was already part of a tradition stretching back into the Arabic and Byzantine past, when the flat style could be used to communicate to workers, and perhaps to the learned readers of ancient treatises on mechanics.[40] But in the fifteenth century, a switch away from logical schema towards the depiction of appearance, followed by the adoption of the viewpoint (before the invention or at least diffusion of linear perspective), resulted in the creation of a style of drawing that could do three things at once. First, it continued to supply enough information to expert makers that a machine could be constructed, even though the new kind of drawing was no more complete than those made in the flat style.[41] Second, the new kind of drawing provided information that was comprehensible to patrons, demonstrating not only the mechanical working of the machine, but its purpose, and its method of use. Third, as is clear from Taccola's notebook, the new method provided a new and powerful tool of investigation. Not a tool of invention,

38 Although its origins may lie in the painterly techniques of Giotto and Duccio. If so, then we might look to explain Taccola's use of his technique in terms of the artists' timely possession of a new kind of tool in a changing context of patronage, where both machine building in the service of the state and humanistic investigations of ancient technologies were extremely serious business.

39 See Goody 1977, and Klein 2003.

40 Again see Lefèvre 2002. But also see Hill 1996, and Jazari 1974.

41 As Popplow points out in his chapter, even Schickhardt's highly detailed drawings intended for his workers left out critical information about gears.

one should be careful to say, since Vigevano was clearly capable of inventing in the flat style, but certainly a tool of more *rapid* investigation of combinations and their use in future contexts. The somewhat halting development of rolling shields in Kyeser's manuscript, as opposed to Tacola's rapid development of several kinds of assault ships, is illustrative.

Another issue raised in the introduction was the "Princes problem," arising from the fact that many early manuscripts were intended for a princely audience. What was the effect of this audience on mechanical drawings? The explanation offered here is that the need to communicate more clearly with patrons drove a change from the depiction of logical, iconic schemas to the depiction of appearance. This development was under way before the invention of linear perspective. However, this was not the only factor in play. The evidence of Kyeser's *Bellifortis* suggests that the method of depiction was not simply borrowed from existing courtly styles of painting and illumination. Rather, as the sequence of rolling shields suggests, the method of depiction had to be worked out, step after step. But as the drawings in Kyeser's manuscript also show, one of the most important constraints on the development of the new style was the need to preserve the ability to communicate parts and arrangement to makers. It is not that there was only one audience for these drawings. There were always two.[42]

This chapter also started with a plea for coming to grips analytically with the evidence provided by the properties of early machine drawing. I've tried to show that there can be a benefit in doing so, in the form of three perhaps surprising results.

One surprise is that, although we tend to regard drawings as evidence of thinking, as extensions of cognitive processes of design in the mind's eye, the properties of early machine drawings actually point more immediately to what might be referred to as a social process of design in which the second person is the real expert, the person who actually determines the final dimensions of the machine that is made. This is a fundamental context of machine design that would remain the same for several centuries after Taccola. Indeed, mechanical drawing would only begin to change in the late eighteenth century when the shift from wood to metal, and the holy grail of replaceable parts, set off a determined attempt to gain control over the hands of machine makers.[43]

The second surprise is the methodological one, in which consideration of the context of construction proves to be crucial to understanding the properties of the machine drawings we see—even though it may well be that the machines were never made. This is important, because we truly need to remind ourselves that design drawings are really about what is to happen after them, not what went before them. It is in this context of what *will* happen that they are made. Surely it ought to be important in trying to interpret them.

The third surprise follows from an examination of Taccola's use of drawings to investigate new kinds of machines, investigations that seem much closer to the cogni-

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42 This only makes sense, since a prince might at any time order a machine from a manuscript to be built, and the authors of drawings were, in most cases, not capable of actually building machines themselves.

43 See the discussions in Alder 1997; Baynes and Pugh 1981; Noble 1984; Smith 1977.

tive processes of design as we normally conceive them. But even here, consideration of construction proves important, leading to the realization that Taccola used these drawings precisely because they meant he did not have to build experimental machines, but was free to innovate as he pleased precisely because he was able to work in the absence of the real constraints of dimensions, materials, labor and physics.

Now on the one hand, it seems to me that the absence of physical considerations in the drawings creates big problems for anyone hoping to conscript Taccola and other early artist-engineers as conceptual builders of the scientific revolution. Neither formal physics nor even intuitive physics are there in the evidence. On the other hand, it seems to me that the lack of physics and the evident impossibility of many early machine drawings has often misled us into thinking that they are merely forms of creative play, fantasy—in other words, just fooling around.

There is, however, another way to look at it. If we bring our expert maker back into the design process, we realize that real constraints are not actually gone, so much as displaced into the future—and, particularly, that they are displaced onto the shoulders of another person. It is the job of the expert builder not only to make the machine, but to make it work, meaning that dealing with the physics was the problem of the builder, and not of someone like Taccola. If so, and if it is the absence of constraints that is the key to innovation on paper, it is possible to suggest that the social situation was the very ground of the individual creativity we see in early machine drawings, and that it was the recognized existence of expert makers (so often disparaged as mere craftsmen) that created the very space in which the Renaissance design of machines on paper was possible.

3

SOCIAL CHARACTER, PICTORIAL STYLE, AND THE GRAMMAR OF TECHNICAL ILLUSTRATION IN CRAFTSMEN'S MANUSCRIPTS IN THE LATE MIDDLE AGES

RAINER LENG

INTRODUCTION

In the late Middle Ages a number of well-known authors created illustrated technical manuscripts that are by now acknowledged not only by technical historians. The sketch-book of Villard de Honnecourt,[1] the *Texaurus Regis Franciae* by Guido da Vigevano,[2] Konrad Kyeser's *Bellifortis,*[3] and the writings of Mariano Taccola[4] are available through editions and widely quoted in scholarly literature.[5] Villard's sketch-book and a part of Taccola's works are recordings with a more or less private character. The other manuscripts are characterized by dedications to high-ranking aristocrats. Guido dedicated his *Texaurus* to the French king Philipp VI (1328–1350), Konrad Kyeser bestowed his *Bellifortis* on two German kings, Wenzel (1376–1400, † 1419) and Ruprecht I (1400–1410), and Mariano Taccola prepared a luxurious copy of *De ingeneis* for King Sigismund (1410–1437).[6]

Hence the best-known pictorial manuscripts can be counted among the courtly literature. As David McGee remarks,[7] the special relationship between the author and the addressee has consequences for the way in which technical items are presented. Being "not really an engineers' but a princes' book," these pictorial manuscripts must ensure understanding. They therefore usually show the whole device in action depicted in scenery or settings. They focus on illustrative graphic presentation, not on physical function or conveying machine building know-how precisely. Provided that there are explicit technical contents to be conveyed, there must have been a further recipient besides the addressee, a real expert who knew about the practical dimensions neglected in the drawings.

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1 For a modern facsimile edition, see Hahnloser 1972; the most important literature is discussed in Binding 1993, 207–224, Barnes 1982 (bibliography), and Bechmann 1993.

2 Vigevano 1993; for author and work see also Hall (A.R.) 1976a, Hall (B.S.) 1978, Hall (B.S.) 1982c, Alertz 2001.

3 Kyeser 1967 from Ms. Göttingen Philos. 63 and Kyeser 1995 from Ms. Göttingen Philos. 64 and 64a; see Berg and Friedrich 1994, Friedrich 1996 and Leng 2002, I 109–149.

4 *De rebus Militaribus (De Machinis):* Taccola 1984b and 1971, *De ingeneis* and the notebook: Taccola 1984a. For the complete work and the most important literature, see Degenhart and Schmitt 1980–82, vol. 4.

5 For instance Hall (B.S.) 1976b, Hall (B.S.) 1982a, see also Popplow 2001, 251–263 and the chapter by David McGee in this volume, both with further references.

6 For the background see Knobloch in Taccola 1984b, 11 and Degenhart and Schmitt 1980–82, IV 21–27.

7 See the chapter by David McGee in this volume.

This specific tradition of explaining technology for high-ranking laymen by means of drawings can be found throughout the entire fifteenth century. Besides the well-known books mentioned, this is also apparent in two unpublished manuscripts dealing with the art of warfare, which were produced for the emperors Sigismund (1410–1437) and Frederick III (1440–1495).[8] Even the first printed technical book—Roberto Valturio's *De re militari*, dedicated to Sigismundo Pandulfo Malatesta (1417–1468) (publ. Verona 1472)—is adapted to the simple rules of explaining machinery in settings.[9] In a large series, not even a step-ladder is drawn without the wall it leans on, nor a pump shown without a stretch of water.

Less known, however, is that a number of pictorial catalogues appeared soon after the year 1400,[10] written by those very skilled expert makers onto whose shoulders authors like Kyeser or Taccola displaced the practical realization of their technical suggestions "without physics."[11] On the "social scale"[12] of authors of manuscripts conveying technical knowledge, these experts are ranked at least two levels lower than Kyeser and the other celebrities mentioned above. And this holds for their audience as well. They are mere craftsmen, but rare specialists in their métier, who put pen to paper for the first time to write down their knowledge in words and sketches. This fact raises a number of questions. How did disparaged tradesman come to writing, given the fact that proficiency in craftsmanship had obviously coped without written manuals for centuries? Did Vigevano or Kyeser supply them with examples, or did they start drawing on their own? Are there significant differences in the pictorial means to those of the technical literature for the courtly context? What were the social circumstances or backgrounds of these catalogues, what function did they serve, which readers did they address? And finally: what kind of expressiveness did their graphic techniques display and how can their development be explained? If technical drawings are part of a universal graphical language to transmit technical knowledge, what is the grammar of this language?

1. THE BEGINNINGS

The beginnings of craftsmen's analysis of technology through writing and picturing are marked by a sample of four illustrated manuscripts, which can be dated to the first

8 Zurich, Zentralbibliothek, Ms. Rh. Hist. 33b, c. 1420–1440, dedicated to emperor Sigismund, and Vienna, Kunsthistorisches Museum, P 5014, c. 1440–1450; for the Zurich manuscript see Grassi 1996, for both directly related manuscripts see Leng 2002, I 221–230 and for a description of the manuscripts, II 315–318 and 417–422.

9 Valturio 1472, mainly a tract about antique Roman warfare, but also with some contemporary devices, was dedicated to Sigismundo Pandulfo Malatesta between 1447 and 1455 and spread across Europe in the form of roughly 20 illuminated manuscripts before printing; see Rodakiewicz 1940 81f., Ricossa and Bassignana 1988, 172, Popplow 2001, 263–266. Almost all of the woodcuttings from the printed edition were used to illustrate the German translation of Vegetius by Ludwig Hohenwang in 1475 (dedicated to the landgrave of Stühlingen), see Fürbeth and Leng 2002, 36–53.

10 Widely ignored for decades and only seldom mentioned, for example in Hall (B.S.) 1979b 21ff. and 127–130; some of these were investigated by Knobloch 1996 in the context of technical drawing. The outstanding figure of Leonardo usually seems to reduce all other authors of pictorial catalogues to the rank of minor figures; for example in Gille 1964 or Parsons 1968.

11 See David McGee in this volume.

12 Hall (B.S.) 1979a, 54.

quarter of the fifteenth century. There are, firstly, two closely related manuscripts, one from Vienna and one from Munich, that contain images of warfare devices. In both cases only the first sheets contain marginal notes or annotations. The Viennese manuscript dates to 1411.[13] The Munich *Anleitung Schießpulver zu bereiten, Büchsen zu laden und zu beschießen*, created in the first quarter of the fifteenth century, presents on only 22 raw single leaves a selection of the Viennese manuscript.[14] The missing drawings may have been lost before binding, such that it also might have been a pattern of the Viennese codex. Characters and style are, on first sight, more primitive. In comparison, a third manuscript, also from Vienna and today kept in the Museum for the History of Art, contains extensive text passages in which both single images and series of plain pictures are integrated.[15] It was compiled between 1410 and 1430. The last is a manuscript from Nuremberg, a fragment of a formerly much larger volume, written between 1420 and 1425.[16] It contains a series of pictures of military devices with explanatory rhymes.

The authors remain anonymous. Neither their names nor the names of their addressees are known. In all four cases the composition is very simple: standard letters, wash drawings, paper instead of vellum, ordinary flexible bindings. Only the Viennese cod. 3069 has a vellum-bound wooden cover. It also contains several pictures from the *Bellifortis*. Therefore we may assume that parts of it were written for an addressee at court;[17] at the very least, the author must have had access to a courtly library. The main part of the codex is based on early craftsmen's material, and even the *Bellifortis* pictures are selected according to technical criteria. All scenes containing mythological or literary items have been sorted out. However, the other three manuscripts of this sample were issued by anonymous craftsmen, i.e. by master gun-makers, who were experts on modern military technology in the fifteenth century. These few early manuscripts constitute the roots of an independent literary species of illuminated manuscripts about military technology that were compiled by experts, away from the courts. This tradition remains vital throughout the whole fifteenth and early sixteenth century. About 50 of them are still preserved today.

The spread of these early pictorial catalogues and of all subsequent master gun-makers' books of the fifteenth century was concentrated in the South of Germany. Although gunpowder technology had been common knowledge in Europe since the first decades of the fourteenth century, and many German master gun-makers served in other countries, no comparable manuscripts have been found in Italy, France or England.

The tradition of master gun-makers' drawings was unique and engendered independently of the older machine-drawing tradition. Only single copies of the Latin writings of Villard and Vigevano were distributed in Italy and France—inaccessible for the uneducated practitioners. Taccola, on the other hand, started with his machine

13 Vienna, Österreichische Nationalbibliothek, cod. 3069; see Leng 2002, II 334–336 and I 172–180.
14 Munich, Bayerische Staatsbibliothek, cgm 600, ed. Leng 2000.
15 Vienna, Kunsthistorisches Museum, P 5135, see Leng 2002, II 319–323 and I 180–195.
16 Nuremberg, Germanisches Nationalmuseum, Hs. 25.801; see Leng 2002, II 266f and I 191–194.
17 See Leng 2000, 25f and Leng 2002, I 179f.

drawings at least two decades after the first master gun-makers' books. The only one from whom they possibly could have benefited was Konrad Kyeser, who wrote his *Bellifortis* six years before the first precisely dated master gun-makers' book in the Viennese cod. 3069 of 1411. But among the four eldest manuscripts, only one has a distant relationship to the *Bellifortis* as mentioned above. But Kyeser's drawings did not inspire the anonymous author: the first part of the manuscript is completely independent. All pictures derived from the *Bellifortis* are later additions to the second part of the codex, and none of them is listed in the opening table of contents on the first two sheets.

Although the master gun-makers' tradition was unique, these craftsmen did not, of course, "invent" drawing in general for their purposes. Some of the characteristic features of the older courtly tradition, such as the use of different viewpoints or the depiction of integral devices on one sheet, strikingly similar conventions or rules, can be found in Villard's sketch-book as well as in Vigevano's and Kyeser's illustrated manuscripts on war.[18] But considering the fact that no master gun-maker had ever seen them (during their first attempts) it is plausible to assume that they used commonly known pictorial techniques of the Middle Ages, which could be seen in any illustrated book, in church glass and murals. These common elements of a pictorial language, together with their very special everyday experiences with single rough drawings for new constructions, constituted the foundation for the first pictorial collections by craftsmen in complete, unique and—as opposed to single sheet files—lasting manuscripts.

2. THE SOCIAL FUNCTION OF CRAFTSMEN'S MANUSCRIPTS

Immediately at the beginning of each manuscript we find drawings or texts with technical content. None of the authors seems ever to have thought it necessary to address his readers or to explain the purpose of his writing. Thus, the social function of these writings has to be inferred by analyzing the social status of the author and by interpreting the text and illustrations. We learn more about the master gun-makers by studying their contracts.[19] There is an increasing number of such working contracts from the 1370s onwards. Most of the master gun-makers had been working as blacksmiths or metal-workers in medieval urban settlements. They left their traditional workplaces in order to specialize and thus to climb the social ladder. They often changed employers, they were very well paid and relatively rare. Lords and cities depended on their detailed know-how of chemistry and weapon technology. Their high mobility, their specific and valuable knowledge were the social basis of the first master gun-makers' codices.

For centuries, technical know-how had been passed on orally in a settled urban tradition. The solitary traveling gun-maker could not rely on such oral tradition. His precious knowledge was always in danger of being lost in perilous military engage-

18 See the chapter by David McGee in this volume.
19 See for the following Leng 1996.

ments. Only by setting down it on paper could master gun-makers ensure that their knowledge be preserved enduringly for journeymen, apprentices and lords. Knowledge, as well as times, were changing: Military technology was developing quickly, instructions for the production of powder and additional substances became increasingly complicated, and their number was increasing. Master gun-makers were no longer able to memorize those instructions. The *Fireworkbook of 1420* had already demanded that "Der Meister sol auch kennen schreiben und lesen."[20] Obviously there had been frauds among the traveling masters. Those who were able to prove their knowledge by producing written evidence were more likely to find employment.[21] For these experts, the way out of the "sub-literate groups"[22] into literacy was not too far and, in any case, of big advantage.

At the beginning there may have been only single sheets, notes about gunpowder recipes, single sketches of technical devices. These early documents of master gun-makers' technology have been lost. Today we can trace them back only by studying the codices that were compiled later from this early material. Looking at the oldest codices one can discover their relationship to those single-file collections.[23]

Apart from the task of passing on innovative knowledge, and from a partial presentational function, these writings had one main social intention: the exchange of technical know-how. Communication among master gun-makers took place by way of books: Master gun-makers mention their colleagues' books, and they find themselves mentioned, too: "… ze einer bedutnws eins andern puochs …";[24] "… diesen sinn den suoch / in dem andern buoch …"[25] Wherever single characteristic drawings can be found in more than one manuscript, conclusions can be drawn about the relationship between these codices and, thus, the exchange of drawings can be assumed.

Certainly the relationships among images of commonly known devices is hard to prove. But the Viennese manuscript cod. 3064, which can be dated to around 1440, is a nice example of parallel intertextual communication: in the inserted gunpowder recipes, the writer names his colleagues who had invented them.[26] It seems that the anonymous author was in close contact to his colleagues in the south of Germany, and they seem to have exchanged recipes quite frequently. The high mobility of master gun-makers and their regular meetings at the major sites of military conflict, for which large numbers of this trade were called together, favored such a literary communication. It is likely that on such occasions sketches were exchanged as well. This

20 Hassenstein 1941, 16: "... wann der stuck souil sind die darzu gehoerendt / die ein yetlicher guetter püchsen meister künden soll / vnd die ein mayster on die geschrift in seinem sinne nit gedencken kann ..." See also Hall (B.S.) 1979a, 47–58.
21 See Schmidtchen 1990 30, Leng 2002, I 105f.
22 Hall (B.S.) 1979a, 48.
23 See Leng 2002, I 151–154.
24 Vienna, Österreichische Nationalbibliothek, cod. 3069, fol. I^{r};—see Leng 2000, 25.
25 Nuremberg, Germanisches Nationalmuseum, Hs. 25.801, f. 15^{r}—see Leng 2002, I 193.
26 Vienna, Österreichische Nationalbibliothek, cod. 3064, f. 9^{v} "magistrum conradum," f. 10^{r} "ulreichs mawrers puluer," f. 10^{v} "mayster johannes von Österreich," f. 12^{v} "also zugelt der marckgraf von rotel Salpeter" (who is also mentioned in Cologne, Archiv der Stadt Köln, W* 232 f. 84^{r} and Frankfurt, Bibliothek des Instituts für Stadtgeschichte, Reichssachen Nachträge Nr. 741, 29), f. 16^{v} "magistrum iohannem," again f. 25^{v} "magistrum iohannem de austria" and f. 26^{v} "secundum iohannem." For a description of the manuscript see Leng 2002, II 331 and I 238.

explains why some characteristic drawings can be found in a number of pictorial catalogues all of which are not mere copies, but individual sketch-books.

The openness with which the master gun-makers apparently communicated their know-how and even inventions is remarkable.[27] Of course, issues of secrecy arose on occasion. The patrons of these experts were interested in spreading their knowledge. In the case of the master gun-maker's death or absence, other servants or citizens should be able to serve the ordnance, too. Therefore some contracts obliged the master gun-makers to teach few persons about shooting and the production of gunpowder.[28] But sometimes this produced undesirable competitors. In 1468 the master gun-maker of Lucerne claimed damages from the council, because some citizens, to whom he was obliged to convey his knowledge, had started producing gunpowder on their own. He complained about the reduction of his income.[29] For such reasons master gun-makers could be interested in maintaining secrecy as regarded their monopoly of know-how within their closer sphere. On the other hand, master gun-makers were cleared for classified military information. No one knew better the attack or defence potential of their patrons. Because of this, the employers themselves were vitally interested in forbidding the conveyance of knowledge, and included strict clauses in their employment contracts. Problems of secrecy occurred for economic or military reasons, but they never seem to have prevented the transfer of technological know-how in drawings among the master gun-makers. The group of skilled experts was too small and too scattered—normally one master gun-maker per city!—for serious competition. It seems they felt themselves to be colleagues obliged to each other by their dangerous and exclusive profession more than as distrustful competitors.

The early production of illuminated technical manuscripts by craftsmen, who were not writing for the courts but for themselves and for their colleagues, is autonomous. It is not influenced by the presentational manuscripts such as those produced by Kyeser and Taccola. No master gun-maker would have understood the ambitious, literary Latin hexameters of the *Bellifortis*! Besides, the audience these craftsmen address is a different one. The drawings serve as a memory aid in the first place, but they also pass on knowledge to journeymen, apprentices, and colleagues. For the first time we encounter texts and pictures that have been produced for communication between skilled experts.

3. PICTORIAL STYLE: PRIMITIVE OR FUNCTIONAL?

The pictorial style, especially that of the early master gun-maker books, often has been contemptuously called "crude" or "primitive."[30] Of course, the style of the sketches is rough. The simple feather drawings couldn't compete with the magnificent book illumination of Taccola or Kyeser. But as they were not meant for courtly

27 For the problem in general, see Long 2001, especially 117–122 about openness and secrets in writings on gunpowder artillery and machines, including some of the above-mentioned manuscripts.
28 For some examples see Leng 1996, 314f.
29 See Hess 1920, 22.
30 Gille 1964, 49f. and even Hall (B.S.) 1979b, 21f.

recipients or addressees, they didn't have to compete with such illustrations. However, if they served the communication between specialists, they had to employ graphic techniques that permitted the transmission of knowledge by means of drawings. In some aspects these techniques did not work smoothly, of course, and cannot be compared with modern visualizations of technical devices. However, this must be considered in light of the fact that these uneducated authors had just stepped out of an oral tradition in the direction of technical literacy.

Even in a first perusal of the craftsmen's manuscripts, one is struck by a significant difference from the presentational manuscripts: only the single device is depicted, and this—in contrast to the courtly manuscripts—without any context, scenery or settings. This didn't mean reducing the pictorial means by dispensing with unnecessary information. While the presentational manuscripts needed an illustrative context to demonstrate the function and the purpose of the apparatus to the courtly audience—all mechanical laymen—this contextual information was not necessary for communication among technicians.

An example taken from one of the oldest master gun-makers' sketch-books illustrates this fact (figure 3.1): The author outlines a piece of artillery with multiple tubes that is meant to improve the slow rate of fire. A scene to illustrate the employment—targets, walls, enemies, as we would expect in the presentational manuscripts—is not necessary. The skilled expert was able to imagine the advantage at first sight. Instead, the raw sketch conveys additional information: A massive block mount is necessary to bear the weight of the tubes. The four tubes must be installed in radial order on a wooden disc and reinforced with two cross-shaped beams. The figure of a gunner revolving the disc explains its function. The tubes are attached to the beams by clamp straps. The disc is fastened on the mount with an iron screw. Thus the horizontal adjustment is variable by means of a spindle. At the bottom of the mount, a device for additional vertical adjustment is indicated, showing that the disc could also be inclined. These indications all give cru-

Figure 3.1. Sketch of a multiple gun using different viewpoints. Drawing from the first quarter of the fifteenth century. (Munich, Bayerische Staatsbibliothek, cgm 600, fol. 13r; Leng 2000, 116.)

cial elements of the construction, enough for another skilled craftsman to figure out and determine the final dimensions of such a weapon on the basis of his individual experiences in machine making. Detailed information about materials, measurements or any special mechanisms could be omitted. In any case, in the age before early industrial mass-production, every craftsman had to develop made-to-measure solutions for each single device to make it work.

By restricting the raw sketch to the essential functional elements of the construction, it was quite possible to convey the knowledge necessary to build a working multiple-tube gun. This functionality of the drawing must be kept in mind by the modern observer, who might experience "visual confusion" because of the drawing's use of multiple viewpoints instead of correct perspective rendering.[31] The mould is shown from a front view, while the disc with the tubes—which is horizontal in reality—is turned about 90 degrees to show it in plain view within the same sketch. Before the introduction of more efficient elevated viewpoints[32] or the spreading of real perspective drawings,[33] the use of multiple viewpoints cannot be judged as pictorial primitivity. On the contrary, it allows a highly effective compression of the data needed to convey technical information by means of one drawing. In contrast to a depiction by modern orthogonal plans, which would demand at least two plans, a late medieval craftsman transmitted the necessary information on only one sheet. And his recipient, used to decoding the multiple viewpoints, had no difficulties extracting all relevant information he needed.

A last comparison demonstrates the difference between craftsmen's and presentational manuscripts (figure 3.2): The early master gun-makers' book in the Viennese cod. 3069 shows a lathe on f. 90^r. The drive, tools, workpiece, even the slide in a variable guide and a storage space for other tools can be clearly recognized. On f. 21^v a more simple device is shown: caltrops, with detailed information about the wooden stakes with iron barbs, and a trick for rendering them harmless. Once more we are confronted with a concentration on the single device, renunciation of superfluous settings and accentuation of functional elements. Based on the material presented in this manuscript, a professional illustrator of about 1450 compiled a codex on warfare technologies for Emperor Frederick III.[34] Here the translation of technical sketches back into a courtly context required a reduction of technical precision in favor of the addition of settings. The lathe is less detailed and was embedded in a workshop scene, and the caltrops had to be integrated in a scenery that shows how they were used to put obstacles in the infantry's way. These drawings appear sophisticated, but they are only useful to a courtly observer. On the other hand, the craftsman, who did not need such contextual information, would miss a detailed depiction of the devices. From his point of view the sophisticated presentation was primitive. He preferred the "crude" but more informative sketches.

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31 See the chapter by McGee in this volume for the viewpoints in Villard and by others.
32 For the development of elevated viewpoints or perspective drawing, see the chapters by McGee and Popplow in this volume.
33 See the chapters by Camerota, Lefèvre, and Peiffer in this volume.
34 Vienna, Kunsthistorisches Museum, P 5014, see above note 8.

Figure 3.2. Top: Devices without scenery. From the *master gun-makers' book* from 1411 (Vienna, Österreichische Nationalbibliothek, cod. 3069, fol. 90^{r} (left) and 21^{v} (right).) Bottom: Illuminated codex for courtly audience with additional scenery. Pictures based on the material of the *master gun-makers' book* from 1411, but dedicated to Emperor Frederick III. (Vienna, Kunsthistorisches Museum, P 5014, fol. 110^{r} (left) and 49^{r} (right); Leng 2002, I plate 12.)

4. DEVELOPING A GRAMMAR OF TECHNICAL DRAWING

With the depiction of complete devices from different angles, the craftsmen and master gun-makers of the early fifteenth century invented a pictorial language that permitted the exchange of technical information. The grammar of this language was simple but functional, reduced to two rules: Show the whole device without any distracting settings, and use as many viewpoints as necessary to convey precise information. The amount and the quality of the transported know-how, however, remained limited. The reader had to take a lot of trouble to translate or even decipher information. Without his own experience as a reference system in the background, these first attempts at a pictorial language had to fall on deaf ears, or rather on blind eyes. There was the serious danger of misunderstanding single elements. With the increasing complexity of technical devices, a sophistication of the rules of the pictorial grammar was inevitable. Analysis, variation, segmentation, and new combinations of single elements were promising means to improve the performance of a pictorial language.

Of course the master gun-makers of the late Middle Ages never expressed in words their primitive rules of communication by means of drawings; only some Renaissance artists attained this degree of reflection.[35] But in an intuitive way they discerned that the once successful process of pictorial communication could be developed further by varying devices, by separating single elements from the integral device, and by extending the catalogue of pictorial rules, in part playfully, in part with concrete intentions. We can find an early example of a prolific attempt to expand the first two simple rules in the already mentioned Viennese manuscript P 5135, compiled between 1410 and 1430. Although the traditional rule of showing only complete devices is predominant, the author in some cases tried out the use of sophisticated pictorial rules. Especially the sketch of the multiple tube we know from the Munich cgm 600 (figure 3.1) apparently was considered to be unsatisfactory. The draft conveyed an approximate idea of the function, but the two viewpoints chosen did not allow the decisive interior mechanisms to be shown in a precise way. Only a master who had built comparable machines before could gain real benefit from this "workshop drawing." But in order to convey the necessary information to a less experienced master or to a journeyman, the precision of pictorial means had to be improved. Distinctly discernible is the attempt to depict separately such components of the whole device (figure 3.3, top left), which until then had been partly concealed or merely indicated. The sketches on f. 93^r–95^r (figure 3.3, top left, bottom left and right) demonstrate such an attempt at pictorial segmentation and variation.

Hidden components or larger parts blocking the observer's view were given their own single sketches for a more precise depiction. In a mental process, the everyday experience of building complete machinery from single components was transferred to improved didactic presentation and the conveyance of technical knowledge among the group of skilled experts. The capacity of pictorial means was increased by adding a further rule of communication: show even the hidden parts in single sketches.

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35 See the chapters by Camerota, Lefèvre, and Peiffer in this volume.

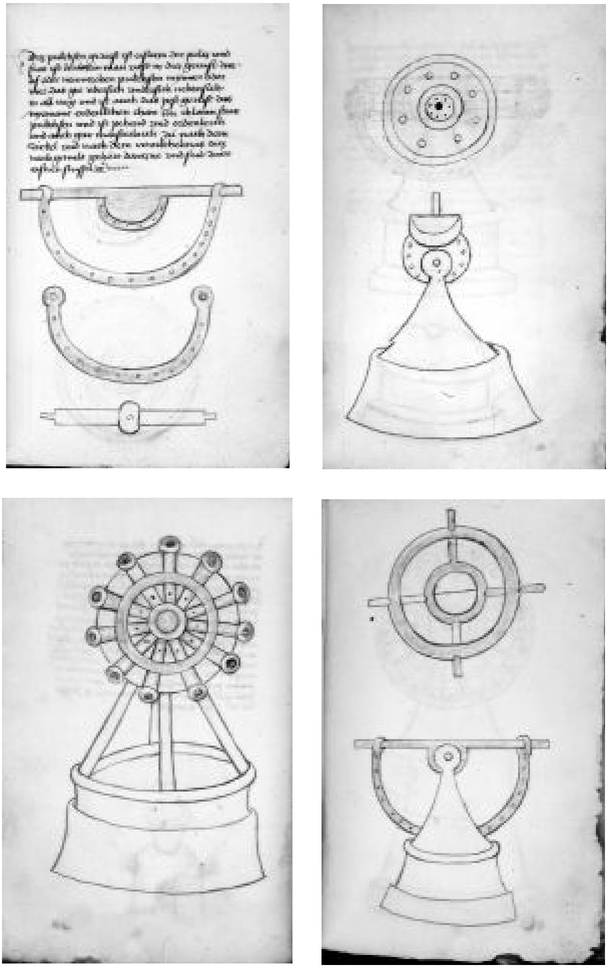

Figure 3.3. Pictorial segmentation and variation, rare examples for depicting single components. From Anonymous, *Pixen, Kriegsrüstung, Sturmzeug vnd Fewrwerckh,* between 1410–30. (Vienna, Kunsthistorisches Museum, P 5135, fol. 55^{r} (top left), fol. 93^{r} (top right) and 94^{v} / 95^{r} (bottom); Leng 2002, I plate 10.)

Although a rare exception among the master gun-makers' books in the first half of the fifteenth century, these few sketches were trend-setting for modern engineering drawings and their custom of dissolving complicated devices into single components and assemblies. Soon after the middle of the fifteenth century a noticeable push in pictorial means is perceptible. Several developments may have stimulated the step to more efficient designs. We can detect technological improvements as well as increased self-confidence among the writing craftsmen, who now began to sign their names on their pictorial catalogues. Gun foundries produced lighter and more powerful tubes.[36] Even conventional arms, the so-called "Antwerk," became more sophisticated by means of improved smithcraft and new woodworking technologies.[37] This resulted in a multiplication of ordnance parks and a great variety of gun types, brought forward especially by the adaptation of gun carriages and foundations for all conceivable purposes. The mounts no longer had to be carved from massive trunks, as we noticed in figures 3.1 and 3.3, but could promptly be composed of prefabricated boards and beams from the wide-spread sawmills.[38] The sophisticated elements of construction are soon reflected in the improved pictorial means of the master gun-makers' books. The increasing number of parts and the more complicated interaction between them obviously reinforced the tendency to enlarge the catalogue of rules for their depiction.

Figure 3.4. Sketch of a multiple gun, using elevated viewpoint and perspective. From Johannes Formschneider, *master gun-makers' book,* Bavaria, third quarter of the fifteenth century. (Munich, Bayerische Staatsbibliothek, cgm 734, fol. 67^{v}; Leng 2000, 52.)

While the sketches of the *Anonymous of the Hussite Wars*,[39] dated to around 1440, hardly knew more than undressed boles, the drawings that the Nuremberg gun-maker Johannes Formschneider created around 1460 show precise wooden constructions in nearly correct perspective view, with detailed depiction of every single beam, pivot hole, nail, and spindle.[40] A comparison between two sketches makes the difference

36 See Schmidtchen 1997, 312–392 and Schmidtchen 1990, 192–210 with further references.
37 Schmidtchen 1990, 210–220 and Schmidtchen 1982, with a large number of figures from master gun-makers' codices.
38 See Lindgren 2000, also with notes about the consequences for pictorial catalogues.
39 Hall (B.S.) 1979b, for the dating of the manuscript and for further literature see Leng 2002, I 231–233.

obvious. Figure 3.1 represents the oldest type: raw construction in multiple viewpoints. The same type of gun (figure 3.4), but half a century later in Formschneiders manuscript, leaves us with a completely different impression. Shape and function of the foundation are clearly perceptible. Even without experience of this special type, but based only on the sketch, a skilled master gun-maker should have been able to build a copy of this multiple-tube gun. Short marginal notes on the purpose of the piece as a turret gun and on the maximum weight of the gun barrels provide further information. But besides more precise technical information, the main development is a new pictorial style. Perspective had changed radically. The alert eye no longer had to rebuild the construction from different viewpoints in a mental process. The new and single viewpoint moved to one point lying on the middle of the axis between the two angles from which the gun had been shown before. That migration enabled the engineering draftsman to maintain a general view of the device while, at the same time, presenting formerly hidden parts.

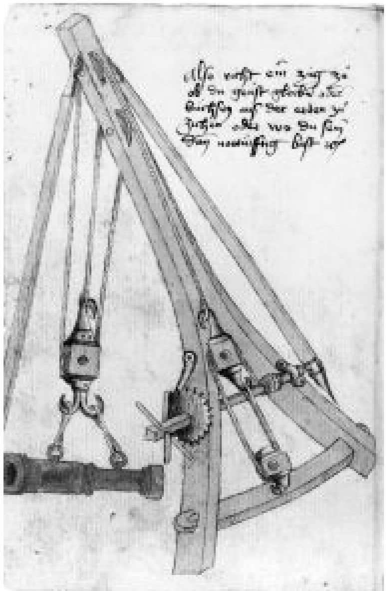

Figure 3.5. Hoisting device with over-dimensional depiction of mechanisms in comparison to the weight. From Johannes Formschneider, *master gun-makers' book,* Bavaria, third quarter of the fifteenth century. (Munich, Bayerische Staatsbibliothek, cgm 734, fol. 61^{v}.)

From then on, the elevated viewpoint and, for the most part, the

40 The first Formschneider manuscript appears to be Munich, Bayerische Staatsbibliothek, cgm 734 (c. 1460), see Leng 2002, II 206–209. On f. 60^{v} the author gave some autobiographical information. He compiled the book by order of the Nuremberg council for his successor, Master Wagmeister. Previously he had served in the Nuremberg armoury for 30 years as a master gun-maker and "adventurer": *Item lieber her wagmeister diese stück hab ich eüch gemacht mer auff fürdrung ewer gnedigen herren dan von des geltz wegen dar vmb bitt ich eüch freuntlichen vnd fleissiglichen mit gantzem ernst Ir wolt euch diese stück enpfolhen lassen sein vnd in rechter guoter huot halten als ich sy dan gehalten hab in meiner huot wol xxx iar in nürnberg ... Johannes formschneider büchsen meister vnd guoter aben teürer.* A more complete catalogue of Formschneider's drawings exists in the later codex Munich, Bayerische Staatsbibliothek, cgm 356 (c. 1480–90), see Leng 2002, II 198–201. Other manuscripts compiled between 1460 and 1500 based on Formschneider's materials can be found in Frankfurt, Stadt- und Universitätsbibliothek Frankfurt am Main, Ms. germ. qu. 14, Gotha, Universitäts- und Forschungsbibliothek Erfurt/Gotha, Chart B 1032, Munich, Archiv des Deutschen Museums, Hs. 1949–258, Stuttgart, Württembergische Landesbibliothek, Cod. milit. 4° 31 and Weimar, Stiftung Weimarer Klassik/Anna-Amalia-Bibliothek, Q 342. For this group of manuscripts see Leng 2002, I 239–249.

optional laterally relocated viewpoint, predominated in nearly all sketches. This came closer to the real perspective already known in Italy. But the conveying of functional principles remained the primary concern, and this sometimes caused violations of the correct use of perspective and scale according to the traditional medieval attitude, so that what was more important could be enlarged. We notice for example in figure 3.5 that Formschneider put special value on the order of pulleys and the correct route of the ropes to make clear the mode of operation of the lifting device. Marginal notes explained the advantage of easily lifting or pulling heavy gun barrels or bells.[41] But more important than the indication of its (evident) use was the over-dimensional depiction of the central elements of pulleys, spindle, gear wheel, and locking spring. Once more, and for good reasons, portrayal of functionality ruled over correct perspective rendering.

With the increasing complexity of mechanical devices, even this pictorial process came up against the limits of feasibility. Again new pictorial means had to be conceived to make the catalogue of drawing rules fit for more intricate tasks. Hoisting devices, for example, were built for ever heavier weights with more and more ropes and pulleys. This could no longer be elucidated properly by means of drawing. In a manuscript from 1524 we find an example for an almost touching but extremely creative attempt to extend and to vary pictorial rules in order to solve this problem.[42] The author was the founder and master gun-maker Christoph Seßelschreiber, who is still widely known for his bronze casting for the mausoleum of Emperor Maximilian I († 1519) in Innsbruck.[43] His book *Von Glocken- und Stückgießerei* contains a set of complicated hoisting devices on f. 69–72. In accordance with the tangled course of the ropes, Seßelschreiber punched the sheets and pulled a number of fine threads through the paper holes representing the real ropes over the portrayed beams and pulleys (figure 3.6).

Of course this was more a didactic peculiarity born out of necessity. But it shows the often playful and inventive method of varying elements of technical communication by means of drawing. However, it proved to be more promising to follow the course of segmentation of machinery we have mentioned as manifested in some sketches from the beginning of the fifteenth century. Even in the comparatively early Formschneider manuscripts we have noticed that the general view of objects appears more and more dissolved. Single parts and mechanisms of central importance for the function are often drawn separately and are sometimes enlarged beside the general view. Figure 3.7 (top) from a Munich manuscript, derived from the Formschneider drawings in the last quarter of the fifteenth century, shows the change in this manner of proceeding.[44] The elevation mechanism hidden in the interior of the gun mould is repeated, dissolved in its single parts, next to the position of its invisible effect. Some

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41 Munich, Bayerische Staatsbibliothek, cgm 734, f. 60v: "Also richt ein zug zu, ob du geust glocken oder büchsen aus der erden zuo zichen oder wo du seiner dann nottürfftig bist. Etc."
42 Munich, Bayerische Staatsbibliothek, cgm 973, see Hartwig 1927, for description and literature see Leng 2002, II 151–154 and I 225–227 with a biography of Seßelschreiber.
43 See Egg 1969, 122 and Gürtler 1996, 88f.
44 Munich, Bayerische Staatsbibliothek, cgm 356, 50f.; see above, note 40.

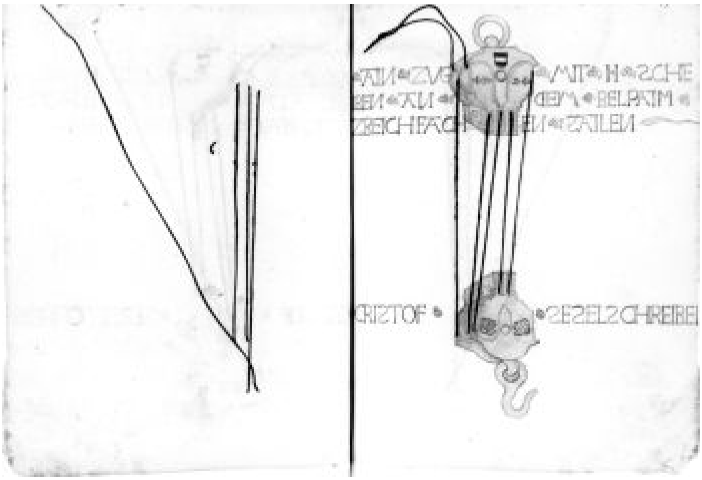

Figure 3.6. Hoisting device with fine threads representing the course of ropes. From Christoph Seßelschreiber, *Von Glocken- und Stuckgießerei,* Munich 1524. (Munich, Bayerische Staatsbibliothek, cgm 973, fol. 70^{v} / 71^{r}.)

marginal notes seemed essential to explain the correct location: "inn das gerüst durch—ausser die stangen." As soon as this manner of depicting separately important but in reality invisible parts was introduced into the syntax of drawing, similar depictions managed to do with reduced forms. The piece of artillery in figure 3.7 (bottom) uses the same gear rack to adjust the height of the tube. Here mere pictorial hints and textual references were sufficient—a process we can call the "introduction of relative pronouns in pictorial grammar."

Later technicians grasped this rule and perfected its handling. The *Buch der stryt*,[45] composed by the Palatinate master gun-maker Philipp Mönch in 1496, who had a particular liking for gear rack and worm-gear drive, shows the separated and enlarged elements of central drive mechanisms on nearly every sheet (figure 3.8).

This tried and tested idea of separating single mechanisms soon seemed to be promoted to a common principle, which again deserved variation and consequent sophistication. Further steps to an enlarged catalogue of rules are apparent in some drawings of the famous *Medieval Housebook*.[46] The identities of both the draftsman of the planets, tournaments and other genre scenes, who decisively influenced

45 Heidelberg, Universitätsbibliothek, cpg 126, for a description see Leng 2002, II 151–154 and I 225–227; Berg and Friedrich 1994, 175, 178.

46 Facsimile and commentary volume ed. by Waldburg 1997; the *Housebook* is private property of the Counts Waldburg–Wolfegg in the castle Wolfegg, Southern Germany.

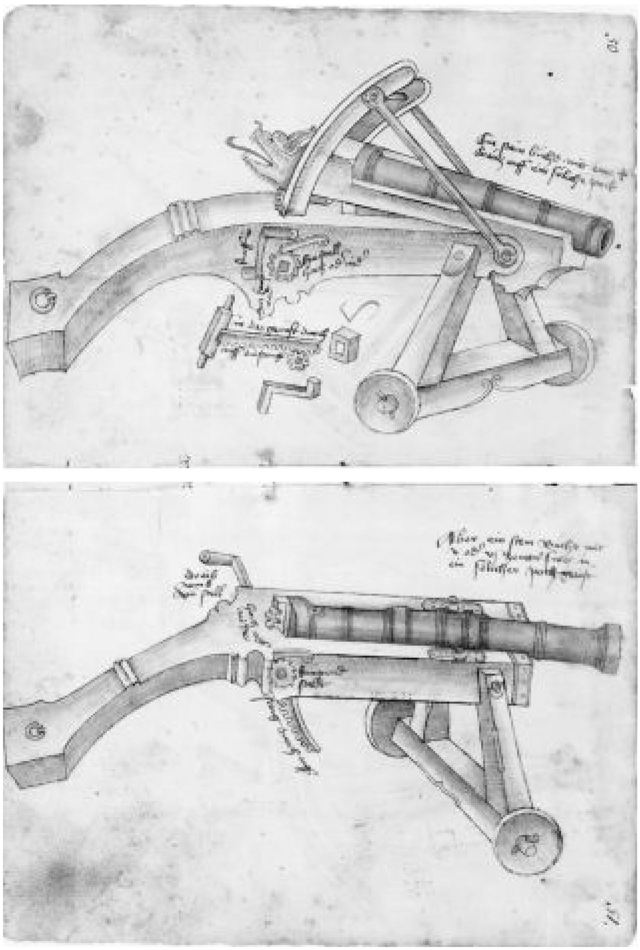

Figure 3.7. Guns and foundations with separately depicted elevation mechanisms and marginal notes. From a later copy of Johannes Formschneider, *master gun-makers' book*, Bavaria, fourth quarter of the fifteenth century. Munich, Bayerische Staatsbibliothek, cgm 356, p. 50f.)

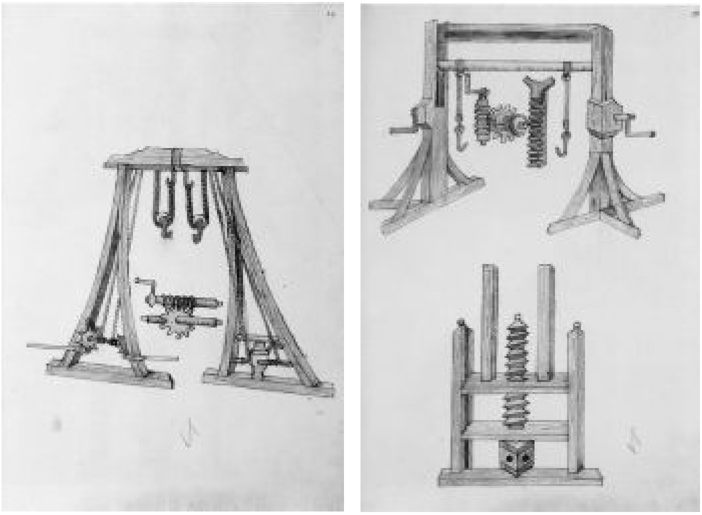

Figure 3.8. Hoisting devices with separately depicted hidden worm-gear driving. From Philipp Mönch, *büch der stryt vnd buochßen,* Heidelberg 1496. (Heidelberg, Universitätsbibliothek, cpg 126, fol. 19^{r} and 20^{r}; Leng 2002, I plate 22.)

the style of Dürer, and that of the patron of the manuscript are among to the great unsolved mysteries of the history of art. Notwithstanding that the courtly contexts put the codex in the milieu of the presentational manuscripts, large parts of it deal with technological questions, produced perhaps by a second, probably engineering draftsman. These mining[47] and warfare materials[48] on the fifth through ninth gatherings are clearly related to the Formschneider drawings. In some sketches the author went beyond the segmentation of mechanisms to something like a modern exploded view, dismantling the machines into all single parts. A battle wagon pushed by six horses, for example, is shown with every detail of the chassis separated (f. 52^{v}).[49] Another battle wagon with a supporting framework on wheels, movable sides and roof is also depicted in all its single component parts (f. 51^{r}). And the sketches of a hoisting device (f. 54^{v}), a thread-cutting lathe with a single slide rest, a tool-holder and a feed in a dovetailed guide, even with a replaceable cutting die (f. 54^{r}) and the components of a traveling screen, equally testify to a true enjoyment of pictorial dismantling (figure 3.9).

47 See Ludwig 1997.
48 See Leng 1997.
49 For the following, see the description of all pictures of the *Housebook* in Waldburg 1997, 42, 48–51.

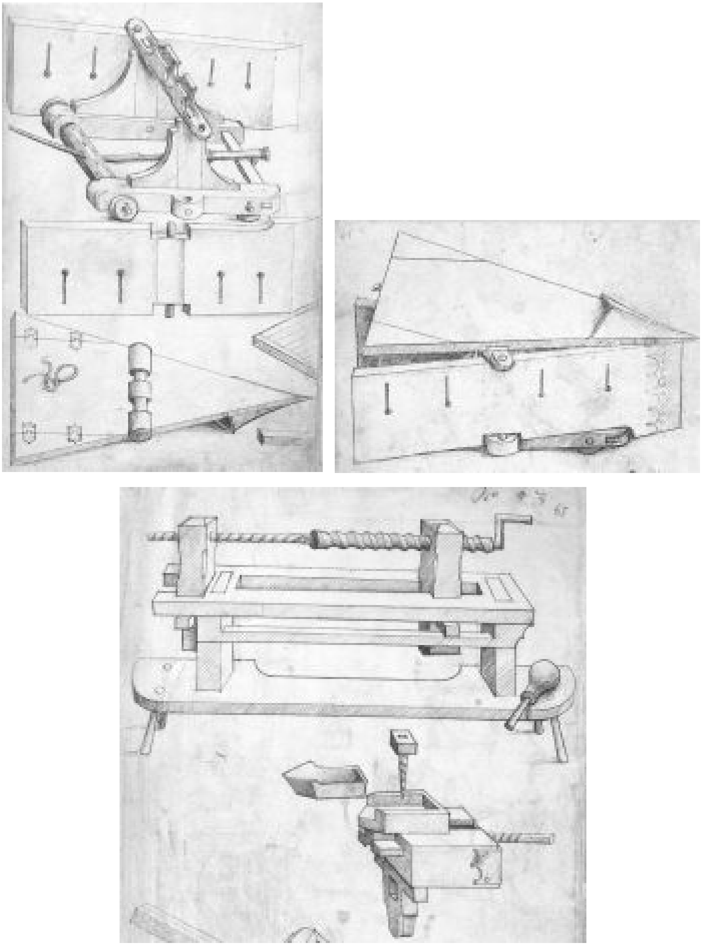

Figure 3.9. Component parts of a battle wagon and a thread-cutting lathe. From the *Medieval Housebook,* Southern Germany, c. 1480.

Other drawing methods became more sophisticated, too. Two sketches of mills, one indirectly driven via a horizontal dropped shaft and pin gears, the other direct via a vertically dropped shaft with bar and counterbalance weight (f. 48^{r+v}) are shown from a perfect elevated and vertically shifted viewpoint. Every single part remains visible to a large extent. Almost nothing is hidden.

Revealing, too, is the pictorial implementation of a large mine with smelting hut, crushing machine and interior for the furrow drain process, which had just been invented at the time (f. 36^r–38^r)[50]. At first an introductory general view shows the complete plan. Then a fictitious round through the fitting-out of the mine follows. The order of the drawings from f. 36^v onwards is absolutely remarkable. Beginning with the source of energy, an undershot water-wheel, the draftsman reveals the whole course of the system of forces following the shafts. Along the main shaft, which drives several side shafts via pin wheels, he leads the observer through the hut. On the way we meet a crushing machine, shaft-driven bellows for the furnaces and a pumping plant (figure 3.10).

Here the pictorial segmentation serves the depiction of various combined technical processes in a large smelting hut, which could no longer be described instructively in one drawing. With the *Medieval Housebook* the segmentation of a pictorial "sentence" about a single device was enlarged and transformed into the segmentation of a complete plant, out of which a pictorial "text" arises.

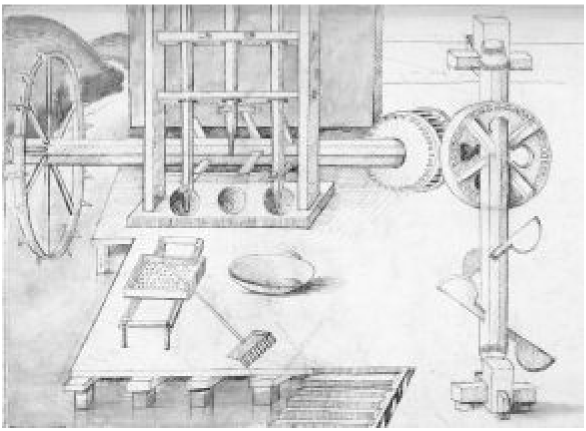

Figure 3.10. Interior of a smelting hut; water-wheel driving a crushing machine with pinwheel transmission of a camshaft indicated, continued on the next sheet with the shaft-driven bellows for the furnaces. From the *Medieval Housebook,* Southern Germany, c. 1480.

50 Described by Ludwig 1997, 127–135.

5. USING THE GRAMMAR

With the introduction of these pictorial elements, the language of technical drawings necessary for communication among outstanding craftsmen in the later Middle Ages was complete. The catalogue of rules as well as the resulting possibilities had evolved continuously since the beginning of the fifteenth century. At first they were restricted to three instructions we can summarize as follows: 1. Show the whole device; don't annoy your beholder by using settings—he is as skilled as you and will recognize the purpose. 2. Use as many viewpoints as necessary to show all important parts and functions. 3. (Seldom used): Only in case of unbearable overlapping, leave out some parts and show them separately.

But at the end of the fifteenth century the grammar of technical illustration had become more extensive and elaborate by trial and error, by creativity and by the sheer necessity dictated by more complicated machines. The drawings following these conventions became more expressive and instructive, even for craftsmen without respective experience. Confined to the essential rules, the new catalogue of sophisticated rules can be paraphrased as follows:

1. Draw simply but in a concentrated manner. Try hard to manage with one sheet of paper for each device, but avoid mere hints that might possibly confuse a less skilled beholder. Therefore draw all particular details distinctly; if necessary, enlarge items that might help to understand the function. Spare your distant colleague's having to work out his own solutions for a result you have already achieved.
2. Do without multiple viewpoints, but pay extreme attention to the one you ultimately choose. Move laterally and upward until the maximum is visible and a minimum concealed.
3. If elements decisive for the function cannot be shown in a general view, then separate them from the context. Draw them enlarged next to their correct position. Form a pictorial "subordinate clause" and join it with a "conjunction" in the form of notes or lines to the "main clause." If such a separately rendered part or mechanism is applied in several sketches, a brief indication will suffice in the following pages.
4. If the machine is so complicated that too many parts will remain hidden and too many necessary pictorial "subordinate clauses" will obscure the whole context, then restructure the "text." Make a series of new "main clauses." Show all parts separately and connect them again by adding a general view in order to enable the beholder to discern their relationship.
5. If you are dealing with such an extended ensemble of mechanisms and machines that all single elements would result in a unfathomable puzzle of sketches, then exploit all your pictorial resources of analysis, segmentation, and structuring. Form a "text." Tell a story. Present a general view as introduction and orientation. Then hold on to sequences of shafts, forces, and functions. There you may fix

other "subordinate clauses" until the whole pictorial "text" fits together seamlessly in your beholder's mind.

Naturally, no craftsman ever was capable of recording in words the grammar of drawing outlined above. But the rules existed in the minds of craftsmen as a common convention for the conveying of technical information by the mere combination of ink lines—it was no coincidence that in the sixteenth century Italian engineers emphasized that geometrical elements serve as an "alphabet of design!"[51] Finally, every system of grammatical signs has to reach a high degree of complexity before it is ripe for a descriptive grammar. But, of course, the language of drawing also worked very well without scientific analysis.

How well it worked and which creative possibilities were opened up by the handful of sophisticated rules is apparent in some manuscripts compiled at the transition from the fifteenth to the sixteenth century. At this time the wealth of drawings had become so great that some extensive collections had amassed in voluminous codices. The council of Frankfurt asked an unknown master gun-maker to compile a collection of nearly all known texts and drawings about military devices, especially those derived from the various Formschneider manuscripts.[52] For this compilation on 178 paper sheets, the anonymous craftsman made full use of all known pictorial means. In particular, the complete spectrum of gun types is represented in all imaginable variations and with all graphical tricks.

But the most amazing consequences of the use of the pictorial grammar are found in two thick and large-format manuscripts, both created in the first decade of the sixteenth century.[53] The Erlangen *Kriegsbuch* and the Weimar *Ingenieurkunst- und Wunderbuch* each contain over 600 sketches on paper, or rather, parchment. The Erlangen codex belonged to the Franconian nobleman Louis von Eyb, the Younger († 1521), administrator of the Upper Palatine territories and counsellor of his Palatine Earl during a period that spanned the War of Bavarian Succession (1504).[54] The Weimar codex, depending on the identification of the patron's coat of arms, was compiled either for the Upper Palatine noblemen von Wolfstein or the Franconian counts von Hohenlohe.[55] Both manuscripts are closely related, as is evident in the duplication of whole series of pictures. Although a partial presentational function can be suspected, the material of nearly all sketches derives directly from craftsmen, especially master gun-makers and mill experts.

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51 For the quotations of Girolamo Cataneo and his pupil Giacomo Lanteri, see the chapter by Marry Henninger-Voss in this volume.

52 Frankfurt, Stadt- und Universitätsbibliothek Frankfurt am Main, Ms. germ. qu. 14, on the front cover *supralibros* with the coat of arms of the city and inscription *DIS BVCH GEHORT DEM RADE ZV FRANCFORT;* for a description see Leng 2002, II 107–110 and I 267–269, plate 23 and 24.

53 For the tendency towards voluminous collections of pictorial catalogues at the end of the fifteenth century, see Leng 2002, I 269–279.

54 Erlangen, Universitätsbibliothek Erlangen-Nürnberg, Ms. B. 26; see for contents and also for the patron's biography, Keunecke 1992–93, and Leng 2002, II 97–100.

55 Weimar, Stiftung Weimarer Klassik / Anna-Amalia-Bibliothek, fol. 328; for contents see Kratzsch 1979, Kratzsch 1981, Hall (B.S.) 1979b, 40f; on the discussion of the coat of arms, see Leng 2002, II 292–296 and Metzger 2001, 253–264. An edition on CD-ROM is being prepared by Christoph Waldburg.

In particular, an extensive series of mills contained in both manuscripts evinces a new desire to combine pictorial elements.[56] Small and large foundations with one or two millstones, main shafts and side shafts, direct and indirect driving, gear wheels and crankshafts, driven by bars, ropes, by hand, by water-wheels or tread-wheels for men or horses, were combined at will. Once craftsmen had learned to dismantle a depicted machine in separate parts and mechanisms as well as to analyse the structure of a "sentence," the task of reassembling them into various combinations was child's play.

Thus, with a restricted variety of forms they were able, as in every language, to produce a nearly unlimited number of pictorial expressions. In return, these multiplying possibilities of combination in depiction encouraged technical experimentation with different drives, measurements and energy sources.

This process of segmentation and the subsequent pleasure in varied reconstruction is best apparent in a large series with several sketches on each of the 15 sheets in the Weimar *Ingenieurkunst- und Wunderbuch*. They present the possibilities of a modular system for the construction of battering instruments or ladders with claws (figure 3.11).[57] Composed of a handful of iron parts such as bars, screws, nuts, claws or rollers, more and more different devices are generated step by step in a nearly endless series of permutations. On these sheets the playful combination of morphological and syntactical elements of the grammar of drawing has indeed produced a complete pictorial text.

6. DESCRIBING THE GRAMMAR

At the beginning of the sixteenth century, the handful of rules for conveying technical information in drawings had produced a large range of pictorial expressions among a small group of specialized craftsmen. But at this point we must note that this process grinds to a halt. Several reasons can be traced. The conveying of technical information over long distances by drawings developed by single innovative craftsmen was supported mainly by traveling master gun-makers. But this special job profile changed at the beginning of the sixteenth century. The construction of guns and foundations was no longer an individual process mastered by only a few skilled experts. Large foundries with preindustrial mass production run by the early modern sovereigns took their place. But with these foundries, the conveying and passing on of knowledge regained a stable sphere, comparable to that in medieval urban settlements before the master gun-makers had left their workplaces as blacksmiths and metalworkers. Hardly any drawings were produced in the foundries. Surely they used rough sketches, too, but there was no need to communicate them or to put them down on lasting media. Moreover, issues of competition and secrecy now became more decisive. Direct oral exchange was desirable and again possible. The production of

56 Weimar, Stiftung Weimarer Klassik / Anna-Amalia-Bibliothek, fol. 328, spread over f. 1^v–38^r and Erlangen, Universitätsbibliothek Erlangen-Nürnberg, Ms. B. 26, f. 124^r–143^r.

57 Weimar, Stiftung Weimarer Klassik / Anna-Amalia-Bibliothek, fol. 328, in two parts on f. 282^r–287^v and 279^r–305^v.

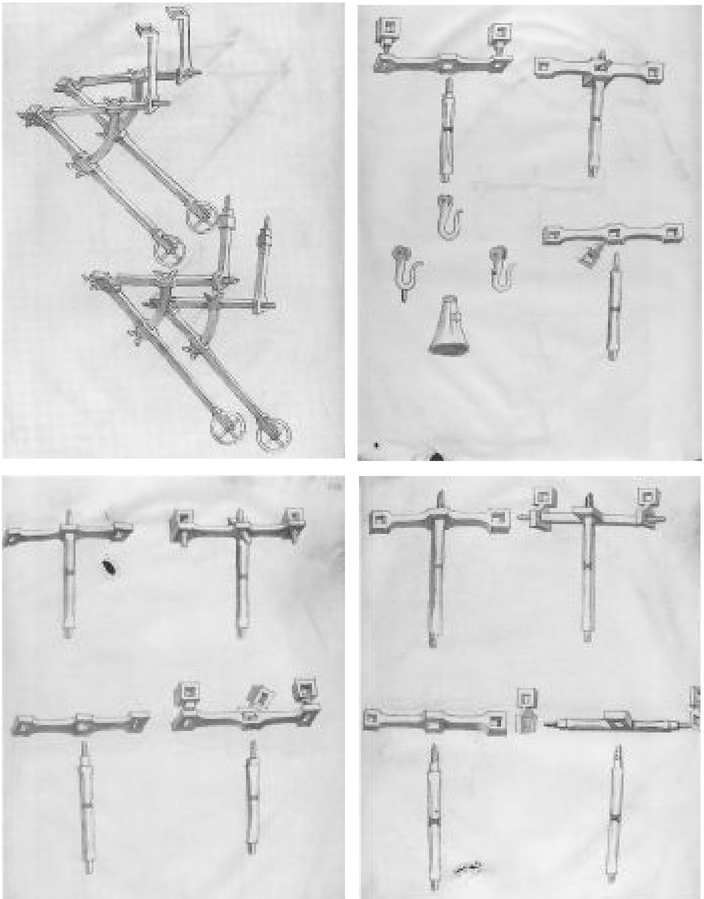

Figure 3.11. Excerpt from a large series of battering instruments composed of a modular system. From Anonymous, *Ingenieurkunst- und Wunderbuch,* Southern Germany, c. 1500. (Weimar, Stiftung Weimarer Klassik, Anna-Amalia-Bibliothek, fol. 328, fol. 282^{v} (top left), fol. 283^{v} (top right), fol. 284^{r} (bottom left) and fol. 285^{r} (bottom right); Leng 2002, I plate 28.)

guns was separated from their professional use. The formerly independent status of the extensively skilled master gun-makers changed to that of a mere piece-laborer.

In the sixteenth century we still notice an extensive production of master gun-makers' books. They no longer dealt with construction, but contain long-winded operating instructions with pictures as mere decorative elements.[58] Illustrations with separated parts or assemblies became rare. They only appear, equipped with precise measurements, in the attempts of some masters in the armouries to standardize the various gun and foundation types.[59]

It can be mentioned in passing that the master gun-makers never attempted to analyse their grammar of drawing theoretically. The only one who dared to present an illuminated full explanation for the construction of a gunner's quadrant by means of a ruler and pair of compasses, an exception admired by his colleagues, was the Upper Palatine master gun-maker Martin Merz († 1501).[60] The inscription on his still extant tombstone in Amberg praises him not only as outstanding master gun-maker, but also as an excellent authority in mathematics and geometry.[61] But his work was apparently ignored by his colleagues; no further copies are known.[62]

The majority of master gun-makers and craftsmen were satisfied with the results achieved—the development of a practical working language of pictorial communication. The step into theory, the description of rules was up to the educated architects and the artist engineers.[63] After their inception in the Italian Renaissance, the circulating printed tracts by Dürer contributed to the spread of combined views and real perspective drawings even among technicians.[64] Not until the end of the sixteenth century did the results of real perspective drawings become apparent both in the sketches of some technicians like Schickhardt[65] and in the wide-spread printed Renaissance Books of Machines.[66]

In conclusion, one pictorial catalogue from the middle of the sixteenth century shall be pointed out as a remarkable instance for the quality technical drawings could achieve against the background of conveying both experiences of practitioners and

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58 For example the *Buch von den probierten Künsten*, compiled by the Cologne blacksmith and master gun-maker Franz Helm in 1535, spread over Germany, especially in the new sovereigns' libraries, in more than 75 extant manuscripts, each with about 60 illustrations; ed. by Leng 2001; for an overview of the numerous other master gun-maker books see Leng 2002, I 330–367.

59 For example, Dresden, Staats- und Universitätsbibliothek Dresden, C 363: *hantbiechlin vnd ausszug von meinen erfindungen*, compiled 1570–80 by the Danzig master of the armouries Veitt Wolff von Senftenberg, see Leng 2002, II 88–91.

60 Munich, Bayerische Staatsbibliothek, cgm 599, f. 66^{r}–101^{v}, see Leng 2002, II 88–91 and I 250–255.

61 Amberg parish church, outside next to the entrance; see the figure of the tombstone along with the inscription in Schmidtchen 1977, 182.

62 One more copy possibly can be identified in a manuscript mentioned by Schneider 1868 129f. in the Library of the Princes of Liechtenstein in Vaduz, although today this manuscript is missing.

63 For their contributions, see Long 1985, and several other chapters in this volume. For Brunelleschi or Leon Battista Alberti see Fanelli 1980, 3–41; Gärtner 1998; on the development of perspective, recently Bätschmann and Giaufreda 2002, 12–18; for Leonardo, see Parsons 1968, 15–93.

64 Dürer 1983 (1525) and 1969 (1525); for the extensive literature, see Mende 1971, 479–483. See also the chapters by Lefèvre and Peiffer in this volume.

65 See Popplow in this volume with further references.

66 For example Besson 1569; Errard 1584; Ramelli 1588; Zonca 1985 (1607); Zeising 1607; Branca 1629; Böckler 1661. For this genre, see Popplow 1998c, 103–124 and Popplow 1998a, 47, 54f.; Bacher 2000 with several figures.

theoretical analysis. In the year 1558 the Nuremberg patrician Berthold Holzschuher († 1582),[67] finance broker,[68] owner of several mines in Tyrol[69] and passionate technician and inventor, drew up his holographic will. He left his eldest son nothing but a bundle of sketches with (as he thought) revolutionary driving methods for carriages and mills.[70] The book was to be kept secret and handed over by the executor to his son after Berthold's death. The heir was asked to call on cities, sovereigns, and kings with these drawings and thus to get rich on his well-deserved user fees.

Here communication by means of drawings had to surmount temporal distances, and Holzschuher by necessity had to ensure that the sketches were perfectly comprehensible without his help. Therefore he displayed the full range of pictorial means available in the sixteenth century. Of course, the complicated gearing of the carriages was influenced by the curious vehicles in Emperor Maximilian's 1526 posthumously printed *Triumpf*,[71] and the style of technical drawing by Dürer's architectural tracts. But the modernity of Holzschuher's drawings is fascinating when compared with actual blueprints.

For all devices we find logical sequences of general views, segmentation into mechanisms and into every single component, precise depiction of gear wheels, axles, and transmissions, everything designed exactly with a ruler and a pair of compasses. Dozens of erased help lines and hundreds of pricks on each sheet bear witness to his detailed efforts. The last sheet of his will, a folded two-page perspective drawing of a large mill moving 16 millstones, constitutes the highlight of the book (figure 3.12). Beforehand every single component was introduced. Additionally, three horizontal sections on the ground level, the shaft system level, and the top view made clear the interaction of framework, components, and the system of forces.

The most amazing fact is that all drawings are completely true to scale. Next to each sketch explanatory notes point out whether a quarter, a half or a complete inch on the paper corresponds to one Roman foot (c. 29,8 cm). Sometimes extra scales are added at the margins of the plans. Character legends join up each component to written explanations. Not only the purpose of the devices, their construction and combination is exhaustively explained step by step, but also the method of depiction. The last will of Berthold Holzschuher therefore can be considered as one of the first modern blueprints containing both a theory of technical drawing and an expressive example of their application.

Holzschuher's sketches remained a never-realized and soon-forgotten curiosity. None of the devices would have worked because he had misjudged gear ratio step-up and frictional losses. But their creation once more shows the social significance of

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67 For his biography, see Koch 1972, 579f.
68 For his financial activities, amongst them a revolutionary dowry insurance, see Ehrenberg 1890 and Krieg 1916.
69 See Kunnert 1965, 246–249.
70 Nuremberg, Germanisches Nationalmuseum, Hs. 28.893; for some short notes and a few figures see Neuhaus 1940 and Rauck 1964; for a description of the manuscript see Leng 2002, II 269–271. The author is preparing an chapter to be printed in the "Mitteilungen des Vereins für die Geschichte der Stadt Nürnberg."
71 Facsimile ed. by Appelbaum 1964, on the carriages, see 90–99.

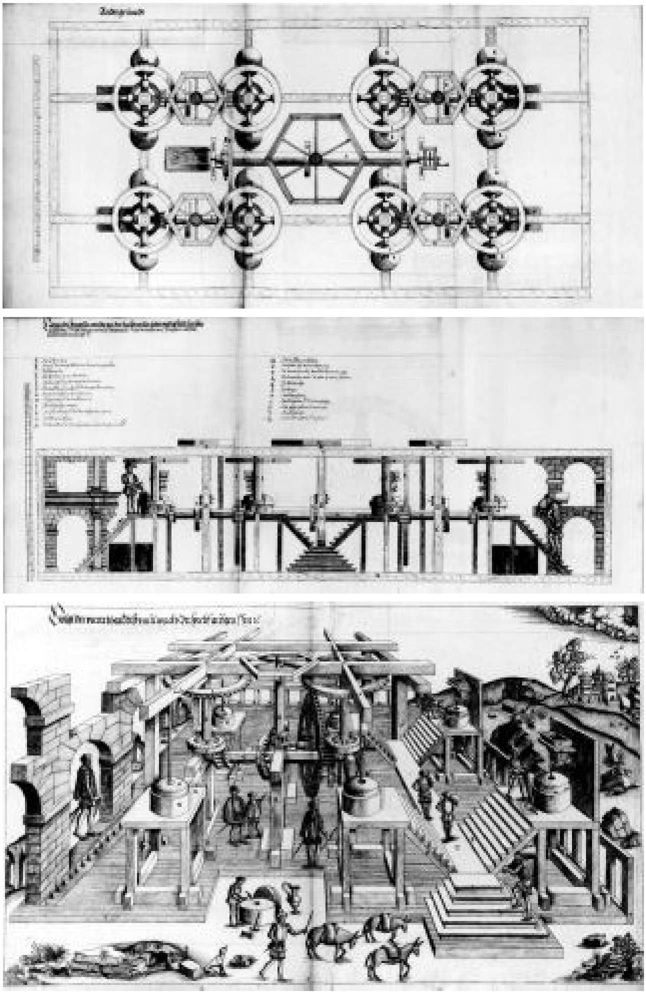

Figure 3.12. Scaled section plans and perspective view of a large mill with a complicated shaft and gearing system. From Berthold Holzschuher, *Last will and book of his inventions,* Nuremberg 1558. (Nuremberg, Germanisches Nationalmuseum, Hs. 28.893, fol. 31^{v} / 32^{r}, fol. 39^{v} / 40^{r}, and fol. 41^{v} / 42^{r}.)

craftsmen's manuscripts. The more skilled experts had to convey increasingly complicated technical know-how over spatial and temporal distances, the more sophisticated and efficient became the basis of communication, with a catalogue of drawing rules serving as a pictorial grammar. Their tentative development and practical testing from the beginning of the fifteenth century onwards led to the creation of some conventions in the following century, which are obeyed in technical drawing to this very day. Thus some fundamental blueprint techniques may be rooted not only in educated early modern engineering, but also in later medieval craftsmen's practice.

PART III
SEEING AND KNOWING

INTRODUCTION TO PART III

The topic of this section is the intricate relationship between technical drawings and knowledge—knowledge those drawings presuppose and/or allude to, invoke, transmit explicitly or implicitly, and/or knowledge, finally, that they help to gain. Besides "literacy" with regard to the different pictorial languages employed, the reading of technical drawings presupposes a spectrum of knowledge of different provenance—technological knowledge regarding both the construction and the working of the depicted device, expertise as regards the practical context of its intended employment, mechanical assumptions or other convictions of how natural things work, and sometimes even mathematical and geometrical knowledge. Accordingly, what these drawings actually convey depends to no little extent on the beholder's familiarity with the represented object and its whereabouts. They tell different things to experts and nonexperts, to the artisans who construct the device, to other experts who employ it professionally, and to lay people who are interested in it for any reason.

Technical drawings thus constitute a sort of focus where different kinds of knowledge come together and intersect. As such a focus, they radiate, in turn, into different domains of knowledge whose relations to the realm of engineering thus became recognizable. They are therefore mirrors of the intellectual world of early modern engineering if one succeeds in reconstructing the ways in which knowledge was connected with them.

The fact that technical drawings presuppose so much knowledge of all sorts is of course due in part to the limited representational capacity given by their restriction to two dimensions, and to the special semiotic shortcomings of each of the different pictorial languages. Technical drawings thus may appear to imply rather than display and convey knowledge. However, such a view not only would be at odds with the general fact that these drawings were an indispensable means of communicating ideas, proposals, solutions, agreements, and the like. It also would miss a dimension of early modern engineering drawings that is of particular significance in this context. Rather than depending mainly on presupposed knowledge, these drawings contributed essentially to the transformation of implicit, tacit knowledge of the crafts into explicit knowledge. They proved to be a chief means of rendering implicit knowledge explicit.

The chapter by Pamela Long studies the ways in which Francesco di Giorgio Martini and Leonardo da Vinci, two outstanding masters of engineering drawing, promoted the articulation of experiences and knowledge of engineering by means of both drawings and texts. Struggling for clarity, both were innovative in coining new terminology as well as in introducing new pictorial formulas, thereby creating an intertwined whole of pictorial and verbal elements that represented practitioners' knowledge with combined forces.

The relationship between image and text in illustrated manuscripts and books on engineering issues, which deserves much more attention than could be paid in this volume, is of particular interest with respect to the advantages and shortcomings of the two media as regards the articulation of technological knowledge. Both Giorgio

Martini's treatises and Leonardo's notebooks demonstrate impressively that such an articulation and explication cannot be achieved by either of the two media alone. Or, to put it more precisely, they show that the capabilities to spell out technological knowledge each of them possesses can only be exploited fully when each elucidates the other. In this field of knowledge, images explain words no less than words images. Furthermore, the combination of verbal and pictorial means of representation opened up new possibilities for connecting the knowledge of this realm of practice with a variety of branches of learned culture. Pamela Long's chapter shows that this opportunity was immediately realized and seized by Giorgio Martini and Leonardo, albeit in quite different manners.

The notebooks of Leonardo also testify to a further aspect of the relationship between drawings and knowledge, namely to the employment of drawings for the obtainment of insights—insights of a more theoretical nature as well as more practical ones. When employed in this way as a means of thinking, technical drawings and knowledge exhibit an interplay that is challenging to both the historical actors and the historian who tries to determine which knowledge was tacitly presupposed, which actually represented, and which gained by drawings used for this purpose.

This interplay is the topic of the chapter by Mary Henninger-Voss, who discusses the employment of plans by civilian and military experts in the service of Venice for consulting and deciding on crucial questions of fortification at the turn of the fifteenth century. When applied for these purposes, a fortification plan resembles an iceberg. For what can actually be seen on the plan is only the visible peak, whereas the bulk of information it contains for an expert is as invisible to a nonexpert as the main body of an iceberg under water. Moreover, since specialists of different expertise interact in such decision-making, military architects and military commanders in this case, the whole range of knowledge presupposed by, connected with, and derivable from a fortification plan is not possessed by any of the specialists, but only by an ideal team of experts representing the state of the arts involved. What is obvious in this case holds for the relationship between technical drawings and knowledge in general: it is mediated by the social relations among the actors involved in advanced technological projects.

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PICTURING THE MACHINE: FRANCESCO DI GIORGIO AND LEONARDO DA VINCI IN THE 1490s

PAMELA O. LONG

In memoriam
Carolyn Kolb (1940–1994)

At the start of his notebook on machine elements and mechanics, *Madrid Codex I*, which he created in the 1490s, Leonardo da Vinci, at the time a client of Ludovico Sforza of Milan, suggests that his treatise would provide a respite from fruitless labors, namely the attempt to discover perpetual motion.[1] "I have found among the other superfluous and impossible delusions of man," he writes, "the search for continuous motion, called by some perpetual wheel." He continues that for many centuries, almost all those who worked on hydraulic and other kinds of machines devoted "long search and experimentation with great expense" to this problem. "And always in the end, it happened to them as to the alchemists, that through a small part the whole is lost." Leonardo offers to give such investigators "as much respite in such a search as this my small work lasts." An additional advantage is that readers will be able to fulfill their promises to others so that they will "not have to always be in flight as a result of the impossible things that were promised to princes and rulers of the people." He recalls many who went to Venice "with great expectation of gain, and to make mills in dead water. But after much expense and effort, unable to move the machine, they were obliged to move themselves away with great haste from such a place."[2]

Leonardo may well be referring to attempts by his older contemporary, the architect/engineer Francesco di Giorgio, to construct water-powered mills in "dead water" *(aqua morte)* by a system powered by continuously circulating water. Yet evidence from several notebooks attests that Leonardo himself on several occasions investi-

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1 I would like to thank Wolfgang Lefèvre for organizing the workshop at the Max Planck Institute for the History of Science that prompted me to write this chapter. Versions of the chapter were presented at the Joint Colloquium, Princeton University Program in History of Science and University of Pennsylvania Department of History and Sociology of Science; and at the Department of History and Philosophy of Science, Indiana University at Bloomington. I thank the participants of these colloquia as well as the members of the Max Planck workshop for their extremely helpful comments which helped me to revise the chapter. I also thank Guido Giglioni for detailed discussions of Francesco di Giorgio's Italian, Steven Walton for discussions concerning Leonardo's cranks, and Bob Korn for assistance with models and "photoshop" enhancement of images.

2 Leonardo 1974, I f. 0^r, and IV 2, "Io ho trovato infra l'altre superchie e impossibile credulità degli omini la cierca del moto continuo, la quale per alcuno è decta rota perpetua"; "co'llunga cierca e sspe[ri]mentatione e grande spesa"; "E ssenpre nel fine intervenne a lloro come alli archimisti, che per una picola parte si perdea il tutto"; "tanto di quiete in tale cierca, quanto durerà questa mia picola opera"; "none aranno senpre a stare in fughe, per le cose impossibile promesse ai principi e regitori di popoli."; "con grande speranze di provisioni, e fare mulina in acqua morta Che non potendo dopo molta spesa movere tal machina, eran costretti a movere con gran fu[ri]a sè medesimi di tale aer."

gated the ways in which such a machine might function. Moreover, the issues of mechanical power and friction that were the focus of such attempts were also essential components in the design and construction of all machines. Leonardo's comment points to common areas of concern among engineers in the late fifteenth century, as well as to differences of opinion about what might work.[3]

The comment on perpetual motion is one of several unrelated introductory paragraphs that Leonardo tried out on the first sheet of *Madrid I*. While he condemns attempts at perpetual motion, he also alludes to the practices of engineers, their construction of mills, and their attempts to find efficacious ways to power them. Yet his treatise is not a practical manual for working engineers, but rather a broad-ranging investigation of machine elements, such as cranks and gears, and their motions, and it includes study of statics and mechanics. Leonardo lived in a culture in which writing books about the mechanical arts and machines had become a significant practice. Clearly a notable influence on his own writing were the treatises on architecture and engineering written by the Sienese architect/engineer, Francesco di Giorgio.

For both Francesco and Leonardo, machines were physical entities that performed certain designated tasks, but they also were significant within larger intellectual and cultural arenas as objects of study within investigations of motion and mechanics. Both men depicted intricate mechanisms pictorially and described their motions in detail. Machines became modalities for understanding certain problems in the natural world, such as motion. The fifteenth- and sixteenth-century development of the genre of "machine books," often created for presentation to patrons, suggests that machines had acquired symbolic value as well, perhaps representing both technological efficacy and the power and authority of princes. Pictorial images of machines within codices on architecture and engineering became increasingly commonplace in the fifteenth century. Yet further study is needed to investigate the *ways in which* machines were pictured in this period, and the uses to which those pictorial representations were put.

In this chapter I examine and compare several of the mechanical drawings of Francesco di Giorgio Martini and Leonardo da Vinci. The two men met, possibly a few years before they were both called to Pavia for consultation on the construction of the Pavian cathedral in 1490s. They shared similar backgrounds. Both were trained initially as painters, both expanded their opportunities for work through the astute use of patronage, both worked at a variety of kinds of tasks—engineering, military engineering, sculpture, architecture, and painting. Both struggled to learn Latin as adults, and both created illustrated treatises and notebooks on a variety of topics, including machines. Leonardo studied at least one of Francesco's treatises as is evidenced by his hand-written glosses on the manuscript copy of *Trattato I* in Florence (Ashburnham 361 in the Biblioteca Mediceo-Laurenziana). Leonardo's list of the books he owned includes a book by the Sienese engineer, although he did not specify which one. Clearly the two men benefited from their meeting and influenced one another.[4]

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3 Francesco di Giorgio Martini 1967, I 148 and f. 35^{v} (tav. 66); and see Reti 1969.
4 See Pedretti 1985; and Marani 1991.

Francesco and Leonardo both concerned themselves with real machines in the context of their engineering and architectural work carried out for their patrons. In addition, in their writings and illustrations, they connected machines to learned knowledge traditions in obvious and not so obvious ways. Nevertheless, the ways in which these two engineer/authors approached machines pictorially, and the ways in which they attempted to join their practical knowledge of machines to traditions of learning were quite diverse. A comparison of their dissimilar approaches allows a better understanding of how mechanical knowledge in the practical sense came to be connected to learned traditions in the late fifteenth century. Both men made these connections by means of illustrated writings, or what in some cases might be better called collections of images inscribed with small amounts of text. As a shorthand way of codifying their differences, I suggest that Francesco di Giorgio created a humanist book in which his interest in engineering finds a place alongside his architectural interests, whereas Leonardo da Vinci created a treatise more directly connected to the scholastic traditions of statics and kinematics.

Each in his own way was carrying on what was a well-established and rich tradition of visualizing machines by the late fifteenth century. There were a few ancient books that contained illustrations of machines—most notably the *De architectura* by the Roman architect Vitruvius, the original drawings of which had been lost, and Greek manuals on siege machines, the so-called poliorcetic treatises, containing beautiful, hand-colored illustrations of siege ladders, rams, and other military equipment.[5] Occasionally machinery is depicted in notebooks and treatises in the medieval centuries, such as a sawmill on one of the sheets created by Villard de Honnecourt in the thirteenth century, and plows and mills depicted in miniature in books of days and in almanacs. Nevertheless, there is an unmistakable expansion of drawings depicting machines and instruments from the late fourteenth century. Examples include the treatise and drawings on the astrarium by Giovanni da Dondi; the treatise on military weapons and war machines by Guido da Vigevano, the machines depicted in Conrad Kyeser's *Bellifortis*; the numerous drawings of guns, gun carriages, and other machines in fifteenth-century German manuscripts; machine drawings in a manuscript treatise *Liber instrumentorum bellicorum* by Giovanni Fontana; the well-known notebooks and treatises of the Sienese notary, Mariano Taccola, which were known to both Francesco di Giorgio and Leonardo; and the military machines depicted in Roberto Valturio's *De re militari libri XII*, by far the most popular military book of the fifteenth century, copied many times and printed in 1472.[6]

Writings about the mechanical arts—painting, sculpture, architecture, fortification, artillery, pottery, mining, and metallurgy, including machines and visual repre-

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5 For a spectacular eleventh/twelfth-century copy of the poliorcetic writings with numerous drawings of military apparatus and machines, see Biblioteca Apostolica Vaticana, Cod. Vat. gr. 1164, esp. ff. 88^{v}–135^{r}. And see Schneider 1912, esp. 4–6 and plates; Galluzzi 1996a, 22; Rowland 1998, 35–37; and Lefèvre 2002, 112f.

6 For an introduction to the scholarly literature and editions of the treatises, see esp. Aiken 1994; Alertz 2001; Barnes 1982; Hall 1978; Hall 1976a; Leng 2002; Long 2001, 102–142; and Popplow 1998a. See also the chapter by David McGee in the present volume. For a lucid essay on attitudes toward the mechanical arts from antiquity to the Renaissance, see Summers 1987, 235–265.

sentations of machines—proliferated in the fifteenth and sixteenth centuries. The complex reasons for this expansion of authorship in the mechanical arts include what historians of technology have called technological enthusiasm, a delight in the technology of machines in itself, regardless of economic or practical implications.[7] Machines were utilized to solve technological problems and to consider more theoretical issues involving statics and kinematics. As they became the focus of pictorial and textual authorship, they became incorporated into literate discourse in a variety of ways. Machines accompanied by visual images of machines and machine parts, including multiple variations of machine elements such as gearing, and of entire machines such as mills and guns, became a significant focus of authorship. While such books had distinct relationships with the actual practice of making and operating machinery and inventing new configurations of actual machinery, it is also true that they entailed a kind of practice separate from material fabrication and operation. Machine drawings do not necessarily provide an undistorted lens into material practices. The relationships of images and textual discussions to material practice is a matter for investigation rather than a given. Moreover, as will be seen in the discussion below, the relationship of "machine books" to the cultures of learning and of practice were never uniform, but rather were complex and diverse from one text and author to another.

1. FRANCESCO DI GORGIO: ENGINEER AND ASPIRING HUMANIST

Francesco di Giorgio (1439–1501) was the son of a poulterer who had left this occupation in the countryside to work for the commune of Siena. Francesco thereby grew up in Siena at the time when the notary Mariano Taccola was still alive; he may well have known the older man and was well acquainted with the latter's engineering notebooks and treatises. Taccola had completed *De ingeneis* in the 1420s and *De machinis* in the 1430s and 1440s. Both were filled with drawings of machines and instruments and often included short commentaries. Taccola, who called himself the "Sienese Archimedes," influenced Francesco in the latter's production of notebooks and treatises. For example, Francesco's earliest notebook, the so-called *codicetto* in the Vatican Library, contains a substantial number of copies from the notebooks of Taccola. Francesco combined his interest in machines and engineering with architecture, including the design and construction of buildings and the study of the Roman architect Vitruvius's *De architectura*. Francesco, who initially worked as a painter and sculptor, went on to become one of the best known architects and engineers of the late *quattrocento*. He worked in Siena, in Urbino for Federico da Montefeltro, in Naples for king Alfonso II, and elsewhere.[8]

Close investigation of Francesco's notebooks and writings have made clear his struggles to learn Latin, an effort that he seems to have made primarily to understand

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7 For technical authorship, see Long 2001, and for technological enthusiasm, Post 2001, esp. xviii–xx and 285–308.

8 For Francesco, see esp. Betts 1977; Fiore and Tafuri, eds. 1993; Scaglia 1992; and Toldano 1987. For Taccola, see Taccola 1971; Taccola 1972; Taccola 1984a; and Taccola 1984b.

the *De architectura*. He eventually was able to produce an Italian translation of the text in the 1480s. An early attempt at writing an architectural treatise and translating Vitruvius can be seen in a manuscript known as the *Zichy Codex* in Budapest, which, Carolyn Kolb cogently argued, includes a copy of an early treatise by Francesco. Francesco's better known writings include two major treatises on architecture, military engineering, and machines. He wrote the first, known as *Trattato I*, a later version of *Zichy*, during his first major sojourn to Urbino from 1477 to 1480 as a result of the interest (and perhaps urging) of his patron, Federico da Montefeltro. He wrote the second treatise, known as *Trattato II*, later, perhaps in the years 1487 to 1489, or as has recently been suggested, even later after his return from Naples to his native Siena in 1496. The precise chronology of Francesco's notebooks and treatises is a complex issue that is the focus of ongoing investigation and scholarly disagreement. Yet all agree that his two major treatises are quite different from each other and that one is earlier than the other. A comparison of the two works as they concern machines underscores these differences. *Trattato I* is the detailed treatment that reflects the concerns of a practising engineer. *Trattato II* sets out the topics according to more general principles, and follows some of the practices of humanist authorship.[9]

Trattato I exists in two copies created by professional scribes. The first, L, created around 1480 to 1482, is in the Biblioteca Mediceo-Laurenziana (Codice Ashburnham 361) in Florence, and the second, T, containing some of Francesco's amendments and additions, created around 1482–1486, is in the Biblioteca Reale (Codice Saluzziano 148) in Turin. The two manuscripts form the basis for the twentieth century edition, edited and transcribed by Corrado Maltese and Livia Maltese Degrassi and published in 1967. As Massimo Mussini suggests, the specificity of the technical details of the drawings of *Trattato I* make it certain that Francesco himself was closely involved in their execution. The most complete version (T) is divided into eighteen sections, or topics, which are not labelled in the original manuscript. Many of the sections represent certain categories of problems or projects confronted by engineers. There is a section on forts, but also one on the bridges of forts and on other types of defense. The chapter on theatres is followed by a separate section on columns. Other sections include practical measurement, lifting machines and mills; ways of conducting water, metals (not present in L), military arts and machines, convents, instruments, bells, towers, and gardens. The abundant drawings are found within the columns of text and in the margins of the sheets and were probably executed by Francesco himself, as was established by Mario Salmi in 1947 by comparison with other of Francesco's known drawings. Although the exact circumstances of the origins of the two manuscripts cannot be reconstructed fully, Mussini's suggestion that they may have been created in Francesco's workshop in Siena is an attractive one.[10]

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9 Francesco di Giorgio Martini 1967. For the *Zichy codex*, see Kolb 1988; and see Mussini 1991a, 82–88, 108–109, 121–124, note 75–79; and Mussini 1991b. For Francesco's translation of Vitruvius, see Fiori 1985; and Scaglia 1985.

Trattato I contains several sections that focus specifically on machines—military machines, cranes and lifting devices, and mills of various kinds. In the following I look at one of these sections, called by the editors, "levers of wheels and mills." This section takes up seven folios or fourteen sheets. The text is written in carefully blocked-out columns, two columns per sheet, by a professional scribe. Interspersed with the columns are box-shaped drawings of mills with their wheels and gears carefully depicted. There are a total of 58 such drawings which, taken as a whole, explain many variations of mills. There are water powered mills including those with horizontal, overshot, and undershot wheels, each in a variety of configurations. There are dry mills powered by animals, mills turned by cranks, and windmills. Taken as a whole, the images reveal two characteristics. First, most of the mills are set into neat, near-square boxes that are aligned with the columns of text. The drawings are made to conform to scribal guidelines, namely the width of the columns of text, square blocks of text and block-like images of mills reflecting one another. Second, it is clear that Francesco is interested in the gears and their interconnections, the axles or shafts, and the ways in which they are powered. For most mills, he provides details of the actual size of the desirable dimensions, such as the diameter of the wheels, gears, and axles, and their thickness.[11]

Francesco begins his section on mills with a discussion of the lifting capacity of a wheel—a discussion that pertains to wheels as to levers in general. He provides a formula for calculating this capacity based on the size of the wheel and of its axle. Take half the diameter of the wheel, he says, and half the diameter of the axle and calculate how many times the second goes into the first. If a diameter of the wheel is 10 braccia and the diameter of the axle is one braccia, you take half of each. Then calculate how many times one-half (half the diameter of the axle) goes into five (half the diameter of the wheel). The result is 10. So, he concludes, every pound of weight on the circumference of the wheel will lift 10 pounds on the axle. A schematic drawing of half a wheel and axle accompanies the discussion (figure 4.1).[12]

Immediately following, Francesco asserts that the lift of wheels for mills and many other kinds of machines are guided by these figures. Nevertheless it is difficult to demonstrate everything that is written, because there is such a great variety of things, one the opposite of the next. Although he has shown what he calls the "Ragione dela lieva," or "Reasoning of the lever," he recognizes that it is insufficient for the great variety of particular kinds and varieties of machines that engineers and others might devise. Therefore, he concludes that it is necessary to make a model. "Many things," he cautions, "to the mind of the architect seem easy and that necessarily succeed, that putting them into effect many lacks are found, which are repaired

10 Francesco di Giorgio Martini 1967, vol. 1, which includes a facsimile of the sheets containing drawings of T. For a brief but masterful description of the manuscripts and the scholarship surrounding their origins and dating, see Mussini 1993. See also Salmi 1947; and Scaglia 1992, 154–159 (no. 62, for Manuscript L) and 189–191 (no. 80, for Manuscript T), although it should be noted that Scaglia's claim that the two manuscripts were created at the monastery of Monte Oliveto Maggiore is without foundation in evidence and has been cogently disputed by Mussini.
11 Francesco di Giorgio Martini 1967, I ff. 33^{v}–40^{r} of facsimile pages, tav. 62–75. See also Marchis 1991.
12 Francesco di Giorgio Martini 1967, f. 33^{v} of facsimile pages, tav. 62.

Figure 4.1. Explanation of lifting capacity of a wheel and mills. From Francesco di Giorgio Martini, *Trattato I*. (Turin, Biblioteca Reale, Codice Saluzziano 148, fol. 33^{v}; courtesy Ministero per i Beni e le Attività Culturali della Repubblica Italiana.)

Figure 4.2. Drawings of mills and explanations of how they work. From Francesco di Giorgio Martini, *Trattato I*. (Turin, Biblioteca Reale, Codice Saluzziano 148, fol. 34^{r}; courtesy Ministero per i Beni e le Attività Culturali della Repubblica Italiana.)

[only] with difficulty. I for myself have seen the experience of a sufficient number of the inventions that here will be demonstrated, not relying on myself [i.e., my ideas alone]."[13] Francesco here shows the calculations meant to determine a mathematical way of measuring the lifting power of wheels, while at the same time underscoring the necessity of practical experience and of making models to avoid costly mistakes.

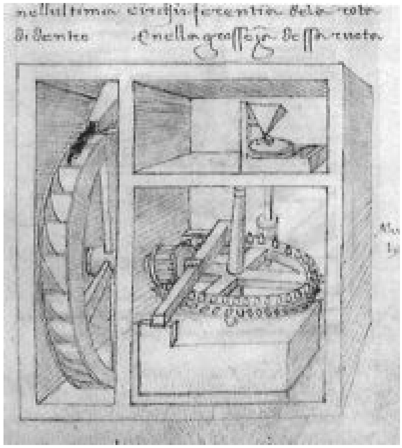

Figure 4.3. Enlargement of overshot mill E (lower right) of figure 4.2.

Francesco then turns to the construction of individual mills (figures 4.2 and 4.3). His treatment consists of a succession of descriptions and drawings of special kinds of mills. He describes and illustrates 58 different varieties. While most are for grinding grain, several serve other purposes. For example, he tells us that mills C and P (the two lower mills in the left column of figure 4.2) are for grinding olives and/or *guado*, the leaves of the plant *crocifer,* which were crushed in mills to produce a blue dye used in the textile industry. They are different from grain mills, especially because they lack a grain chute and feature an arrangement of the mill stones in which one stone sits vertically on top of the horizontal stone, an arrangement that differs from that of grain mills in which two horizontally placed stones turn against each other. Francesco also describes suitable locations for particular mills. For example, mill B (the top drawing in the left column of figure 4.2) is a small mill powered by a crank designed for a small fort, a convent, or a house. He tells readers that it is designed to grind a variety of different kinds of grains.[14]

In his descriptions of wheels, Francesco carefully explains to what type of mill it belongs and then goes on to describe the wheels and gearing, giving what he considers to be the appropriate measurements, and other details such as the best number of rods on lantern gears. In the case of water-wheels, he specifies which kind of wheel is appropriate for specific situations given the availability of water. If the water is some-

13 *Ibid.,* 141–142, and f. 33^{v} of facsimile pages (tav. 62), "molte cose all'animo dell'architetto paia facile, e che riuscir il debba, che mettendolo in effetto gran mancamenti in essi truova, in ne' quali con difficultà reparar vi pub. Io per me delle invenzioni che qui demostrate seranno, d'assai buona parte, in me non confidando, spirienza ho veduta." The phrase "raggione dela lieva" is written above the diagram of the half wheel and axle on f. 33^{v} of the manuscript.

14 *Ibid.,* 143–145, f. 34 of facsimile (tav. 63).

what sporadic, overshot wheels are better, whereas undershot wheels are okay for situations where a continuous, strong flow of water is available.[15]

Take, for example, mill E, the one on the bottom right of figure 4.2 (enlarged in figure 4.3). The mill is powered by an overshot water-wheel. The water spills from a funnel on top of the wheel, turning the wheel. On the shaft of the wheel is a lantern gear, which rolls over the vertical teeth placed around the top circumference of the horizontal crown gear wheel. In the back, another lantern gear is attached to the shaft, which turns the mill stone. This lantern gear rolls on the horizontal teeth of the same wheel. Francesco has also drawn the mill's wooden frame and the grain funnel into which the grain to be ground into flour is to be poured, and the millstones. The latter two components are drawn in disproportionately small sizes.[16]

Here is what Francesco tells his readers about this mill powered by an overshot water-wheel:

> Similarly the present figure of the mill in every place where it can have contingency with little water, it is easy to make, because the capacity to lift is great and free, as is apparent, and I through my own trial have made and seen. Let the water-wheel be 16 feet high. The lantern gear of the axle of this wheel, a diameter of two feet and [with] 16 rods. The toothed horizontal wheel places its large teeth where the lantern gear of the water-wheel revolves over the large teeth on the top circumference of the toothed wheel. And in the width [rim] of this wheel, [are] the small teeth, and because the lantern gear of the millstone makes them turn, given their thickness and their size, and the outer circumference of the wheel where the teeth are, the millstone goes quickly and with facility. And let the diameter of this wheel be 8 feet, its axle short in the manner of an acorn, made [thus] because it goes more strongly. The lantern gear of the water-wheel, let its rods be [made] in the manner of rollers, that its motion light passing lifts strongly enough, thus as figure E demonstrates.[17]

Francesco thus informs his readers that the mill is good for situations with an uncertain or uneven water supply because of its great power. (He recognizes that overshot water-wheels are more efficient than undershot.) He describes the actual workings of the machine and provides some of its dimensions. His writing as a whole is important for its extensive development of technical vocabulary and this passage illustrates that. Taken as a whole, his writing presents by far the most extensive technical vocabulary of any previous author, something that makes his texts unique and important sources for the study of mills and other contemporary technologies.[18]

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15 *Ibid*.
16 *Ibid*.
17 *Ibid.,* 144–145, f. 34 of facsimile (tav. 63), "similmente la presente figura del mulino in ogni loco dove dependenzia avere possa con poca acqua facilmente è da fare perché la lieva è grande e libera, siccome manifestamente si vede, e io per pruova fatto e visto l'ho. Sia la ruota dell'acqua d'altezza piè sedici. Er rocchetto dello stile d'essa ruota diamitro piè due e vergoli sedici. La ruota dentata per piano posta i denti grossi dove er rocchetto della ruota dell'acqua ripercuote nell'ultima circunferenzia della rota di dentro. E nella grossezza d'essa ruota i minuti denti, i quali er rocchetto della macina girando, per la spessezza loro e la grossezza e ultima circunferenzia della ruota dove i denti sono, veloce mente e con prestezza la macina va. E sia el diamitro d'essa ruota piei otto, el bilico suo corto a guisa di ghianda fatto perché più saldamente e con fermezza va. Er rocchetto della rota dell'acqua, a guisa di rulli i suo' vergoli sia, ch'el moto suo leggermente passando assai più forte leva, siccome la figura E demostra."
18 Calchini 1991.

In *Trattato I* Francesco provides a detailed written account, illustrated by numerous drawings, of his architectural and engineering knowledge. Mills were essential elements of many building projects from castles to forts, and they were also essential to any town or city. Taken as a whole, Francesco provides numerous examples of variations. Knowledge of such variations was essential to any practising architect/engineer of his time. Presented with variations of geography, power supply and potential use, the master must devise or decide upon a mill with particular characteristics for specific sites. Yet Francesco was not writing a manual for other practitioners. Rather, he was detailing his knowledge of practice in a codex at the instigation of his patron, Federico da Montefeltre, for the benefit of a more elite readership. The professionally copied text and carefully drawn images, on vellum bound in leather, consisted of a display of engineering knowledge for patrons, not a manual for use by other practitioners. Nevertheless, Francesco's careful detailing of numerous variations points to his close identification with the concerns of practicing engineers, a group that included himself.

Francesco's later treatise, Trattato II, is extant in two copies, one in Siena (S.IV.4), known as S (the earlier version) and another, Magliabecchiano (II.I.141) in Florence, known as M. Trattato II is organized differently from Trattato I, with fewer chapters, the addition of learned introductions, and more frequent references to ancient texts such as the *Naturalis historia* of Pliny and the *De architectura* of Vitruvius. Some of the more general topics of Trattato I survive as chapters in the later treatise, for example, the city, temples, fortresses, and harbours. Corrado Maltese, the editor of the 1967 edition of the treatise, supposed that it was the work of a humanist hired to revise Francesco's earlier treatise. The more recent investigations of Mussini, however, underscore that the treatise is the work of Francesco himself, with only short introductory passages added by another. This later treatise reflects Francesco's greater knowledge of Vitruvius. In addition, he provides more general treatments. He has moved from detailing many particular mills in Trattato I to more general accounts of far fewer mills, an approach more suitable to a readership of patrons and university-educated men.[19]

Trattato II is divided into seven books or *trattati*, as Francesco himself called them. There is a preamble, a book of common principles and norms, and then treatments of houses, ways to find water, castles and cities, temples, forts, ports, and a section on machines, "Machines to pull weights and conduct water, and mills," and then a conclusion. The material on machines that was scattered among various sections of *Trattato I* has received a new treatment in this treatise in one section only, section seven, which concerns lifting machines of various kinds and mills. In all, eighteen machines are displayed in drawings and described, a radical reduction from the earlier work, which contains drawings and descriptions of 58 mills alone, and

19 Francesco di Giorgio Martini 1967, I xi–xlviii; Mussini 1993, 358–359. The Magliabecchiano manuscripts contain several other works by Francesco as well—the *Trattato* is on ff. 1–102, Francesco's translation of Vitruvius on ff.103–192, and on ff. 193–244^{v} a collection of drawings of military machines and fortification designs.

treats many other machines as well in separate sections, including lifting cranes and military machines.[20]

Offering a rationale for this new approach, Francesco explains: "Thus indeed I will provide a drawing *(la figura)* of some mills, so that through those, other similar ones may be able to be discovered by readers."[21] He thereby suggests that readers will be able to read about one kind of mill, seemingly understand its principles, and then discover other types. At the same time, it appears that his simplifying strategy involves more than simply wanting to present a more general and rationalized account of mills. This becomes apparent when he immediately plunges into a lengthy biographical lament: "More and more times have I thought about not wishing to reveal any of my machines," he says, "for the reason that I have acquired knowledge of that with my great cost of experience and grave inconvenience, leaving in part the things necessary to life." Nevertheless, experience has shown him "the effect of ingratitude." People do not realize, he continues, that "experience cannot be acquired truly without long time and expense, and independent of other useful cares." When they desire a machine or instrument, they see the design, and "seeming to them a brief thing, they scorn the fatigue of invention." Worse, Francesco concludes, they claim these inventions as their own.[22]

Francesco immediately turns from his bitter complaints to a story from antiquity, which he draws from the tenth book of the *De architectura*. Vitruvius tells the story of two engineers, Diognetus and Callias. The Roman architect relates that the people of Rhodes decided to withdraw support from their tried and true engineer, Diognetus, and to give it instead to a newcomer named Callias. Callias had arrived and presented a lecture in which he displayed a drawing or model *(exempla)* of a spectacular defensive machine, a crane that could sit at the top of the wall of the city and pick off siege machines by lifting them up and rotating them around and then placing them inside the walls, rendering them ineffective. However, soon King Demetrius the Besieger arrived at the island with a fleet of ships and a huge siege machine, which he set up immediately. As the assault continued, the people became increasingly frightened of their prospective defeat and enslavement, while Callias demurred that the machine for which he had shown a drawing in his lecture could not be made big enough to deal with Demetrius's huge siege machine. After much begging, a lucrative contract, and the supplication of the city's virgins, Diognetus was persuaded to return to his position of city engineer. He saved the city by instructing the Rhodians to breach the wall in the area of the siege machine. He had them throw large quantities of water and sewage on the ground around the machine, so that it got stuck in the mire. Finding his siege machine inoperative, Demetrius the Beseiger decided that the Rhodians were unbeatable, and promptly sailed away.[23]

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20 Francesco di Giorgio Martini 1967, vol. 2.
21 *Ibid*., II 492, "Si ancora di alquanti pistrini metterò la figura acciò che, per quelli, delli latri simili da li lettori possino essere trovati."
22 *Ibid*., II 492–493.
23 *Ibid*.; and Vitruvius, 10.16.3–8.

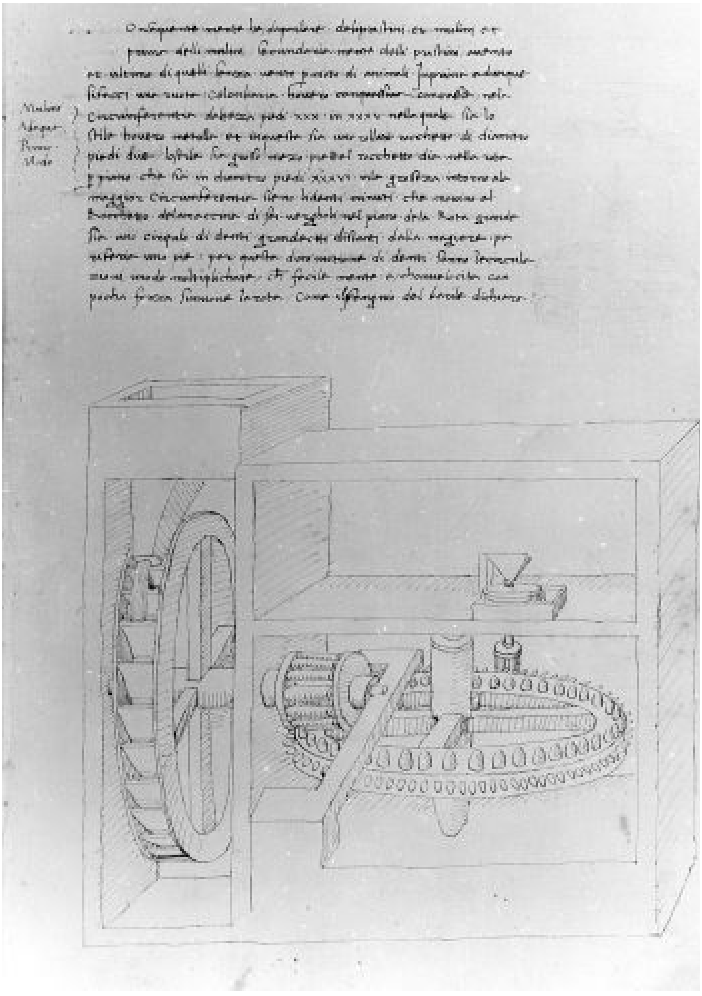

Figure 4.4. Overshot mill. From Francesco di Giorgio Martini, *Trattato II*. (Florence, Biblioteca Nazionale Centrale, Codice Magliabechiano II.I.141, fol. 95r; courtesy Ministero per i Beni e le Attività Culturali della Repubblica Italiana.)

The story underscores the importance of loyalty to a trusted architect and engineer, and it also suggests that actual practice is far more important than the theoretical knowledge presented in a lecture. Its appeal to Francesco undoubtedly also rested in its vivid account of the role of a drawing. By means of this ancient story, Francesco underscores the point that neither models nor drawings can be easily transformed into an actual working machine, primarily because they do not forewarn about problems arising due to different scales. While Vitruvius used one word, *exempla*, to explain what Callias displayed to the Rhodians, Francesco uses two—"some models and designs of machines" *(alcuni modelli e disegni di macchine)*. It is best not to trust those who have merely taken a model or drawing from someone else, such as, Francesco might as well add, those who had stolen drawings from him. Drawings cannot substitute for actual machines, nor can an "ignorant and presumptuous" architect like Callias stand in for the experience of an "ingenious and expert architect" like Diognetes—or indeed, like Francesco di Giorgio himself.

Francesco then proceeds to a discussion of machines. Of the eighteen machines he treats, there are six kinds of *argani*, or lifting machines comprised of capstans or windlasses, four kinds of *viti*, machines for lifting or pulling using screws, a water pump, and eight mills, including six dry mills or *pistrini* (including a windmill), and two water-mills or *mulini*, one powered by a horizontal wheel and the other powered by an overshot water-wheel.[24]

Francesco discusses only one type of overshot water-wheel mill, which he draws in a large size on a single page underneath the text describing it (figure 4.4). This machine is similar to the overshot wheel discussed above, one of five (E) on the sheet in *Trattato I* (figure 4.2, lower right and figure 4.3). His description in the later treatise is similar, but not the same as that for the analogous machine in *Trattato I*. He gives the dimensions of various parts such as the water-wheel and the crown-wheel. He explains that the wheel turns easily because the vertical teeth on top rim of the crown-gear are larger than the horizontal teeth on the lateral rim, and there are fewer of them. The smaller teeth on the lateral rim of the crown-wheel turn the smaller lantern wheel attached to the axle that turns the millstone. The drawing clearly shows the difference in size and number of the two rows of teeth on the crown-gear wheel. It also shows the different sizes of the two lantern gears, the one on the axle of the water-wheel being larger than that on the millstone axle. Francesco explains that these differences in the size of the teeth in the crown-wheel and the size of the lantern gears are what make the machine turn easily. It is notable that in his earlier drawing of the analogous machine such differences are not apparent in the drawing, nor does the explanation concerning ease of motion appear in the text. This is an example then, of using one machine to explain a more general principle (that of the gear differential), instead of enumerating varieties of mills suitable for diverse situations.[25]

The changes that are apparent between *Trattato I* and *Trattato II* are indicative of Francesco's efforts to write a more learned treatise, efforts that were influenced by

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24 Francesco di Giorgio Martini 1967, II 495–504 and tav. 317–331.
25 *Ibid.*, II 500–501; and f. 95 (tav. 325). Cf. *ibid.*, I 492–493 and fol. 34 (tav. 63).

humanist practices of authorship. Rather than showing a large variety of particular machines, he discusses a smaller number chosen on the basis of types (for example, as we have seen, he discusses only one overshot water-wheel mill). He devotes more space and larger drawings to each type. He also begins each chapter with a general introduction, and refers often to ancient passages relevant to the topics that he treats. In Francesco's treatises, it is notable that the engineer becomes visible as an individual who suffers the hardships of invention, and works as a practitioner. Francesco decries the thefts of invention and the patrons' reliance on drawings alone rather than on the engineer's extensive practice. In *Trattato II* especially, the human engineer emerges as a striking figure. This is in part a result of Francesco's engagement with the *De architectura* and other ancient texts, and in part the result of his engagement with contemporary humanist writings such as the treatises of Alberti, Valturio, and Filarete.[26]

2. LEONARDO DA VINCI AND THE ELEMENTS OF MACHINES

In the early 1490s when Leonardo da Vinci (1452–1519) was working on *Madrid I*, he had extensive contact with Francesco di Giorgio when they were both consultants on the cathedral of Pavia. Possibly they had met earlier, in the late 1480s. Leonardo's project of a treatise on machines and mechanics clearly came about at least in part because of the influence of Francesco di Giorgio. In turn, Francesco was influenced by the earlier Sienese author of machine writings, Mariano Taccola. Francesco copied some of Taccola's text into his own notebooks, especially the small, early notebook referred to as the *codicetto*. Similarly, Leonardo copied some of Francesco's text into *Madrid Codex I*. And as mentioned previously, Leonardo put his own notations on several sheets of the manuscript copy of *Trattato I*, Ashburnham 361, now in Florence. Clearly Francesco di Giorgio and Leonardo influenced each other; their work contains common elements, but can also be distinguished in fundamental ways.[27]

Madrid I (Ms. 8937 of the Biblioteca Nacional in Madrid) consists of two notebooks that, it has been argued cogently, were bound together by Leonardo himself. In the first half, up to folio 92, the folios progress forward in normal fashion from left to right, written of course in Leonardo's characteristic mirror script. It consists of a treatise on practical mechanics and elements of machines. Leonardo numbered this half of the codex on the rectos of each folio through the first six quires. The second half of the work progresses backwards from the back of the treatise toward the centre—from right to left. It consists of the more theoretical part of the work—a treatise on the science of weights *(scientia de ponderibus)*. Leonardo numbered this half in mirror numbers on the verso starting at the back and going towards the front (folios 95–191 in the current foliation). All the sheets of both halves are the same size, all bear the same recurring watermark, and the whole contains only two protective flyleaves.[28]

26 See Grafton 2000; Long 2001, 122–133; Filarete (Antonio Averlino) 1972.
27 Galluzzi 1991; Johnston 2000, 24–105; Pedretti 1985; Tocci 1962; and Zwijnenberg 1999, esp. 35–46.

Madrid I contains many illustrations of machines and devices, which in part come out of Leonardo's studies of motion, but which are also grounded in his practice as an engineer. In the 1490s Leonardo was a client of the Sforza family in Milan, patronage that he acquired primarily on the basis of his abilities as an engineer and productive artist. Sforza patronage was bestowed on a group that included both learned humanists and artisan practitioners including Leonardo. The Sforza created a court that produced painting, sculpture, and buildings to their greater glory. They also strenuously involved themselves in the armaments industry and in canal building undertaken to improve transportation and agriculture primarily for commercial purposes. Court patronage, material production, and commerce were closely interrelated endeavors.[29]

The first half of *Madrid I* is filled with beautiful drawings of machines and devices, and machine parts surrounded by detailed textual explanations concerning how they work and how they demonstrate the ways in which the natural world works. It treats the motions of toothed wheels, springs, clock escapement mechanisms (what Leonardo calls *tempi*), continuous and discontinuous motions of various sorts—counterweights, chain gears, screws, pinions and wheels, endless screws, lifting devices, mills, keys and locks, and crossbows, among others. There is evidence that Leonardo carefully planned his treatise. In an examination of the manuscript in Madrid, Marina Johnston has recently discovered that the drawings were made first by creating preparatory drawings with metal or bone point before ink was applied. Other evidence of careful preparation includes sheets in the *Codex Atlanticus* in Milan that contain drawings that are repeated in *Madrid I* as well as blocks of text, crossed out, that are carefully written into *Madrid I*. It is clear in many cases that Leonardo drew the images first and then wrote around them. Detailed investigation of the specific pictorial and textual content of single sheets in the codex is particularly fruitful since many of the sheets function more or less as self-contained units on a single topic or on two or more related topics.[30]

For example, on folio 0 verso are two lines stating the subjects at hand. A single line on the right reads "On the Placing of the Figure" *(Del posare della figura)* (figure 4.5). On the left is the two line heading: "Book entitled of quantity and Power" *(Libro titolato de quantità e potentia)*. Below these two headings is a paragraph, which (I surmise) Leonardo added at a time later than when he created the rest of the page, inserting a comment into an available space. This comment concerns a nine year old boy named Taddeo who played the lute in Milan on Sept. 28, 1497 and who is considered the best lute player in Italy. Below the passage about Taddeo is drawn the figure of a crank mechanism. The crank itself can be seen in the back of an appa-

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28 Leonardo 1974, vol. 3 (Reti's introduction to the codices); and Johnston 2000, 24–105, who provides a cogent analysis of the structure of the manuscript that adds significantly to Reti's foundational discussion; and see Maschat 1989.

29 For the Sforza court as it developed, see Catalano 1983; Chittolini 1989; Ianziti 1988; Lubkin 1994; and Welch 1996. For Milanese canal building, see Parsons 1968, 367–419.

30 Leonardo 1974, I f. 9^r, IV 24–25 (springs, clock escapements, continuous and discontinuous motions); I f. 10^r, IV 27–28 (chain gears); I f. 15^r, IV 39–40 (screws); I f. 19^r, IV 47–48 (endless screws); and I f. 46^v, IV 89–90 (mills). For a discussion of the organization of the work as a whole and of individual pages, see esp. Johnston 2000, 24–105; and see Zwijnenberg 1999, 100.

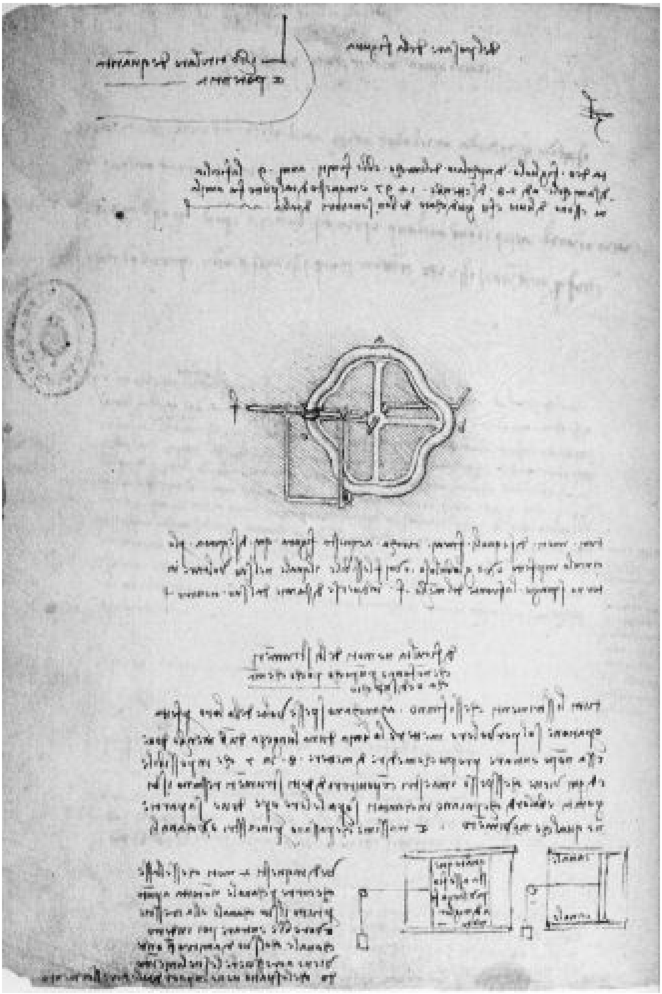

Figure 4.5. The "globulus" and other mechanisms with grooves. Drawing by Leonardo da Vinci. (*Codex Madrid I,* fol. 0^v; courtesy Biblioteca Nacional, Madrid.)

ratus in which a rectangular frame is moved in and out from the centre of a quasi-circular form with irregularly curved edges. Leonardo writes that

> you will be able to perform all unequal motions as shown by this drawing with the art of the imperfect circle, which could also be called globulous or flexuous. This instrument will, in its revolutions, drive the front of member f to varying distances from its center.[31]

The heading underneath this passage labels the topic of discussion "Of instruments that perform poorly because of a little something that is at fault or in excess." In the ensuing paragraph, Leonardo notes that the performance of an instrument is often hindered "only through wishing to put the line of a length of a part where it cannot enter." It is as if one would pretend to put 8 into 7 which is impossible. From this situation, he continues, "the constructors of said instruments remain dismayed, stupefied, that they seem spellbound over their work when it involves movement." He notes that this especially occurs when the movement is to take place in fittings or channels. At the bottom of the page, Leonardo sketches two examples of apparatus in which a weight hung over a pulley is dragging a board through grooves or channels. He notes that if the board does not touch the channel at every point through the entire groove, "it grows and would be able to enter then in said channel with its diameter." Hence "its line comes to grow in a way that the space is not capable of such growth." The sketch shows two figures of boards in channels. The figure on the left displays a longer board, which contains writing that informs that the facility of movement will improve as the length of the board increases. A second sketch on the right shows a shorter board further back on the apparatus. The channels or grooves are labelled. The board is placed at the start of its course along the channels, and because of its shortness, will, according to Leonardo's textual note, have more trouble being pulled along.[32]

Viewing the page as a whole, all of its elements seem at least in part related, with the exception of the comment about Taddeo, which was probably added later. One general theme concerns apparatuses that contain elements, which move along in grooves or channels. Such mechanisms are powered either by a crank as in the globulous circle, or by descending weights as in the two figures at the bottom of the page. Like Francesco di Giorgio, Leonardo is concerned about slight discrepancies or small glitches in the apparatus, which cause it to function with less than optimum efficiency. Leonardo's interest in efficient function is also evident in his observation that the length of the board being pulled in grooves affects the way in which it works. This conclusion may or may not be intuitively obvious—it seems likely that Leonardo had constructed this and similar mechanisms with slight variations to test the ways in which they worked.

Leonardo's textual descriptions of the mechanical elements on this page strike me as being notably awkward. This conclusion can be evaluated only by reading the Italian; Reti's English translations (which I have sometimes modified in my own English

31 Leonardo 1974, I f. 0^{v}, IV 2–3.
32 *Ibid*.

citations) at times present more coherent and often more abstract restatements than the Italian seems to warrant. Leonardo seems continually challenged to find appropriate words to describe the workings of machine elements, as well he might given the lack of a developed technical vocabulary for machines in the written Italian of his day. Yet Leonardo's own part in developing such a vocabulary seems much less extensive than Francesco di Giorgio's. Leonardo's textual descriptions are essential to his illustrations and his illustrations are crucially necessary to understanding his writing. Particularly taking into consideration that Leonardo attempts to discuss the ways in which his devices work, text and drawings, respectively, would be relatively incomprehensible one without the other.

Leonardo is interested in the actual motions of the apparatus and his statements appear to be based on actually constructing many of the elements and causing them to move. I draw this conclusion from the fact that throughout *Madrid I* he describes hundreds of minute local motions of numerous machine elements and their variations. For instance, he refers to the difficulties of cranking the rectangle back and forth in the globulous circle, and he notes that a longer board in the grooves moves more easily than a shorter one. Although it is presumptuous to assume to know the extent or limits of Leonardo's own intuition as it concerned the motions of elements of machines, it seems improbable that he or anyone else would understand intuitively the direction and nature of all the small variations of motion, and the difficulties of all the various elements of machines that he discusses without trying at least some of them out. And we know that Leonardo was extensively involved with actual engineering projects such as canal building and large-scale casting and that he hired German artisans to help him construct apparatus. Moreover, he frequently refers to his own experience.[33]

At the same time, Leonardo here often carefully separates his investigations of local motions and particular elements of machines from actual whole working machines, despite the fact that his ability to invent and draw whole machines is spectacularly evident on some folios of this codex and elsewhere, especially in the *Codex Atlanticus*. Yet he does not seem interested in presenting a brief for engineers, but rather in pursuing an investigation of local motion and the operation of particular machine elements. His is a contribution to a learned investigation, which may well also have practical implications. He often fails to mention the actual possible uses of machine elements—and in fact, many of them (such as the globular crank with moving rectangle described above) may have been invented solely as an experimental apparatus to explore the local motion and operation of particular elements.

On the following folio Leonardo treats cranks joined by connecting rods, and their motions (figure 4.6). The heading for the sheet comprises a single word, "Questions" *(Quisiti)*, reminiscent of scholastic practices. The question he has in mind follows: "Why does the flame rushing to its [natural] place take the shape of a pyramid while water becomes round in its descent, that is, at the end of the drop?" After pos-

33 *Ibid.*, III 40–41 for Leonardo's references to the various German artisans that he hired; and for explicit references to experiments, I ff. 77^r, 78^r, and 122^v, and IV 179–180, 183–184, and 325–366.

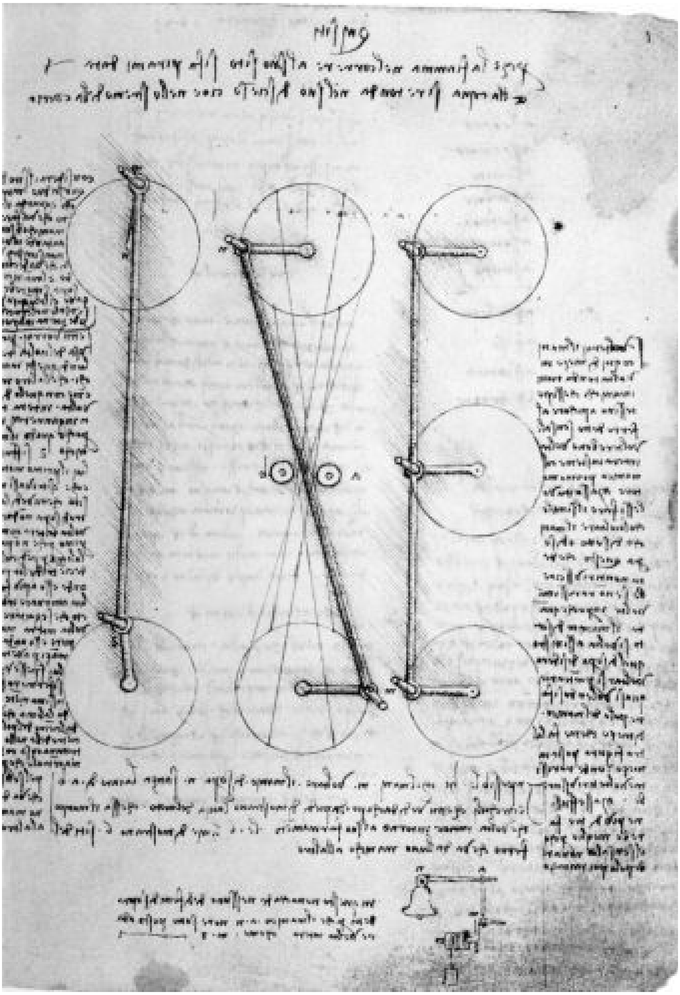

Figure 4.6. Crank Mechanisms. Drawing by Leonardo da Vinci. (*Codex Madrid I,* fol. 1[r]; courtesy Biblioteca Nacional, Madrid.)

ing this question involving the differing shapes of two different elements as they seek their natural place, Leonardo launches into a discussion of what might be called the shape of motion of a device consisting of two cranks joined by a connecting rod. He illustrates the same device twice within a chronological time sequence, first on the left, and then in the middle after the crank has been moved. The depiction on the far right shows a variation with three cranks.

Concerning the apparatus on the left, Leonardo writes:

> Consider the position of these two cranks and observe the motion made by the lower crank in its movement to the left. You can see that it is moving down; the upper crank would be raised, but the length of the *sensale* [connecting rod] does not permit it, and as a consequence the crank returns. Should you wish to turn the other in a complete revolution by turning one of the cranks with the aid of such an instrument you would be deceiving yourself. Inasmuch as crank S would be the prime mover, being in the same position as can be seen, the upper crank would not be able to make a complete revolution like the first one because it can not overcome the perpendicular line that unites the centres of the axles. Therefore this crank will have more facility to turn back than to make a complete revolution. But if by chance the crank advances, it will stop violently when the line that unites the centre of the axle with the centre of the ring takes the same position as the central line that goes through the *sensale*, uniting both cranks.[34]

He thus describes the exact movement of the cranks and the connecting rod, assuming that the motor is the lower crank marked S. The central figure shows where the two cranks would normally end up, assuming that the lower crank in the image on the right is moved to the left, or in a counter clock-wise direction. Normally the top crank moves to the 6 o'clock position and then reverses itself, ending in the position shown in the middle image. Occasionally the top crank goes all the way around. This is what Leonardo says in his text, this is what he shows in the two images, and indeed, in a model that I made of this apparatus, this is what happened.

The central image shows the same apparatus that is shown on the left, with the cranks in different positions—the positions where they would normally end up when the bottom crank is moved in a counter clock-wise direction. Leonardo uses this central crank apparatus to make further statements about possible motions. In the paragraph directly underneath the three images, he notes that it would be impossible for the lower crank m to turn the upper crank n without the aid of a and b—he is referring to two pegs or rollers in the centre. He tells us that the circles show the movement of the cranks as they turn on their bearings and that the six lines show the six positions of the iron rod that connects the cranks. On the right side of the page, Leonardo discusses the three cranks attached by connecting rods. He says that if you turn the cen-

34 *Ibid.,* I f. 1^{r}, IV 5–6, "Considera il sito di questi due manichi, e guarda il moto che vol fare il manico di sotto, andando in verso man sinistra. Vedi che va declinando, e 'l manico di sopra si vorebe alzare e lla lungheza del sensale nol concede, onde torna indirieto. Se ttu vorrai, per causa del voltare l'un di questi maniche, che ll'altro insieme con quello dia volta intera, tu ti ingannerai. Inperochè qua[n]do il manico S si ffussi lui il primo motore, e che e' si trovassi nel sito che ttu vedi, l'altro di sopra non dara volta intera come 'l primo, perchè non passa la linia perpendiculare de' cientri de' lor poli. Onde esso ha più facilità a ttornare indirieto che seguitare la volta intera. Ma sse pure esso andassi inanzi per aventura, lui spesse volte si fermerà fortissimamente, quando la linia che va dal cientro del polo al cientro dell'anello, sia tutt' una cosa colla linia centrale che passa pel sensale che va dall' uno manico all' altro."

tral crank around, all the cranks that might be connected with the *sensale* would make a complete turn, going along with the central crank, which is the prime mover. However, if you attempt to turn all the cranks by turning the upper or lower cranks, "you would succeed only very seldom." He explains that when the lower crank turns to the left, the upper crank wants to turn in the opposite direction, almost as if the middle crank were an axle as in the centre figure, the rollers or pegs function as an axle. It would work better, Leonardo adds, if you would use more than three cranks, because then the *sensale* would have no axle in the middle.[35]

Here Leonardo is thinking about different arrangements of cranks and connecting rods, the ways that they would move, and whether they would function well or poorly, depending upon small variations. At the bottom of the page he provides a drawing of a very different apparatus with a crank that swings back and forth. Here none of the problems mentioned above would occur, he says, because the crank a n is not supposed to make a complete revolution as is m s.[36]

These two folios, albeit representing only a very small number of the hundreds of devices and elements of machines treated in *Madrid I*, provide a window into Leonardo's interests and methods. First, he explores specific kinds of mechanisms and devices. On the folios discussed here, he treats two kinds—first, mechanisms that move in channels, and second, cranks. Leonardo uses textual explanations and visual images in his investigations. Both are essential and I believe that they are on par with one another. In most cases here and throughout the codex, neither the text nor the image is fully understandable without the other. Leonardo experiments on paper with many specific variations. Certainly in at least some instances he also constructs actual devices. But even on paper, he creates structural variations and describes in detail the ways in which direction of motion and facility of functioning are affected by variations in structure. It is evident that he places natural elements (such as the drop of water and the flame) and mechanical elements within the same conceptual world, and believes that investigating the one can help illuminate the other. Yet Leonardo's interest in issues of friction and facility of motion are indications that, while philosophical issues such as the nature of motion fascinate him, he has one foot firmly planted within the arena of engineering practice.

In significant ways, *Madrid I* in itself constitutes a kind of investigation. Rather than presenting the *results* of "research"—it *is* the research. This is the case because for Leonardo, drawing and writing taken together constitute not only the tools of investigation, but the investigation itself. He devises a number of different variations as images on paper, and thinks about the ways that these variations influence direction and ease of motion. He is interested in both structural variations of different kinds of devices, and in the local motions that result from those variations. Such variables have profound practical manifestations. Leonardo is also interested in them as part of his general study of motion, a study, which is local and observational, which

35 *Ibid*.
36 *Ibid*.

sometimes must have involved making models, and which uses the creation of visual images as a way of both thinking about and explicating variations.

Leonardo's devices function on some level as observational tools. He constructs various devices and then observes and describes how they move, in what direction and how easily. It seems probable that he actually constructed or had constructed by others at least some of the devices that he draws. We know that he frequently hired German artisans to make machines and devices for him. He also mentions from time to time his own experimentation.[37] I believe that some of the motions that he describes probably could only be learned by making the device itself and observing how it could be made to move.

3. FRANCESCO, LEONARDO, AND THE VISUAL CULTURE OF MACHINES

Francesco di Giorgio and Leonardo shared similar interests that include machines and mechanisms. Although Francesco treated whole machines, while Leonardo often treats machine elements, the contrast is not as great as might appear at first sight. Francesco often focuses on only certain parts of the machines he discusses, such as the water-wheel and gears. He provides drawings of whole machines but is primarily concerned with certain working parts. Both men focus on how well the machine works—the factors that encourage or inhibit easy functioning. For both men, such concerns come out of their engineering practice. Both present variations, although Leonardo goes much further in his investigation of variations as a general focus, rather than as part of utilitarian machines.

By considering their respective treatises as a whole, the larger context within which each frames his treatment of machines becomes clear. From this point of view, there are fairly striking differences. In his *Trattato II*, Francesco's chapter on machines is part of a treatise on architecture and fortification. The Sienese architect worked on his treatises while he also learned Latin and worked to understand and translate Vitruvius's *De architectura*. He was familiar with the earlier treatises on architecture by the humanist Alberti and the architect Antonio Averlino called Filarete. His view of architecture includes a framework in which human proportions determine the mathematical symmetries of an ideal architecture, which reflect the ideal proportions of cosmos. His anthropomorphic ideal architecture was informed by both ancient and near-contemporary writings as well as by contemporary architectural practice.[38] Francesco attempted to write a humanist treatise on architecture and fortification in which machines played an essential part.

Considered as a whole, Leonardo's treatise has been described as a bifurcated work in which the two halves mirror each other. The second half, starting from back and moving to front, that concerns statics and kinematics, mirrors the first half that treats elements of machines, including numerous variations, and their motions. The

37 See footnote 33.
38 Millon 1958; Saalman 1959; and Tigler 1963. For the larger tradition of architectural authorship to which Francesco contributed, see Long 1985.

second half includes discussions of passages from the writings of Jordanus of Nemore and other late mediaeval writings on statics. Leonardo connects his discussions of machines with the an interest in statics and kinamatics, placing his work within this earlier tradition but also departing from it in his fundamental treatment of machines and mechanisms and their many variations, and in his investigation by means of both writing and drawing. On many sheets, including the one concerning cranks discussed above, Leonardo introduces statements concerning the motions of natural entities—here they are drops of water and flames. He seems to suggest an analogy between mechanical and natural motions; at the very least, he is introducing the two within the same framework and on the same page. Elsewhere in the treatise, such analogies are far more explicit. In general *Madrid I* investigates connections between the natural and the artifactual worlds with implied or explicit comparisons of natural and mechanical movements.[39] Such small-scale comparisons and analogies are quite different from Francesco's large-scale microcosm/macrocosm framework connected by mathematical proportions. Both authors compare the natural to the mechanical worlds, but in very different ways.

Francesco di Giorgio and Leonardo da Vinci both undertook projects of self-conscious authorship on topics of mechanics, mechanical arts, and machines. For both, visual images played a fundamental role in their authorial practices. In different ways, their writings were connected to engineering practice on the one hand, and to learned culture on the other. Their treatises reflect the fifteenth-century transformation of the status of drawing and painting from mechanical to liberal art, a transformation far from complete in the 1490s. The fact that visual images could play such a fundamental role in treatises that aimed to contribute to learned culture, and the fact that machines themselves were embedded in these learned treatises, signals a broad cultural transformation in which the mechanical arts and visual images have become newly significant within the larger culture, including the culture of learning. Both Francesco and Leonardo contributed to this trend in part because their images of machines and machine parts in themselves constituted virtuoso displays of technical facility. Those images also functioned as explanatory tools within detailed discussions of machines, written discussions relatively unfamiliar to any of the diverse cultures of learning in the fifteenth century.

Within their project of authorship Francesco and Leonardo articulated, both visually and in writing, the tacit knowledge of machines that they had acquired within the compass of engineering practice. The process of authorship itself clearly required each author to make visible and to articulate what he knew tacitly—to rationalize that tacit knowledge to make it understandable to practitioner and non-practitioner alike. We can suppose that the process of articulation encouraged the authors themselves to think about both the principles and the working of the machines that they described. This process is evident in Francesco's explanation of the gear differential, and is evident in Leonardo's drawings as well. Leonardo's experimentation by means of drawing machines and writing about them is grounded in his tacit knowledge of how

39 Kemp 1991.

machines are made and how they work. What is not always clear is the location of the line between what he knows tacitly and what he learns from the process of drawing and writing about mechanisms. Both authors make the tacit knowledge of their own practice available to readers, including learned readers who would not have possessed such knowledge.

In this chapter I have compared Francesco di Giorgio and Leonardo's treatments of machines, and pointed to their shared culture of engineering practice, and their common experience of functioning within client/patronage relationships. I have pointed to their substantial intellectual similarities and their significant differences. What I have not done is suggest that Leonardo's work represents an "advance" over that of Francesco di Giorgio's. Rather, each author identified with and developed his writing within the context of diverse strands of contemporary learned culture—Francesco within the context of humanist writing, Leonardo within the context of scholastic traditions. I would suggest that their two diverse ways of relating to and advancing learned culture point significantly to transformations within that culture.

Neither the writings of Francesco or Leonardo represent a point within a linear development to "classical" mechanics. At least in some cases, as Michael Mahoney has emphasized, classical mechanics as it developed in the seventeenth century moved toward increasing use of mathematics and greater abstraction.[40] Quite in contrast, both Francesco and Leonardo were interested in local motion, specific force and power, in overcoming small, real glitches, and in overcoming friction. Their conceptualizations of machines remained firmly grounded in engineering practice and in detailed, complicated images that were ready referents to actual or at least imaginable machines.

While their writings did not lead in a direct line to classical mechanics, they did make a contribution to the development of empiricism and experimentalism, which were central to the emergence of the new sciences in the subsequent centuries. Francesco's and Leonardo's practice of authorship in itself is significant in this regard. Both men utilized their knowledge of engineering practice and their skill in the visual arts to explicate machines and machine parts within treatises inspired by noble patrons and connected to traditions of learning. Their writings contributed to a culture of knowledge in which instrumentation, tools, machines, and indeed, drawings, came to play a crucial role in legitimating knowledge claims about the world, including its natural and its mechanical components.

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40 See the chapter by Mahoney in this volume; see also Mahoney 1985 for his initial discussion of this idea.

5

MEASURES OF SUCCESS: MILITARY ENGINEERING AND THE ARCHITECTONIC UNDERSTANDING OF DESIGN

MARY HENNINGER-VOSS

> The office of the architect is to make the fortress secure from the four prime offenses: because in the ordering of our fortifications we will always have before our eyes, as the principle goal, that it should be secured from battery, from scaling [the walls], from shoveling [trenches], and from mines. And as the major part of the offense comes from artillery, so the principle defense will be from the same.
>
> Galileo Galilei, Trattato di Fortificazione

As a teacher of military engineering, Galileo began with compass constructions of basic geometric figures for the purpose of design. Immediately, however, he reminded his students that the design was to meet the particular goal of defense. For that "we will always have before our eyes" the offense that would never have been depicted directly in the design.[1] While historians of technology have made much of the engineer's visual knowledge, the "eye of the engineer" in sixteenth-century military engineering was not necessarily confined to what was present on the surface of the design itself. How then could a design be judged? What sorts of knowledge were required to visualize not only the building itself, but its probable success?

There is a difference between seeing an engineer's drawings and reading those drawings. It is perhaps because of this essential "reading" metaphor that we so often talk about a "lexicon" of design elements and the evolution of different "grammars" of drawing. I wonder however if our metaphors are not misleading us, re-encoding the act of interpretation of engineering drawings with a self-enclosed act of language recognition, rather than pointing us toward the difficulties in higher interpretation of meaning. In sixteenth-century formal treatises, engineers themselves often compared good design favorably to the thick description of words. A good model made transparent the window between the project design and the project's success. The actual ways in which designs of large public projects were interpreted and judged, however, belie the transparency of "good design." Engineers' designs interest me precisely because they are representations not of linear discourse, but of complex relationships between things—some of them depicted and measurable, and some of them unseen, though often reduced to a measure of something that is depicted. In the largest, most public engineering projects of sixteenth-century Italy—those concerning fortifications and hydraulics—it was the meaning of those measures and the nature of the

1 Galilei 1968a, 84.

relationships between design factors that became most hotly contested in the evaluation of designs.

The whole point of Eugene Ferguson's work, *The Eye of the Engineer,* was simply to root engineering practice in the design process.[2] Ferguson delved neither into the cognitive act of design, nor into the social act of design interpretation and realization. His message was simply to emphasize the nonverbal design function over the development of theoretical constructs in engineering, and to catalog the engineer's use of design over the past five centuries. He roots the forms and shapes of modern engineering practices, for good or ill, in the military engineering of sixteenth-century Italy and seventeenth-century France. It is the "fortress mentality" that remains for Ferguson the (perhaps unfortunate) heritage of engineers, the zealous accounting of "inputs" and "outputs."[3]

The milieu of Italian military engineers will also be my focus here. I am not interested in finding there a root of modern engineering, however. Or if I am, I am more concerned with the attributes of design that the twentieth-century theorist J. Christopher Jones outlined.[4] Like many scholars, Jones traced modern technique to a Renaissance design-by-drawing method that replaced a craft approach. Jones emphasized that the design allowed an engineering style that could overcome the limitations of the single craftsman's practices. The design enabled large, complex engineering tasks to be divided and coordinated; it stored tentative decisions for one part of the design while other parts were developed; it served a predictive function for what would work. I am less interested here in the particular assertions Jones has made or not made, but want to emphasize that these features of design all relate to a context in which technological tasks *are* large, evolve over years, and require a minimization of risk. As David McGee has shown, Jones's depiction of the "craft approach" may not be importable to early modern technical work;[5] however, Jones does offer two insights that may illuminate the large-scale engineering projects of that period. First, when designs are made by teams—as they certainly were for cinquecento fortifications—we must change the way in which we view design production. Second, the control and judgment process is set up to predict the success of a thing that does not yet exist, based on the performance of things that do. Jones shifts the discussion, appropriately, to what a design *does* at particular stages of the process.

Engineers' drawings do things; they do things in particular social contexts, and to be useful rely on an implicit human organization and economy of knowledge. As so many authors in this volume have shown, what a drawing means depends both on the techniques that convey that meaning and the place that the drawing has within the design and construction process. This essay draws on work in both these avenues, but is an investigation of how meaning was constructed around predictive operation for the planning of cinquecento fortifications. In their models' scaled dimensions, military engineers encoded meanings that depended on an "architectonic" understanding.

2 Ferguson 1992.
3 *Ibid.,* 70.
4 Jones 1970.
5 McGee 1999, 209–214.

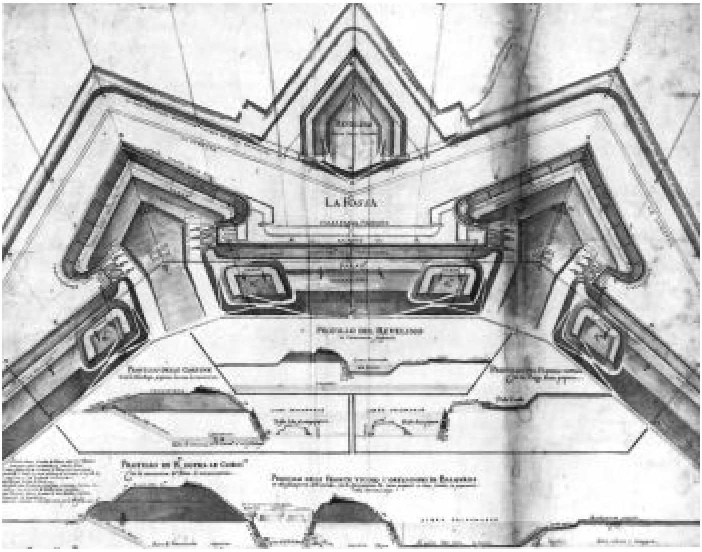

Figure 5.1. Plan and profile for two bastions at the fortifications of Palma, 1595. This is among the most finely rendered plans, in fine wash, neatly labeled. The plan should give the width and length of walls and thus proportions between firing positions; the profile should provide the width and height of the walls, as well as its composition (dirt, rock fill, etc.) and the proportions of wall, scarps (embankments that fortified walls), and trench. From Filippo Besset di Verneda, Parte di pianta e corrispondenti profile di tutta la fortezza di Palma, 1595. (Venice, Archivio di Stato, Raccolta Terkuz, n. 42.)

This architectonic understanding was teleological in that it tied form strictly to function; it was an understanding that placed the forms depicted into the larger context of the terrain to be defended, and sought to determine structure within the parameters of gunpowder warfare. While engineers challenged each other to "see" on the surface of the design the relationship between projected defense and probable offense, the "reading" of the design required not only eyes, but work—intellectual, technical, and sometimes physical—both on and off the plane of the design. The predictive meaning of a design was seldom transparent. Rather, it was the site of negotiation, controversy, and investigation. As we examine how fortifications models took on meaning, easy divisions between theory and practice dissolve.

What sort of drawings are we talking about? While measured drawings may be rare for sixteenth-century machine drawings, fortifications plans are littered with numbers and calculations, from the preparatory to the final drawings. In large part we will be looking at the construction and evaluation of models in the context of realiza-

tion, to import Marcus Popplow's category, and these invariably included measurements.[6] The words "model" and "design" were employed interchangeably in both published architectural treatises on fortification and in documents pertaining to government administration of fortification works. The substitution of one for another cannot always be counted on since models could be made in different media—either a two-dimensional representation on paper, or a three-dimensional construction made of wood, gesso, or some other material.[7]

Architectural historians have assumed that presentation models were usually of the three-dimensional sort, although they also point out that Pietro Cataneo's mid-century architectural treatise, in which military architecture is prominent, encouraged students to develop perspective techniques so that perspective drawings might serve as a cheaper, more portable substitute.[8] The historiographical assumption in either case is that those unaccustomed to looking at architectural designs would see the project better in a small but fully constructed version. However, the letters we have that accompanied the models (which often we do not have) seem to refer to paper designs; and yet very few perspective drawings of projected fortifications are found in state archives. Further, both princes and military leaders had a high degree of fortifications knowledge. While I think it is likely that at some point a constructed three-dimensional model was required by governmental commissions, most discussion over models appears to have taken place over simple ground plans and profiles, and their measurements. Although perspective drawings also were included in teaching manuals for military engineers, the ground plan and profile received greatest attention, and Galileo insisted in his own treatise that with these all the dimensions of a fortification could be known[9] (figure 5.1). Another reason that most of our documents appear to refer to ground plans may have been that even fine ground plans could easily be copied and sent to the various decision-makers by pricking the outlines over another piece of paper (figure 5.2). Here I will only be concerned with these measured, scaled, drawn plans.

Models were not simply maps, nor were they simply sketched-out ideas. Drawing may be the nonverbal representation of information or ingenuity, but the design model is comparable (but not reducible) to mathematical solution-finding. In this way, we can see that engineers worked with a certain number of "knowns" and an array of "variables" in order to project a design that would work under all foreseeable conditions of attack. The process of identifying these knowns and variables was divided among the men who created the design and those involved in the control and evaluation of the design. The military design model was constantly moving between

6 Popplow in this volume.
7 Scheller 1995; Goldthwaite 1980; Rosenfeld 1989.
8 Cataneo 1554, book 1, f. 1; Adams and Pepper 1986, 190; Adams and Pepper 1994, 64; Ackerman 1954.
9 Severini 1994; Pellicanò 2000, 113–115. Filippo Camerota's studies of projection techniques in this volume, testify to perspective techniques that would have been measurable and useful in the context of fortifications planning, and yet I simply do not know if these techniques were used by military engineers in the planning process during the sixteenth century. In general, perspective drawings as part of presentations of fortifications plans (as opposed to reconnaissance, commemorative, or mnemonic uses) only become conspicuous in the seventeenth century.

spheres of knowledge. The design had to be seen now with the eyes and tools of proportion, now with the eyes and experience of military strategy, and now with the eyes and analyses of mechanical physics.

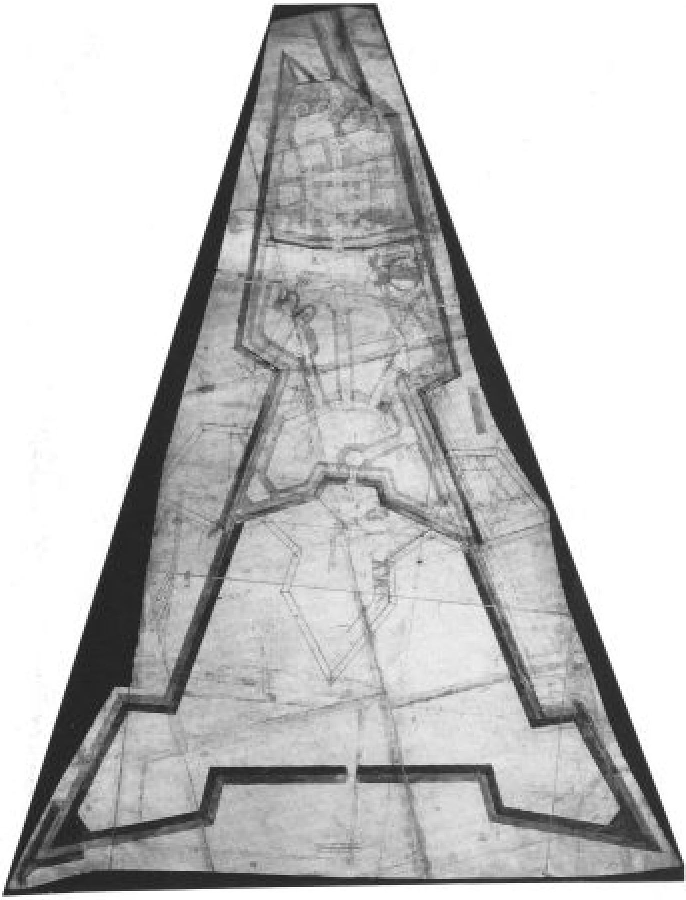

Figure 5.2. Plan of the lower fortress, called San Cataldo, Perugia, Rocca Paolina, 1540. The plan is color-keyed and has been extensively pricked at angles, an indication that it served as a basis for copies From the workshop of Antonio da Sangallo. (Florence, Gabinetto dei Disegni e Stampe, 272 Architettura r.)

The hierarchies around fortification models also meant that many design features that had less to do with the gross necessities of strategy were handled at other levels of the process. The sketch-books of men like Antonio da Sangallo or Michele Sanmicheli, premier military architects of the first half of the century, show a broader palette for the study of fortifications, including layered measured plans that developed over time, perspective studies, mapping studies, sketches, and so on.[10] These however do not seem to be those drawings that elicited official comment. Questions of embrasure sizes and ventilation chimneys that could allow smoke to escape from enclosed cannon platforms seem to have elicited little comment from the higher-ups.[11] The execution of fortification interiors was probably left up to head stonemasons.[12] The spare plans submitted to governments, however, encoded in their measurements a broad array of architectonic relationships that only could be evaluated with compass in hand, and with a view to the experiences and sciences of gunpowder warfare.

I present here a series of vignettes that address the question: what constituted a good model in cinquecento military engineering? How could designs predict the utility of a not yet existing structure? I have examined textbook models and training, an episode of disagreement between engineers, and the directions to which theoretical considerations behind designs could lead (i.e., to a "new science" of artillery and fortifications). These vignettes show different levels on which the models could be inspected. These include adherence to the mathematical practices that were fundamental to fortifications designs; the reduction of vast arrays of political and technical factors into debates over measure, and the philosophical investigation of artillery concerns relating to design decisions.

1. WHAT GOOD ARE MODELS? FROM VASARI'S "LIFE OF BRUNELLESCHI" TO BUONAIUTO LORINI'S "DELLE FORTIFICATIONI"

Mastery of design had a special place in architectural practice. It was the first mark that distinguished the "architect" functionally and hierarchically from the masons and carpenters.[13] To use the terms of fifteenth- and sixteenth-century Italian art theorists, *disegno* had to be an embodiment of *ingegno*; to judge design was also to evaluate the mental tools that fashioned it. This *ingegno* referred both to imaginative fantasy and to the rules and sciences (perspective, knowledge of nature, etc.) that controlled expression in *disegno*. For figurative arts and civil architecture, there remained a tension between the imaginative and the scientific aspects of *ingegno*.[14] In military architecture, expression of fantasy was obviously limited, and the array of sciences

10 See especially Adams and Frommel 1994.
11 E.g. see the correspondence of engineer Francesco Malacrede and generals Pallavicino and Savorgnano, in Venice, Archivio di Stato di Venezia, *Archivio proprio Contarini*, B. 8, ff. vii^r to xvii^r.
12 Adams and Pepper 1994, 62.
13 Pevsner 1942; Ackerman 1954.
14 Kemp 1977; Kemp 1990; Puppi 1981.

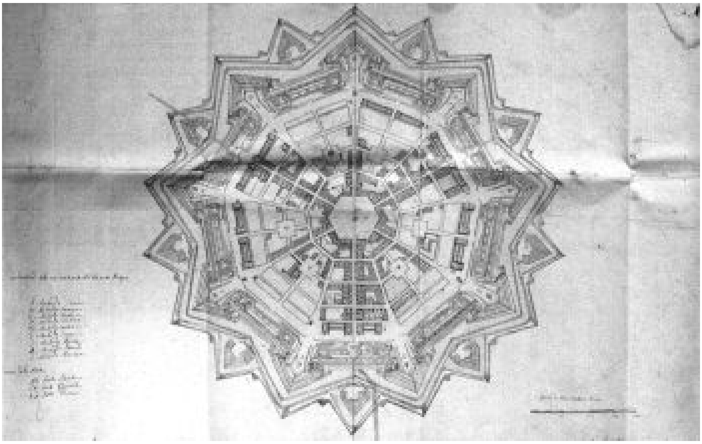

Figure 5.3. An ideal circuit of walls. The bastioned curtain wall is surrounded by a star-shaped “covered way,” and in between the rays of the stars are rivelin outworks, additions popular in the seventeenth century. Pianta di Palmanova. (Venice, Biblioteca Museo Correr, Cod. Cic. 3486/IV.)

and practices in which the architect was to be conversant ranged from geometric construction to tactical savvy to artillery aim.[15]

There is a reason why sixteenth-century military architects began to identify themselves more exclusively as “engineers,” and distance themselves from civil architects who dealt in “the secrets of the statue-makers” guild.”[16] Military architecture was completely reconfigured in the first decades of the sixteenth century. Due to the demands of mounting and withstanding the new gunpowder weapons, high crenellated walls and round towers gave way to a polygonal circuit of walls and spade-like bastions (the enceinte known as the *trace italienne*), and a variety of platforms, trenches, outworks, earthworks, countermines, and scarping techniques (figure 5.3). The design of fortifications became more closely associated with the design of machines, and engineers began even referring to the fortifications system as “la macchina.” The new fortifications offered little leeway for aesthetic choice or even traditional knowledge, as the innovative new architecture was to meet the requirements of gunpowder weapons.[17] At the same time, military engineers mediated between stonemasons (and legions of unskilled laborers), military leaders, and

15 Hale 1977; de la Croix 1963, 30–50; Lamberini 1987 and 1990; Manno 1987; Fiocca, 1998, *passim*.
16 Lorini 1597. See also Hale 1977. I have noticed the same phenomenon in the roles of the Florentine court where Bernardo Buontalenti and Ammanati are both called “architetto” or “ingegnero” in alternate years.
17 Wilkinson 1988.

governmental patrons while their training became ever more associated with mathematical arts. Increasingly through the century, fortifications engineers became members of large teams of professionals, beyond the cooperation and contributions of workshop assistants.[18] While workshop practices still thrived around figures such as Bernardo Buontalenti in Florence and the Genga family in Urbino, military engineers increasingly made their primary alliances among military leaders who had charge of engineering.

It was perhaps for these very reasons that the great sixteenth-century proponent of the dignity of arts of *disegno*, Giorgio Vasari, paid relatively little attention to military architecture. In his *Life* of Francesco di Giorgio Martini, he even lamented that Francesco had become too carried away with military concerns. While Vasari wanted to found design in a "science" that adhered to Aristotelian notions of theoretical knowledge, he also characterized design as a manifestation of inscrutable genius. Vasari's *Life of Filippo Brunelleschi* unwittingly lays out the tension between scientific clarity and skillful genius in models made for large public works. The tale is paradigmatic of many tales told in the history of this field.

In any large-scale building project in which government administration was involved, the model appears as an object that is at once clear and contested. That is, the design might clearly show what is to be constructed, and even how. But the predictive quality, whether the actual structure will "work," was in question when innovation was introduced. This is demonstrated in Vasari's *Life of Brunelleschi*. The story illustrates the way in which models served as manifestations of the inventiveness and knowledge of the architect, even as, paradoxically, judgment of them could be a problematic issue. According to Vasari, Brunelleschi secretly worked up models for the building of the duomo on the Florentine Cathedral, including detailed studies of lifting machines and scaffolding and a complete scaled wood model. These he produced at opportune moments. The wardens of the Cathedral, however, remained ever hesitant to award complete authority to Filippo, and the models produced by rival architects continually confused them, even as the cupola rose daily under Brunelleschi's direction.[19]

Vasari's *Life of Brunelleschi* is a story of heroic *ingegno* tied to meticulous *disegno*. Brunelleschi's designs and models are in fact testaments to his *ingegno*. Without them, we would not be convinced how much of the construction of the duomo lay in the direct control of the architect, and how much was contributed by the master stonemasons, carpenters, and metal workers on the job. Vasari makes it clear that every detail of the construction owes its genesis to a sketch or model from Brunelleschi's hand. Or rather from his head, since Vasari emphasizes that the workers were "poor men who worked by their hands," and that work could not proceed without Brunelleschi's direction. Rather, Brunelleschi taught the artisans everything—he constantly made models for the stonecutters and ironworkers, we are told, "from wood, wax, or even turnips." Control over the models meant control over the project from

18 Lamberini 1986.
19 Vasari 1963.

top to bottom, and if Vasari could link the design function to high intellectual activity, so much the better.[20]

Models in Vasari's story have a transparency of meaning on one level. For example, Vasari recounts the story in which Brunelleschi refuses to show his designs for putting the cupola on the Cathedral without pillars or other supports during the wardens' convocation of architects, since then the other masters too would know how to do it. Within the time frame of the *Life*, however, models as a class of object appear to have had a murkier aspect. The model of Brunelleschi's rival, Lorenzo Ghiberti, was rewarded with the incredible sum of 300 ducats, but, in Vasari, exhibits little or nothing of use. The models of the other masters are imperfect, and only confuse those whose "fickleness, ignorance and lack of understanding" led them to favor other contenders. But how does one escape such ignorance? How does a warden or patron *judge* a model?

Vasari's story only vaguely accounts for even Brunelleschi's knowledge—his years of study in Rome, where he "drew every sort of building." However, as Vasari himself would tell us (and fifteenth-century contemporaries such as Leon Battista Alberti), Brunelleschi's architecture of the Florentine Duomo is a signal chapter in the innovative approaches fifteenth-century architects took in creating a renaissance world competitive with the ancient one. It was a feat not only in the management of geometrical form, but of the weighty stones that went into its construction.[21] Whether such knowledge could have been "seen" in Brunelleschi's models in and of themselves, as Vasari seems to suggest, is another matter. The Florentine architect's own advice to the "Sienese Archimedes," the civil and military machine designer Mariano Taccola, suggested only that Taccola should not "squander his talent" explaining his inventions to ignorant persons who would only keep them from being heard "in high places and by the right authority." Reveal them only to an "appropriate gathering of men of science, philosophers, and masters in the mechanical art," Brunelleschi exhorted.[22] This is hardly an exhortation to "craft secrecy" as it has sometimes been interpreted, but how such a gathering might be constituted in the fifteenth century, except through the informal coming together of friends, is less clear. An institutional framework for men learned in design and its sciences was in fact to be part of Vasari's vision in his establishment of Florence's *Accademia del Disegno* in 1563.[23]

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20 Obviously the sort of models that Brunelleschi might have made with wood scraps and turnips are not related to the sort of scaled drawings we address here. However, it is instructive that Vasari employs these stories to build up a general profile of Brunelleschi's design abilities. While Vasari had his own professional motives for presenting Brunelleschi in this light, even in the generation following Brunelleschi's death, Tuccio Manetti also heroized Brunelleschi's models, and his story of Brunelleschi's turnips are the source for Vasari, as are many other elements of the account. Howard Saalman's conservative reading of the documents related to work on the Duomo, however, brings a healthy skepticism to the view of Brunelleschi's enormous superiority over other craftsmen builders. See text and notes of Manetti 1970. Both Manetti and Vasari referred to models as a way of underscoring Brunelleschi's authorship of the building and differentiating him from other building professionals.

21 See Settle 1978.

22 Battisti 1981, 20–21.

23 Barzman 1989; Wazbinski 1987; Ward 1972; Kemp 1990, 132–135.

The *Accademia del Disegno* was in fact a kind of quasi-state institution for the training and organization of painters, sculptors, and architects—men who sought release from the confines of crafts guilds, and whose distinction was precisely their knowledge of design. Patronized by the Medici Dukes, the *Accademia* served as the primary consulting agency on all engineering projects and public works.[24] Officers were appointed directly by the Duke, and the academy fostered military as well as civil architects. Vasari's own nephew, the academician Giorgio Vasari the Younger, collected numerous designs pertaining to military engineering, from various measuring instruments to artillery pieces.[25] Galileo's friend and academician, Ludovico Cigoli, wrote a treatise on perspective that featured military application in its iconography, and the illegitimate Medici sons, Don Giovanni and Don Antonio, were associated with the *Accademia del Disegno* in part because they were expected to be skilled in military architecture. In 1595, when Don Giovanni was serving Emperor Rudolf II in the Hungarian wars, he sent home to Florence for "good engineers and men of design."[26] Buonaiuto Lorini, a Florentine who later became a senior engineer in the employ of Venice, was also encouraged by the Dukes of Florence to master design and to pursue a military career.

Lorini in fact published a treatise, *Delle fortificationi*, which outlined the expectations of a good working design. The purpose of the treatise is emphasized through the inclusion of a dialogue between a count delayed at Zara (in Dalmatia) and Lorini himself. It centers around the problem of determining which plans will succeed, and what qualifications of an engineer are requisite. Design, Lorini suggests, is necessary to all arts, but especially to command.

> On *disegno* depends the true understanding of all things: it enables one to show that great perfection which the *ingengo* of a man may have, who both imitates the wonderful works made by nature and by art, and also shows everyone, and makes them understand each of his concepts. And therefore *disegno* is of such value that whoever masters it can truly say that it is very easy to express perfectly any work that he wants to put forth. This is because through this [*disegno*] are shown all inventions and their elements—approving the good and emending the poor. But also the sites of the country, that is the land, the sea, and whatever natural and artificial features are at work there, are all represented on a simple piece of paper, whether you make your demonstration as things really are, or should be. Also, being able to see the design of things would not only be useful, but necessary—particularly with regard to explicating and making understood our concepts, as it would be. For example, if you wanted to present and make understood the construction in a city that has been done, or needs to be done, simply with words, it would turn out impossible not only to be able to judge its perfections and imperfections, but also to know its true form. On the other hand in *disegno* made with measure, one can make this demonstrable.[27]

This lengthy passage exposes much about expectations on models and the work they were to do. Lorini conceives of *disegno* as a process, which allows one to represent

24 Ward 1972, 111–116.
25 The younger Vasari's manuscripts are found in the Biblioteca Riccardiana, Florence.
26 Florence, Archivio di Stato Firenze, Carte Alessandri: Archivio Privato di D. Giov. De Medici, Filza 8, f. 47–49.
27 Lorini 1597, 32. On Lorini at Zara, see Promis 1874, 640.

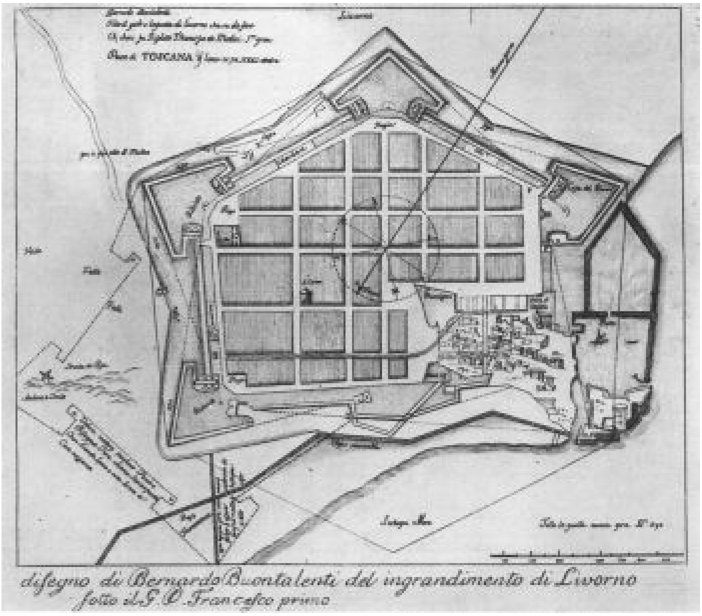

Figure 5.4. This printed plan of Bernardo Buontalenti shows the topography surrounding the city of Livorno, the existing fortifications, and (in dotted line), Buontalenti's suggestion for the enlargement of the city walls. (Anonymous 1796.)

both concepts and things as they really are. *Disegno* here is especially valuable because concepts and things, invention and nature, can *interact together* on the simple piece of paper (figure 5.4). While this may be a specific interpretation of the role of the architect, Lorini highlights here the importance of clarifying one's invention through design. The design can say what words cannot. It can synthesize details so that proper proportions and relations can be seen. Alternatively, a designer might separate details into elements that can be more clearly delineated. Here design takes on the characteristic of transparency. One can *see* true forms; perfections and imperfections are manifest. And yet, Lorini will maintain, such knowledge of design is founded not only on mathematical relations but on practice as well. Design is a kind of touchstone of theory and practice, and should demonstrate the inventor's depth of knowledge in each.

But how would all of this answer the count's initial dilemma: there always seem to be different opinions, and much opposition whenever a fortress is built. How can

there be any rules for fortification beyond the various opinions of the different inventors? Lorini is quick to reply that the engineer, like the physician, relies on both theory—("having its foundation and every perfection from mathematics")—and practice.[28] But how can evidence of "practice" also be read off the design? Indeed, this is the unvoiced problem that peeks around the confident statements of a great number of military engineering treatises. How is the local, and individual, knowledge of an engineer's practical knowledge to be assessed, and how should it enter the design process? One answer implicit in Lorini's treatise is that opinions on fortifications are expressed as design, and there are rules for design, expressed in measure. Measure captures not only the theoretical, mathematical feature Lorini constantly alludes to, but it often appears in engineering treatises as a kind of boiled-down experience of practice itself. That is, measurement often appears to stand not only for a particular act of measure, but also for repetitive experience itself.

Lorini's patron, the fortifications governor and artillery general Giulio Savorgnano, claimed to have spent his fortune in the service of the Venetian State so that "I leave nothing but the great quantity of designs and honorable writings that are to my merit."[29] Indeed one of the most copied of Savorgnano's writings is a list of rules by which design could be judged. When Savorgnano wanted to outline rules for the princes and counselors who would decide among designs put forth by soldiers, engineers, and captains, he warned,

> To be able to have a solid judgment on this science [of fortification] is not so easy, nor is it enough to know how to design on paper *cortinas* and bastions in different ways, since there could be proposed many forms and different materials to the Prince who must make the decision. For this reason, the counselors reason well when they hear one [plan], but then when they have to compare those that disagree, the counselors that have never been to war remain confused and irresolute.

Savorgnano followed up with 25 rules for the judgment of fortifications. As it turns out, the guidelines this artillery general and superintendent of fortresses provides to ameliorate a lack of battle experience serves itself as a kind of measuring stick to be held to each proposed design. Almost every one of Savorgnano's rules is a parameter of measure that must be met. For example, the first rule is that the distance between the flank of one bastion to the outer angle of the next (or the protruding point of the bastion) should be 200 passi so that enemy artillery will not be able to shoot at the inside of the bastion from the point of the counterscarp without facing defensive fire. The second rule defines the flank of a bastion at 30 passi since that is how much room would be needed to operate two cannons.[30]

Experience, like the landscape itself, could be represented in measure. It is in this way that designs offered to place before one's eyes what did not yet exist, in relation to both the landscape that did exist and the dictates of experience. Through design, as

28 *Ibid.,* 54
29 Venice, Archivio di Stato di Venezia, Mat. Mist. Nob., B. 5, letter dated 15 May, 1592.
30 Giulio Savorgnano, "Regule delle fortificatione Moderna di Ill.mo et Ecc.mo Sig.or Giulio Savorgnano," Venice, Archivio di Stato di Venezia, Mat. Mist. Nob., B. 13, f. 3^r–3^v. See also Manno 1987.

Lorini suggested, the experienced engineer could determine the best proportions of the parts of a fortification plan, and how it would perform in given situations. But of course the final determination of what plan was "best" did not fall to the military architect. Models were the centerpiece to a circuit of opinions in which the engineer had only one voice, and *qua* engineer (as opposed to as a military officer), not a terribly significant voice.[31]

Models were the first reports sent to senior commanders or engineers, or sometimes the only thing they would see before making their own emendations. So Francesco Maria della Rovere I, the Duke of Urbino and master architect of the Venetian fortification system before his death in 1538, obtained the "models and measures" for fortification at Vicenza from a m[aestro] Agostino so he could prepare to go over the details when he reached Vicenza himself.[32] Another fortifications expert in the employ of Venice, Sforza Pallavicino, apparently made a number of suggestions on models sent to him for various projects in progress within Venetian territories, sometimes without ever having seen the site at all. He had only the designs to rely on.[33] This was of course routinely the case for the final decision-makers, especially in the case of Venice. An entire department of government, the *provveditori alle fortezze*, was instituted primarily to look after and track the flow of these models, and records relating to their progress.[34] Over 40 senators were expected to review the models of one of Michele Sanmicheli's contested fortification projects, and presentations were conducted regularly in the Senate.[35] Hence Savorgnano's "rules."

2. EPISTEMOLOGICAL ANCHORS: CARTOGRAPHIC SURVEY AND GEOMETRIC DESIGN

Measured design was the bond between the experience of the engineer and his pretensions to science. It was also the chain of good faith that bound the lieutenants, commanders, junior and senior engineers, and the counsels of government.[36] It was the means by which decisions that could affect the lives of hundreds of men, or thousands of people, could be made at a distance of hundreds of miles. It is not coincidental that military engineers tried to impress on their patrons the epistemological foundation of their practice—not merely for rhetorical effect, but often in conscientious earnestness. Fortifications designs were as much a product of human imagination as any other building, but they had to fit both a real present physical reality (the terrain of the place to be fortified) and a future possible reality (the eventuality of siege). Military engineers had one oar in cartographic methods, which promised to reduce to scale exactly physical landscapes and standing walls, and another in the geometric methods, which allowed the design of "perfect" forms of desired propor-

31 Morachiello 1988, 45–47.
32 Promis 1874, 109.
33 Venice, Archivio di Stato di Venezia, Mat. Mist. Nob., B. 7, ff. 10^r to 20^r.
34 J. R. Hale 1983b.
35 Venice, Archivio di Stato di Venezia, Sen. Terr, Reg. 33, ff. 70^v–71^r and 135^v–136^r. See also Hale and Mallet 1984, 250; see Lamberini 1987.
36 Lamberini 1987, 17.

tionalities. Textbooks extolled the virtues of these practices, and could also serve as a kind of book of models of various structures and techniques that could be adapted to particular sites.

The method of Sanmicheli is illuminating. A report on work he did for Duke Francesco Sforza II related,

> On arriving, Maestro Michele several times went round the site measuring it, then he got down to drawing, first in the form it has now, then, after making a separate sketch showing the additions he proposed for making it as strong as possible, he made a further survey to check the levels of the ground with a view to an effectual defense against mines.

Only then did Sanmicheli supply "some finished drawings" to the Duke.[37] It appears often to have been the case that military engineers drew designs directly on maps of surveyed sites. One such production in the Marciana Library shows the outline of the old city walls in pink, the ones recently constructed in green, and the architect's proposed amendments of new bastions and a star-shaped rivelin in yellow.[38] The tendency could be universalized. "Where steps the military engineer steps the chorographer," wrote the military engineer Lelio Brancaccio.[39] Indeed, survey and mapping techniques had long been important to the architect's tool box, and was one of the largest areas of overlap between architects and mathematicians. The practical mathematical abacco tradition was common training for early modern Italian artificers, and both mathematicians and architects knew survey techniques.[40]

Measuring instruments, most of which relied on simple triangulation, made measuring by sight easily accessible. Most were adaptations of the astrolabe or quadrant, but tracts on measuring instruments multiplied, and these were mostly written by military engineers, or authors catering to a military audience. There was the *verghe astronomiche* of the engineer Antonio Lupicini, the mathematician Niccolo Tartaglia's instrument in *Nova Scientia,* Mutio Oddi's *squadra* and *polimetro,* and numerous others. Lorini suggested readers turn to one of the compendia of measuring instruments, Cosimo Bartoli's Modo di Misurar (figure 5.5).[41] Measuring by this choice of sighting instruments must have been quite a boon to professionals for whom survey was a vital part of any project, and reconnaissance of enemy strongholds was a high priority.[42]

The seriousness with which this task was taken is testified by an episode in the tutelage of Buonaiuto Lorini. Lorini had been sent to Bergamo by his patron Giulio Savorgnano to make some designs of the city. He proved headstrong and "impatient," as Savorgnano discovered when he reached the city. Lorini had mis-measured a curtain wall by 30 *passi*, and was off on some of his angles by five or six degrees. Employing a particular quadrant, Savorgnano had "busted his brains" to show the

37 Hale 1977, 18.
38 Venice, Biblioteca Nazionale Marciana, Ms. Cl. 7, no. 2453 (10493). This appears to be a late seventeenth-century manuscript detailing information on the fortifications of Venice.
39 Lelio Brancacio 1590.
40 Rossi 1998; Veltman 1979; Adams 1985.
41 Lupicini 1582; Tartaglia 1537; Oddi 1625 and 1633; Bartoli 1564.
42 See for example Adams et al. 1988.

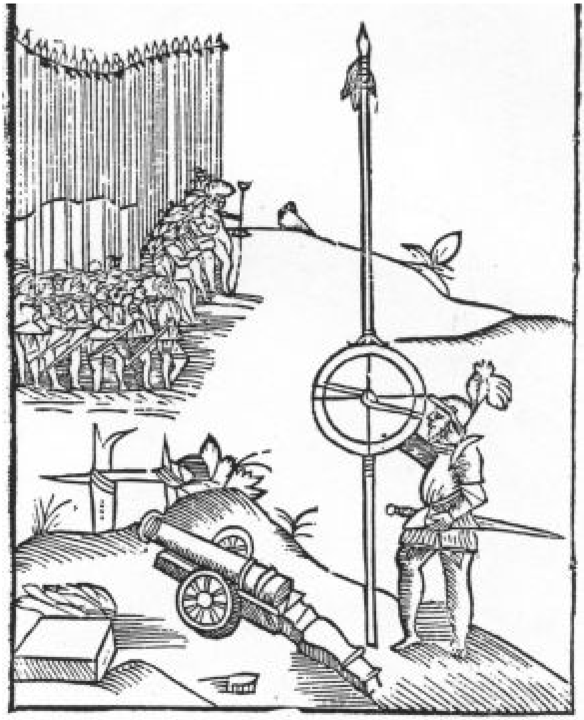

Figure 5.5. Instrument for measuring angles. From Bartoli 1564. Bartoli offered a compendium of mathematical techniques of measurement.

younger engineer his errors. He had Lorini re-do his model, and Buonaiuto produced some excellent designs. Full of fury, and horrified that some of the faulty designs had reached the patrician Giacomo Contarini, Savorgnano wrote the patrician, "If I were an absolute ruler like the Duke of Florence, I would have [Lorini] castrated, as they do to unmanageable horses who become good once they are castrated."[43] But it was probably Lorini's hands that Savorgnano wanted to rein in. Despite Savorgnano's rage over Lorini's faulty measuring, he probably needed the Florentine's facility in drawing in order to make fine, detailed plans. Lorini was one of the most talented of apparently numerous men of design Savorgnano kept as "familiars."

43 Venice, Archivio di Stato di Venezia, Archivio Proprio Contarini, B. 19, dated 6 August 1586.

If the cartographic survey formed a basis of models, the geometrical construction of the architect's projected fortification on top of it was the most prominent feature. In teaching design to young architects and *cavalieri*, this aspect of the practice also became freighted with theoretical references. Girolamo Cataneo emphasized that this "most divine, most certain, most useful science of numbers and measures" was the basis of all arts, but especially military discipline, in which it was needed to order battle, design fortifications, and understand artillery.[44] Cataneo's repeated association of his design methods with the great "Platonic philosopher" Euclid was also featured in the engineering treatise of Cataneo's pupil, Giacomo Lanteri.[45] We can see the geometrical progress of Antonio de Medici, the natural son of Francesco I of Florence, under the tutelage of the court mathematician, by don Antonio's copy-book. The compass marks etch the geometrical construction of regular polygons, from a triangle to sequentially greater-number-sided shapes. The final exercise is a pentagon on which regular bastions at each angle have been constructed. Ostillio Ricci, better known as Galileo's teacher, also taught his pupils, pages at the Florentine court, from Leon Battista Alberti's measuring tract, and is associated with a work on the instrument known as the *archimetro* (figure 5.6). And so it came to pass that mathematicians—like Cataneo, and Galileo's teacher, Ostillio Ricci, and Galileo himself—became teachers of aspiring young architects and well-born soldiers.

The emphasis on geometry and measurement—discussion of which was invariably studded with references to certainty and to the philosophical heritage of this knowledge, sometimes going back to the Chaldeans—accomplished two things. First, it served to put architecture on a par with liberal arts; this marked architects above common workmen, and made the duties of architecture more consistent with the education of noblemen and leaders. Second, knowledge of geometrical elements served, as Lanteri has Cataneo say, as an "alphabet" for learning design.[46]

Knowledge of measuring instruments and the Euclidean basis of the geometric design of fortifications was the knowledge shared between engineers, their military patrons and their sovereigns, regardless of their own war experiences. At top levels, engineering was often seen as a principal aspect of the role of a captain general or of an artillery general.[47] Fortification design was a topic of avid interest for princes such as Francesco Maria della Rovere and Charles V, who corresponded and disputed about the topics of where to plant artillery, where to put the gate in a curtain wall, or how to cover flanks with fire from bastions.[48] Numerous other princes were also fascinated by the new fortifications methods, as often indicated by the prefatory remarks of engineering treatise writers. Some had a personal stake in the matter, as was the case for Cosimo I of Florence whose control over fortifications in his own territories was a hard-won political right granted by the Hapsburg emperor who had supported

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44 Girolamo Cataneo 1584, III f. 2^r.
45 Lanteri 1557. Euclid the geometer was confused with the Platonic philosopher Euclid Megarensis throughout the sixteenth century.
46 Lanteri, 7.
47 For example, see in Savorgnano's collection Venice, Archivio di Stato di Venezia, Mat. Miste Nob., B. 14, f. 1^v; Collado 1606, first dialogue.
48 Promis 1874, 114

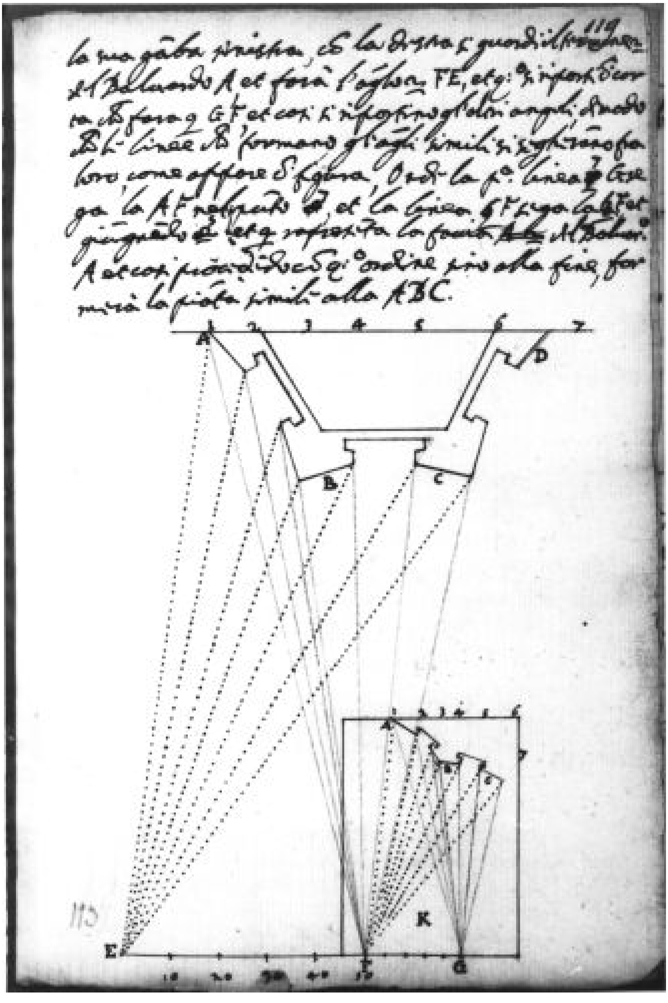

Figure 5.6. The fortification is reduced exactly to scale. From L'Uso Dell'Archimetro, attributed to Ostilio Ricci. (Florence, Biblioteca Nazionale Centrale Firenze, MS Magliabechiana VII 380, f. 118 v; courtesy Ministero per i Beni e le Attività Culturali, all rights reserved.)

the Medici rule.[49] Others were themselves military figures, as were della Rovere and Sforza. Near the end of the sixteenth century, academies for young *cavalieri* also multiplied. The intellectual content of these schools centered around the first six books of Euclid, and a plethora of measuring tracts.[50]

Geometry gave military architects clarity if not soundness of design. They shared a mode of communication and a lexicon of images. Even in the relatively early treatise of the architect Pietro Cataneo (no relation to Girolamo), the fortifications take on their "orders" from the number of sides of their enceintes. In Cataneo, these are always depicted as regular polygons.[51] In large part, this is because a number of other design choices are predicated on the sides of the enceinte, including the shape of the bastions. Thus the "orders" of fortification came from the number of sides. Even Giovan Battista Zanchi, the seasoned captain who extolled primarily practice in his treatise, followed such an ordering, and suggested that those wanting to learn fortifications spend some time with an "architectural manual of models."[52] These would provide not only schematic geometrical forms, but often perspective drawings "to show all that should be built." With the proliferation of military engineering manuals, copy-books incorporated designs from numerous sources.[53]

However, even the most geometrically-inclined textbook of military architecture (including Lorini's) would warn its readers that all fortification had to suit the site, and take full account of possible means of offense and defense. Survey and geometrical construction were the propaedeutic arts required by anyone who expected to produce or judge fortification designs. They were, of course, insufficient for planning models. What mattered was how well adapted to the task the designer had utilized fortification elements—bastions, cannon platforms, exits, trenches, curtain walls, casemates, covered ways, and more. The heights, wall thickness, bastion angles, distances could all be measured, but had to be measured by knowledge outside design. Nothing shows this so well as the written opinions about these designs.

3. WHAT DESIGNS ARE GOOD ENOUGH? CONTOURS OF CONTROVERSY

Textbooks and mathematicians offered instruction on how the military architect might achieve clear, measured models. What made the design good and why it was good, however, usually needed to be explained. Because the design itself should ideally have had a scaled one-to-one correspondence with the landscape, standing structures, and proposed structures, the model could act as a kind of substitute object. This meant that other engineers and commanders could assess the relation of the planned fortification to surrounding landscape features, and that they could in effect mount assaults and defenses on it. That the measurements should stand the test of this sort of

49 Diaz 1976, chap. 2.
50 Hale 1983c and 1983d; Torlontano 1998; for contemporary reference see Oddi 1625 and 1633.
51 Pietro Cataneo 1567.
52 Zanchi 1554, 56.
53 Oxford, Bodleian Library, Canon Ital. 289, "Regole di Fortificatione" is one such copy-book. It is comprised almost entirely of designs copied from Lorini, Tensini, Brancaccio, and others.

analysis shows us that the knowledge embodied in a design was much more extensive than a simple lexicon of forms. But that knowledge was almost never of a certain nature, despite its expression in the "certain science" of geometry, and therefore designs also embodied opinion.

Fortifications textbooks consistently regaled engineers with precepts on how to derive (*cavare*, literally "to dig out') the form of a fortification. The form is derived from the landscape; the defense is derived from the offense, the length of wall between bastions is derived from the distance artillery shoots; the angle of bastions is derived from the number of sides. That is, nearly every dimension is designed in reaction to conditions or factors that may or may not be represented. One had to see not just a model, but the way walls might hold up to the expected barrage of artillery, and the maneuvers the design would make possible given various tactics of the enemy. And of course, costs were measured, too. As a whole, the fortifications design had to be inspected as a machine design might, since the fortress increasingly became machine-like in that all elements had to relate to each other for mutual defense.[54] Unlike most machine drawings of the sixteenth century, however, fortifications were drawn to scale, and usually included actual measurements.[55] Measurement became the terms in which debate could be conducted. All measures—the length of curtains, area of bastions and platforms, depth and width of moats, among many others—also had to be related both to environmental factors and to each other, and that is where "seeing" and controversy crossed paths. Here I will look at just one exchange in a debate over the goodness of a design that was a single salvo in a long battle between rival fortifications experts.[56]

After the Venetians lost Cyprus, they were faced with dwindling control over their Adriatic possessions. The fortification of Corfu and Zara became incendiary topics in the late 1570s. Corfu was routinely figured as "the principal frontier of our empire as well as of all Christendom," and Zara was the Venetians' chief trading and naval base on the Dalmatian coast.[57] Venice had its top fortifications experts and engineers working on these projects, and also hired, as many other states often did, an outside inspector and consultant. Ferrante Vitelli was given leave from his post as Governor-General for the Duke of Savoy to be able to inspect the strategic Venetian sites where stronger defenses were to be erected, and to review the models for construction. Savorgnano was also involved in the Corfu and Zara projects and sided with the Governor-General, Sforza Pallavicino, against Vitelli's opinions. Pallavicino had been involved with the Corfu fortifications early on; he had sent his opinions on a first phase of fortification there before the fall of Famagosta (1571), and had by 1578 made plans for expanded defenses at Zara.[58] The two experts' criticism of each other would become bitter and very public. Already in October of 1578, Pallavicino had

54 Hale 1983a; Manno 1987; Wilkinson 1988.
55 See McGee 1999; Poni 1985; Marchis and Dolza 1998.
56 See debate over Palmanova in La Penna 1997.
57 Hale and Mallet 1984, 443–444.
58 Venice, Archivio di Stato di Venezia, Mat. Miste Nob., B. 7, ff. 18^{v}–20^{r}. For Pallavicino's career, see Promis 1874, 447–463; Mallet and Hale 1984, 302–308.

expressed disgust with Vitelli's own designs for Corfu, and the Governor-General was further irritated when he received from the Venetian Signoria Vitelli's criticism of his model for Zara along with a re-worked design. What designs were good enough? According to Vitelli, Pallavicino's was not.

Vitelli had "corrected" a number of elements, Pallavicino wrote back to the Signoria, in a way that would only end up being less effective, and much more costly. Vitelli had not "seen" that most of the problems he addressed had already been accounted for in the design. For example, Vitelli objected that one could enter a bastion through the opening in the trench where the gate was, "but did not see that the cannon platform near the lobe of the bastion (*orrechie*), and the platform on the bastion, could not be entered," since they were covered from the point of the opposite bastion—"a feature he didn't have on his fortress in Corfu." Moreover, according to the plans sent by the Signoria, the lobe that Vitelli had re-designed for the Grimano bastion did not give direct [access] to the inside of the structure, "as your Serenity can measure with a line, and will be able to know whether I'm telling the truth or not." Also, Vitelli had written that the shots from the bastion Santa Marcella would not be longer than 230 passi, and so would not cover the entire length of the wall. First of all, Pallavicino pointed out, the distance [between Santa Marcella and the next bastion], even if it was a little out of range for the archebuses [to strike] "in full force," was still within range for the archebuses to do some damage. More importantly, "the defect of this distance" was already corrected for in his designs by the *dente* (jagged protrusions) in the curtain, which would only be 160 passi away. "I don't know if he did not see it, or if it slipped his mind," Pallavicino noted. Also, the Signoria will note that in Pallavicino's original design, there is a *piazza* for at least three artillery pieces, which will also be well-protected by the *dente*, being higher. Vitelli had criticized Santa Marcella for being too low, too short, and the front angle too acute. However, the shortest side was 30 passi—the same measure as a side of the Citadella bastion in Vitelli's Corfu design; Santa Marcella's longest side measured 60 passi, and the angle was obtuse.[59]

Pallavicino's reply to his Venetian employers was an attempt to convince the Venetian patricians to see as he did by drawing their attention to the significance of the measures in his model, and how they relate. We can see here that measures are not so clear-cut, even apparently in comparison to guidelines like Savorgnano's. Several things are of note. First, Pallavicino and Vitelli continually refer to actions that take place on the design as if it were a real structure. The exception to this is at the one point when Pallavicino wants measurement to be a truth-telling exercise, and invites the senators to measure for themselves on the plane of the design whether there would be the necessary access to a bastion. Second, some measurements sometimes appear to have some obvious meaning that all the actors must have recognized. In particular, the length of the short side of the Santa Marcella bastion (30 passi) had appeared in Savorgnano's rules as the minimum room necessary to mount two cannons on a bastion. The fact that the angle of the bastion is obtuse also relates to what

59 Venice, Archivio di Stato di Venezia, Mat. Mist. Nob., B. 9, ff. 44^r–52^r.

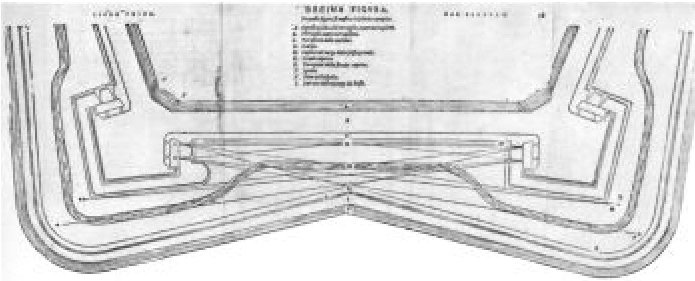

Figure 5.7. Lines of fire on a fortification plan. While these lines were regularly drawn on models in textbooks, private collections, and battle representations, they are seldom found drawn directly on a design submitted for a commission. From Girolamo Cataneo 1584.

was by 1578 a precept. A wide angle on a bastion gave it much less vulnerability at the tip where there could be a spot blind to defensive fire. On the other hand, when the difficulty of the length above 230 passi came up, Pallavicino first responded with a consideration of relative force with which an archebus shot could be made at that length. As Antonio Manno has pointed out, artillery shot was the unit in which a fortification was measured. However, the variety of positions and kinds of artillery, combined with their variable impact for various reasons, meant that this unit could not be a single standard. Nevertheless, in the language of design, measurements were the terms by which questions of tactics could be addressed (figure 5.7).

Vitelli, as a veteran inspector, had his own list of considerations for planning fortifications. It ran over thirty pages long, and was an attempt to enumerate everything that should be looked at during a site visit, and how to imagine every possible means of offense and defense in every possible landscape, under every possible administrative condition.[60] What is interesting is that what could often come out at the other end of this process—the examination of an exhaustive array of variables and considerations—were rules (measures) for judgment found in the writings of Savorgnano, Pallavicino, and Orsini.[61] That is, the "rulebook" measures could only be an approximate control on the infinite uncertainties of tactics and artillery performance.

Direct measure in and of itself did not decide anything; it only provided the basis on which one could form an opinion. Savorgnano commented on one of Vitelli's presentations to the Venetian Signoria, "These engineers of yours, whom I like to call "soffistici," want to make everything sound black-and-white, and do not understand the principles of things that make designs and not words, so that with a compass in hand, one can make them see their errors, or if they design something well, they

60 Vitelli's "Istruzioni" are transcribed in Promis 1874, 606–638.
61 Orsini's rules are collected in Venice, Archivio di Stato di Venezia, Mat. Mist. Nob. Pallavicino's military precepts are included in Ruscelli 1568, fol. 55–56, and Promis 1874, 447–462.

deserve praise."[62] Whatever may be the faults or merits of a plan, Savorgnano suggests that no decision can be rendered at all without the design. Only then could one study, with compass in hand, how well proportioned the design was, how well "dug out" from the site and how well "derived" from enemy offense. The compass was as necessary to "reading" the design as were the eyes, but neither eyes nor compass could reason about the design without words.

The process by which a design was finalized could be a drawn-out affair, and often depended on the internal organization of the state *vis a vis* its technical corps.[63] By 1582, four years after the original exchange between Vitelli and Pallavicino, the question of the Corfu fortifications became an embarrassment. Savorgnano and Pallavicino had in the intervening years lost no opportunity to criticize Vitelli, privately and publicly. Savorgnano had apparently even tried to get the mathematician Guidobaldo del Monte to publish a critical analysis of the Corfu fortifications.[64] The Senate declared that two provveditors-general should go to Corfu with the engineers Lorini and Bonhomo "because for many years now it has been known to all rulers and in all places that various objections have been raised to the fortifications in Corfu." Venice wanted to have these differences of opinion resolved "for the benefit of Christendom and ourselves."[65] Both Lorini and Bonhomo were clients of Savorgnano, and it would be their job to help the provveditors see in stone what could not be decided in design.

4. WHAT IS GOOD DESIGN? PRINCIPLES

The first person to claim in print that one might be able to judge the design of a fortress based on "principles of things that make designs" was the practical mathematics teacher, Niccolò Tartaglia. Tartaglia attempted to turn the arts of artillery into a mathematical science in his *Nova Scientia* of 1537.[66] He extended his researches into military arts, and published a series of didactic dialogues in *Quesiti et Inventioni* nine years later.[67] Tartaglia expressed the dream of judging designs merely by looking at them in the first dialogue of the sixth book of *Quesiti*. The first question revolves around the fortifications of Turin. The military engineer and artillery general Gabriel Tadino asked Tartaglia, was this not proof that "the *ingegno* of man has reached at present the most sublime level that it is possible to reach?" Although admitting that he has no experience of fortification at all—indeed has only seen a few—Tartaglia replies that the *ingengo* of a design is known only by the form, not the material. Having only his mathematical wits to rely on, Tartaglia had to insist that whatever the strength of the material construction, the flaws of the design itself were manifest in

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62 Opinion on Vitelli (and other engineers employed by Venice) in Venice, Archivio di Stato di Venezia, Mat. Mist. N., B. 11, f. 210, quoted in Manno 1987, 228.
63 For Florence, see Lamberini 1986, 1987 and 1990; Casali and Diana 1983, Fara 1988.
64 Henninger-Voss 2000, 248–250.
65 Venice, Archivio di Stato di Venezia, SS. Reg. 83, 58–58^{v} (16 Feb), quoted from Hale and Mallet 1984, 444.
66 Tartaglia 1537.
67 Tartaglia 1546.

form only. Form by itself could add to the "strength" of the walls. To Tadino's insistence that Turin is as well fortified as any city could be, Tartaglia answers, "In this your reverence is greatly deceived."[68]

Tartaglia's analysis of the Turin fortifications is primarily based on its quadratic form and the proportionate smallness of its bastions. In order to make the heaviest strikes against a fortification wall, the cannonballs must strike perpendicular to the wall. With a quadratic design, there would be ample opportunity to do so. The small, acute bastions could only offer inadequate covering fire to the curtain walls. On the one hand, it is difficult to believe this was such disappointing news to Tadino as Tartaglia represented it to be. The quadratic form was already out of favor among fortifications designers, and Tadino had a long distinguished career as a fortifications superintendent and artillery general for Venice, Malta, and the Holy Roman Empire.[69] On the other hand, the Turin fortifications purportedly gave the city the aspect of an austere stronghold, and were something of a conversation piece.[70] In the 1540s new four-sided fortifications were still being built, or at least refit, such as those at Pistoia. However, by the next generation of engineers, four-sided fortifications would be entirely condemned.[71]

It is not so clear whether this rule of fortification was spread more by participation in the profession or by books such as Tartaglia's. In 1554, Giovan Battista Zanchi authored the first published architectural treatise devoted entirely to fortification.[72] He drew heavily on Tartaglia, so that even the small "map" of the Turin walls, represented as a perfect square with tiny bastions, seems to have been drawn from the mathematician's earlier book (figure 5.8).

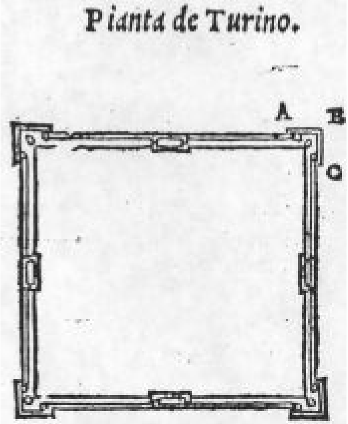

Figure 5.8. The plan of Turin from Tartaglia 1546.

An interesting aspect of Tartaglia's exchange with Tadino in *Quesiti et inventioni* is that Tartaglia constantly refers the general back to his drawing board. In the questions following the discussion of Turin, Tartaglia suggests that there are six "conditions" for an unassailable fortification. Tadino challenged Tartaglia "who can neither design or plan nor build a model" to explain them. Yet after naming each (for example, that no enemy be allowed to shoot perpendicularly to any walls), Tartaglia invari-

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68 *Ibid.*, 64^r–65^r.
69 See Promis 1874, 41ff.
70 Pollak 1991, chap. 1.
71 E.g. Savorgnano in Venice, Archivio di Stato di Venezia, Mat. Mist. Nob., f. 13, see 2^r; Alghisi 1570.
72 Zanchi 1554.

ably refers the solution (and the explanation itself) to what could be seen on a model. Tadino protests that he has never seen a fortification that could not be battered perpendicularly; Tartaglia replies, "You could see in a design how this could be achieved."[73] Tartaglia claims that the bastions and *cortina* could be designed in such a way that the enemy would be forced to plant their battery only in a disadvantageous spot near the bastion. Tadino promises to return in a few days with "a plan designed by my own hand" that would fulfill these conditions. Tartaglia expresses his confidence in the general: anyone advised by Tartaglia could make such a design, much less Tadino "who is the height of *ingegno*."[74] As it turns out, however, the mathematician "who can neither plan nor build a model" has one, and slowly he begins to entice Tadino with it. After Tartaglia states his third and fourth conditions (that ordered battle arrays be vulnerable to defensive fire in four directions and that if the enemy does batter down a wall, it would be too dangerous for them to enter), Tadino "begins to imagine" how this could be done. "But I want to consider this better, and make a little model, because in making models a thing is better clarified, and then we will see if my opinion is the same as yours." Tartaglia answers again that in Tadino's design he will probably discover Tartaglia's own models. The general adds that "the practice, reasoning, and dispute on a subject makes one discover many things … and one becomes introduced to new particulars that afterward he can think over and easily discover."[75] The process of design itself reveals relationships, but that process is itself—at least in Tartaglia's hopeful depiction of it—tied up with a larger discourse in which novelties are exchanged and examined. By the end of this series of questions, Tartaglia coyly agrees that he will only give up his models to Tadino himself. What he gives in his original 1546 edition, however, is primarily a list of dimensions: the cortina should be 7 feet wide up to 10 feet high, then tapered to 2 feet, but with a counterscarp of 8 feet …

Only in the next edition of *Quesiti* (1554) did Tartaglia provide some actual designs, addressed as a *gionta* to the philosopher Marc Antonio Morosini. These designs are rather clumsy, but do put into publication some of the newest ideas in fortification, including a bastion with a lowered cannon platform that appears to have been derived from one designed by Antonio da Sangallo for a Roman fortification (figure 5.9). Tartaglia, who apparently counted a number of architects among his disciples, admitted that this design was in fact supplied to him by a student. A later sixteenth-century annotator of the *Quesiti* noted that the very same bastion was to be found in the treatise of Carlo Theti, and that Theti had probably stolen it from someone else.[76] The annotator bemoaned the state of such "novelties." I think, however, such comments misunderstand the nature of Tartaglia's "inventions."

73 Tartaglia 1546, f. 65^{r}.
74 *Ibid.*, f. 66^{r}.
75 *Ibid.*, f. 66^{v}.
76 Exemplar in Museo della storia della scienza in Florence, once believed to have belonged to Galileo.

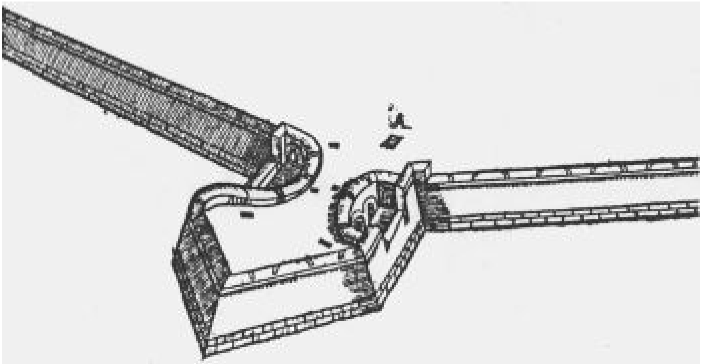

Figure 5.9. A plan of a bastion reproduced in the 1554 edition of Tartaglia 1546. The bastion appears to involve the innovations of Antonio da Sangallo from a few years before.

Tartaglia prized not his own design ability, but his ability to reason over designs, and put them in a discourse that could analyze and mathematize the design problem. His own work on ballistics with which the major portion of *Quesiti* is concerned could have had real design effects. For example, in the first book, he attempted to prove that a cannon placed in a valley an oblique 100 passi bellow a fortification would make more damage shooting upwards than a cannon placed on a hill directly across from the wall and only 60 passi away. This of course would hardly be an expected result. Tartaglia analyzed the common gunner perception that perpendicular hits caused the most damage in mathematical terms of positional gravity. By referring readers to weights on a balance, Tartaglia showed that the angle of impact mattered less than the speed and trajectory of the cannonball. Thus the mathematician claimed to correct faulty extensions of practical knowledge as in the case above.[77] Evidence that such studies *did* become considerations of designers is indicated by some of Tartaglia's plagiarists. When Walter Ryff published large portions of Tartaglia's books translated into German, it was as part of a large quarto volume that was meant to serve the total geometrical architect, and was published as one section in larger treatments of perspective and architecture.[78]

This study of artillery impact through philosophical traditions of analysis was one that would be developed for the remainder of the century by mathematicians up to and including Galileo.[79] Cataneo's Euclidean "alphabet" was the basis of design—

77 Henninger-Voss 2002.
78 Ryff 1547 and 1558. This was the second and third part of a large volume that also included other architectural treatises that drew on mathematical sources. A facsimile edition exists: Hildesheim/New York, 1981. I owe this reference to Rainer Leng.
79 Laird 1991.

indeed the opening pages of Lorini's and Galileo's fortifications tracts summarized the first half book of Euclid—but architectonic thought for fortifications required not only knowledge of the proportionate relationships that one could see, but also begged ones (like ballistics) that had to be analyzed in separate study.[80] It was nevertheless design's foundation in the practical mathematics of design and measure that no doubt gave a practical mathematics teacher like Tartaglia the foothold to present himself as an advisor to a man like Gabriel Tadino, a man so distant in profession and so far Tartaglia's superior in status, experience, and education.

It is impossible to say with any surety what the precise manifestation of these theoretical excursions on the impact of moving weights could have been on fortifications design. The military architects Francesco de Marchi and Jacopo Fusto Castriotto both corresponded with Tartaglia. Castriotto opened his letter floridly praising Tartaglia's *ingegno* in every science, but particularly "artillery and its effect … and the fortification of cities and castles." He begged Tartaglia's opinion on an enclosed design and discourse.[81]

On the other hand, a less impressed Giulio Savorgnano, who had been featured as one of the interlocutors in *Quesiti,* had his dwarf send Tartaglia some further questions, "to make him figure out some delightful things."[82] The questions are almost entirely questions of what the cause of some effect is, like why the compass does not point to true north, or puzzles, like whether a mortar will have a greater effect if it is shot from a height down, or from a low position upwards. They are questions that either no one knew, or that Savorgnano seems already to have known either from his own battle experience, or from the vast number of experiments he performed with artillery at his own castle, Osopo. They are challenges to Tartaglia's claims to discover causes of what is already known by experience, and to be able to know by mathematical analysis what other men find out only through direct experience.

As a way of "ratiocination" that predicted the outcome of experience, Tartaglia's books were useful as teaching aids for readers of little experience, and even as a guide for those who had much. They became standard references for artillery manuals for another century. Even in eighteenth-century France, Tartaglia's relatively crude conceptualization of projectile motion appears to have been employed by common gunners, while Academy-trained artillery officers and designers were trained in the new rational mechanics that had developed out of the Galilean and Newtonian traditions. And the merits of this mathematical education were still unclear.[83] Tartaglia himself would have thought of himself as helping soldiers "understand the principles of things that make designs." But it is also clear that Tartaglia wanted to affect the understanding of practice not only of men like Gabriel Tadino and Giulio Savorg-

80 For identity of the Lorini–Galileo tracts, see Lamberini 1990.
81 Urbino, Urbino Università, Fondo del Commune, B. 77, fasc. VI, dated 27 Dec. 1549.
82 The questions are copied in Venice, Archivio di Stato di Venezia, Mat. Mist. Nob., B. 13, ff. 55^{v}–57^{v}. They are apparently misdated, however, as 1542, since question 26 (57^{v}) begins "I saw what you said about granulated [gun]powder," a discussion that could not have been "seen" until the *Quesiti* of 1546. For serious views of the relation between Savorgnano and Tartaglia, see Manno 1987; Ventrice 1998; Keller 1976a.
83 Alder 1997, 93 and *passim*.

nano, but also of decision-makers—such as patricians and secretaries to princes—who were constantly called upon to judge design, but who often had no other lens with which to view them other than the teachings of experts and books.[84]

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Recent studies of engineering history have focused on the visual basis of the "knowledge" of the engineer, and have been skeptical of the contributions "theory" has made to engineering practices. Eugene Ferguson, for example, specifically relates specialized training and theoretical argumentation to the engineer's need to legitimize his services before patrons.[85] It seems to me, however, that the intervention of practical mathematicians like Tartaglia was a natural extension of the design practice of architects, which was itself a mathematical practice. Certainly Tartaglia saw this as one point of entry for his services. Even if men like Giorgio Vasari had consciously attempted to heighten the status of design professionals by uniting them and claiming a theoretical basis for their practice, that practice had for a long time been sophisticated geometrically, as Wolfgang Lefèvre and Filippo Camerota have so amply shown, right down to the art of the stonemason. What changed in sixteenth-century military architecture was that the design process was shared by a far greater variety of men; models were evaluated within a more developed hierarchy (indeed were central to new bureaucratic agencies); and that the nature of its goals were tied to the concurrent, but exterior, developments in strategy and artillery. The spheres of knowledge broadened which, in the terminology of Michael Mahoney, had to be "bootstrapped" in the construction of fortifications. While engineers continued to employ a language of "seeing" in their opinions on designs, their drawings referred in their measure, execution, and planning to an ever more complex "architectonic" synthesis of geometrical techniques, strategic imagination, and newly emerging sciences of weights in motion.

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84 [Henninger-]Voss 1995; Cuomo 1997.
85 Ferguson 1992, 65–73.

PART IV
PRODUCING SHAPES

INTRODUCTION TO PART IV

The topic of this section is the origins and development of the geometric techniques used for drawings and geometrical constructions employed in the process of planning and producing devices or buildings. Thus, it also discusses the origins and development of such techniques as an integral part and epitome of the professional knowledge and skill of engineers and architects.

Early modern depictions of machines, on which most of this volume's chapters focus, do not immediately make accessible the wealth of drawing techniques employed by the engineers and architects of the age. Apart from a few orthographic plans of machine parts, since the days of Taccola machines were usually depicted in a style that looks like a kind of naïve perspective rendering at first glance. No geometry seems to be needed for such creations. On closer inspection, however, it becomes obvious that these depictions not only use deliberately advantages of rendering in perspective, in particular as regards the choice of the viewpoint, but often employ different projection techniques such as military perspective. When one starts to pay attention to these technical details, these seemingly naïve depictions of machines appear embedded in the whole range of geometrical projection techniques that were partly rediscovered, partly reinvented, and in any case autonomously developed and elaborated in the Renaissance—such as perspective projection, orthographic projection, combined views, oblique projection, special projections in the realm of cartography, and stereometric constructions for special architectural purposes. Taking into account as well that engineering, architecture, and, as figures like Leonardo, Dürer, Raphael, and Michelangelo prove, even fine arts were trades not as secluded in this age as later, it becomes clear that the different drawing techniques developed in close connection. They form a whole of techniques and procedures for the production of technical drawings developed by and available to the broad community of engineers, architects, and artists.

The chapter by Filippo Camerota traces in detail the development of the most important methods employed for technical drawings—linear perspective, military perspective, shadow projection, double projection, and stereometry. The chapter's emphasis lies on the codification of each of these methods, that is, the codification of both a method's theoretical principles and the procedures and techniques that can legitimately be applied in its pursuit.

The distinction between drawing methods or styles such as perspective or orthographic projection, on the one hand, and of techniques and procedures through which they were realized, on the other, deserves attention. The whole of drawing methods available to engineers, architects, and artists consisted not only of a variety of styles, each of which had different advantages and shortcomings. It also was a many-faceted whole as regards the techniques and procedures used to achieve a rendering in each of these styles. To give an example: to trace a true perspective rendering of an object, one can construct it according to the rules of geometry and geometrical optics—if one has sufficient command of these rules and of the construction techniques required by them; if not, one can use mechanical or optical aids and achieve the same result.

Techniques and procedures of drawing thus can be distinguished according to the knowledge that they require on the part of the draftsman. It makes sense to speak of learned drawing techniques in contrast to more practical ones. To a large extent, this distinction between learned and practical drawing procedures coincides with that between learned geometry and practitioners' geometry. In our context, this distinction is of particular significance in two respects—with regard to the roots of the different drawing styles and techniques, on the one hand, and, on the other, with regard to the actual employment of these different techniques by engineers, architects, and artists in their professional practice.

The chapter by Wolfgang Lefèvre treats the question of the roots. It studies Antonio da Sangallo's and Albrecht Dürer's invention of the combined views technique, a method of plan construction on which modern plan constructions essentially rest, and traces its learned background as well as its roots in medieval practitioners' geometry. The chapter by Jeanne Peiffer discusses the practical employment of drawing techniques of a learned character. In pursuing the fate of highly developed drawing techniques elaborated by Dürer among artists of sixteenth-century Germany, and further up to their mathematical codification by Desargues at the beginning of the seventeenth century, it traces a history of a gradual divorce of learned and practical drawing procedures.

6

RENAISSANCE DESCRIPTIVE GEOMETRY: THE CODIFICATION OF DRAWING METHODS

FILIPPO CAMEROTA

The need for drawing as the visual expression of ideas forcefully emerges in the first treatises of the Renaissance, particularly in the field of architecture. As Leon Battista Alberti wrote, "the drawing will be a precise, uniform representation, conceived in the mind … and brought to completion by a person endowed with ingenuity and culture."[1] It appeared obvious, and not only to a humanist such as Alberti, that drawing was considered a cultivated language, accessible only to those who knew its basic rules. According to Filarete, "knowing how to read a drawing is more difficult than drawing," so that, while there were many "good masters of drawing … if you ask one according to what rule he has drawn a certain building … he will be unable to tell you."[2] A good architect, then, had to be both "practical and knowledgeable," in the words of Francesco di Giorgio;[3] that is, he had to possess an adequate *dottrina*, or knowledge of the disciplines essential to his profession; he had to have *ingegno*, or inventive talent and the ability to solve technical problems; and he had to know *disegno*, or drawing, in order to correctly represent the product of his ingenuity and to convey his thought immediately and unequivocally to others. These are the basic premises underlying the extraordinary efforts expended by Renaissance theoreticians to codify the rules of graphic language that laid the foundations for what, near the end of the eighteenth century, Gaspard Monge was to call "descriptive geometry."

This work of codifying continued over a rather long span of time and was of course favored by the spread of printing and the rise of the academies. In this chapter the various stages of codification in writing will be examined in chronological order, grouped under such general headings as painters' drawings, military engineers' drawings, the projection of shadows, architectural drawings, and those of stonemasons.

In the *corpus* of Renaissance treatises, the intention of instructing an architect, an engineer, or a "practical and knowledgeable artist" emerges from the structure conferred on the development of the themes, which range from elements of practical geometry and workshop expedients to geometrical demonstrations based on Euclid's *Elements*. Considering the great abundance of *ingegno* in the construction sites and workshops—as brilliantly demonstrated by works of art and architecture—the theoreticians' objective was that of forming a *doctrine* of drawing that not only would serve to refine the techniques of the profession but also, and more importantly, to raise the intellectual status of those whose art was grounded in geometry. This need

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1 Alberti 1975, I.1, transl. from Alberti 1988.
2 Filarete 1972, 157–158.
3 Francesco di Giorgio Martini 1967, VI 489–490.

was felt not only on the artistic but also on the technological level, where verbal descriptions of the complicated mechanical devices that drove worksite and war machines were now intentionally corroborated by the great descriptive value of drawings. This significant change with respect to medieval tradition is well documented in the manuscripts of Mariano di Jacopo, known as Taccola, and Francesco di Giorgio, where a progressive perfecting of techniques of representation may be noted, undoubtedly favored by the progress then being achieved in the field of pictorial perspective.[4] Francesco di Giorgio's appeal to the need for *drawing* as the visual expression of *ingenuity* is echoed by the even stronger admonition of Leonardo, who in one of the Windsor folios points out the inadequacy of words to describe the works produced by man: "But let me remind you that you should not stammer with words … since you will be greatly surpassed by the work of the painter."[5]

1. PAINTER'S GEOMETRY: THE DEVELOPMENT OF LINEAR PERSPECTIVE

The first attempt to establish rules and principles for geometric drawing was made by Leon Battista Alberti, who between 1435 and 1436 composed two works, closely linked and addressed primarily to painters: *Elementa picturae* and *De pictura*.[6] The title of the first book explicitly recalls the important treatise by Euclid, which had been the chief reference point for the mathematical sciences since the Middle Ages. Euclid's *Elementa* was a fundamentally important text in both the universities, where it furnished the geometric basis for studies on optics, astronomy and the other arts of the *quadrivio*, and in the abacus schools, where practical mathematics was taught to youths who were to go on to commerce and the crafts. During the fourteenth century the abacus schools underwent notable development, especially in Florence, and among the pupils were youths destined to become architects, painters, goldsmiths or sculptors. Filippo Brunelleschi, for instance, was one of them.[7] In composing his "Euclid for painters," Alberti thus addressed himself to a public already familiar to some extent with the principles of practical geometry. His indications serve to construct, by relatively simple steps, plane figures both in their true shape and diminished through perspective. It was only the humanist presumption of being able to explain the problems of drawing without drawings that kept these instructions from being immediately comprehensible. In this sense Alberti could be one of those "most worthy authors" mentioned by Francesco di Giorgio who, having chosen to explain their concepts "with characters and letters … and not by figured drawing," had made the content of their works obscure to many, being "rare those readers who, not seeing the drawing, can understand it."[8] In spite of its lack of drawings, *Elementa picturae*

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4 See McGee and Long in this volume. On problems of representation in the treatises of Taccola and Francesco di Giorgio, see also Galluzzi 1996b, 24ff.; and Lamberini 2001, 3–10.
5 See Francesco di Giorgio Martini 1967, VI 489–490; and Leonardo, Windsor, Royal Library, K/P 162^r (the passage is quoted in Galluzzi 1996b, 84).
6 *Elementa picturae*: see Alberti 1973, III 112–129; and Gambuti 1972, 131–172. *De pictura*: see Alberti 1973, III. For an English-language edition, see Alberti 1972.
7 See Manetti 1976, 52. On abacus schools, see Van Egmond 1980; and Franci and Toti Rigatelli 1989.
8 Francesco di Giorgio Martini 1967, VI 489–490.

was designed to furnish the rudiments necessary for an understanding of *De pictura*, where the themes dealt with, explains Alberti, "will be easily understood by the geometer. But he who is ignorant of geometry will understand neither these nor any other discussion on painting."[9]

The *De pictura* has no drawings either, but in this case the work is more conceptual than practical. Apart from his explanations of the method for constructing a floor in perspective and for using an instrument to draw from life, Alberti is interested mainly in defining the optical-geometric principles that govern the plane representation of three-dimensional space. The practical procedures were in fact already familiar to painters. Alberti had been stimulated to write on this subject expressly by the perspective skill of artists such as Brunelleschi, Masaccio, Ghiberti, and Donatello. On the plaster underlying the *Trinità* in Santa Maria Novella, for instance, Masaccio had left incised a practical but impeccable method for drawing the perspective of a foreshortened circle within a square; while in the *Nativity* of San Martino alla Scala, Paolo Uccello had left a fine example of what is today termed "distance point construction," which was at the time a practical way of representing the progressive decrease in size of a perspective floor directly on the painting, employing three vanishing points, one central and two lateral.[10] What was lacking was a text allowing artists to explain the reasons for their mode of operation, an "entirely mathematical" book—stated Alberti—which would explain the abstract concepts of geometry through the language of painters. Since the painter "studies only how to counterfeit what he sees" (I.2), Alberti is concerned with explaining the manner in which the eye perceives the reality around it, how the lines of sight measure sizes "almost like a pair of compasses" (I.5-6), how the Euclidean theory of similar triangles explains the relationship between real and apparent size (I.14), how in order to construct similar triangles it is necessary to cut the lines of sight (I.13), and how, in final analysis, "painting is none other than the intersection of the visual pyramid" (I.12). Comprehension of this last concept is entrusted to a fitting metaphor, which describes the painting "as an open window through which I look at what will be painted there" (I.19).

The way in which these concepts could be usefully transformed into a method of representation is explained by Alberti through a procedure that critics have always interpreted as an abbreviated form of the more laborious construction attributed to Filippo Brunelleschi.[11] Basically, Alberti seems to have developed a painters' method through which perspective could be drawn directly on the painting, thus avoiding the long architects' procedure that called for the use of preliminary draw-

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9 Alberti 1973, III 53.

10 The method used by Masaccio consists of foreshortening the square inscribed in the circle, dividing each side into the same number of parts, and joining each point to the matching one on the contiguous side. This traces a series of tangents to the inscribed circle, which immediately appears in elliptical form. The traces of this construction method are clearly visible at the base of the capitals on the front columns. The construction method adopted by Paolo Uccello to foreshorten the perspective floor consists instead of tracing three points on the horizon line, one central and two at the ends, of joining the divisions of the ground line to those of these points, and tracing the parallel straight lines identified by the intersections of the three bands of converging lines. This construction method can be seen in the sinopia of the fresco now in the storage deposit of the Uffizi; see Camerota 2001, 93.

11 See Grayson 1964, 14–27.

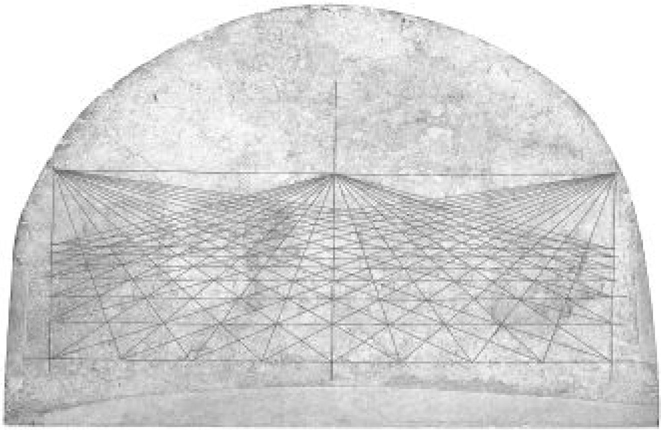


Figure 6.1. Perspective scheme constituting the sinopia of Paolo Uccello's *Natività*. (The fresco and its sinopia are now in the Uffizi Gallery, Florence.)

ings in plan and elevation. With Alberti's method, the principle of intersection of the visual pyramid could be applied to the construction of the flooring alone, which, being conceived as a modular grid, furnished the measurements for all of the objects appearing within the depth of the perspective field; the sinopia of Paolo Uccello's *Natività* shows how this was a real pictorial requisite (figure 6.1). Piero della Francesca developed this construction method still further, teaching artists to transpose points from the plan to perspective utilizing only the central vanishing point and a diagonal line.[12] The three points used by Paolo Uccello were to find a sort of codification in the work of Jean Pelerin le Viator, who in the first illustrated printed book on perspective (1505) described the methodical use of the so-called "tiers points," two lateral points similar to those used in architecture to draw the curve of a pointed arch.[13] The "third point" was not to be more appropriately termed the "distance point" until the writings of the great codifiers of the sixteenth century: Albrecht

12 Piero della Francesca 1984, I.28–29.

13 Viator 1505, in Ivins 1973. In the work of Viator "tiers points" are often used to construct buildings or interiors in an angular view. The term "tiers point" also appears in the notebook of Villard de Honnecourt (see Hahnloser 1972, 115–116) and is used to indicate the center of the compass in drawing a pointed arch. To trace this type of arch, the architect first draws the springing line, then points the compass at each end of the line, with the opening equal to the length of the line, and traces two arcs of a circle which intersect on the central axis. The "third points," i.e. the centers of the compass at the ends of the line, are also the points of convergence of the joint lines between the stone or brick ashlars of which the arch is built.

Dürer, who used it to teach how to operate according to the "the shortest way;" Sebastiano Serlio, who explained, not without some uncertainty, its relationship with "Albertian" construction and Giacomo Barozzi da Vignola, who was to make it the basis of his "second rule of perspective" (figure 6.2).[14]

The "first rule"—the "longest way" in Dürer's words—was instead represented by the method attributed to Brunelleschi by Giorgio Vasari, namely the methodical use of the plan and elevation views as graphic documents necessary to measure the intersection of the visual pyramid with the picture plane (figure 6.3).[15] Since this was

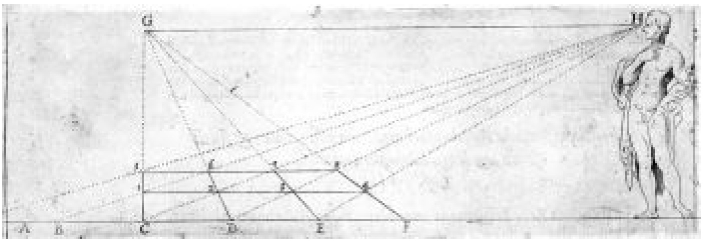


Figure 6.2. Second rule of perspective (G, vanishing point; H, distance point). Woodcut by Giacomo Barozzi da Vignola. (Vignola 1583.)

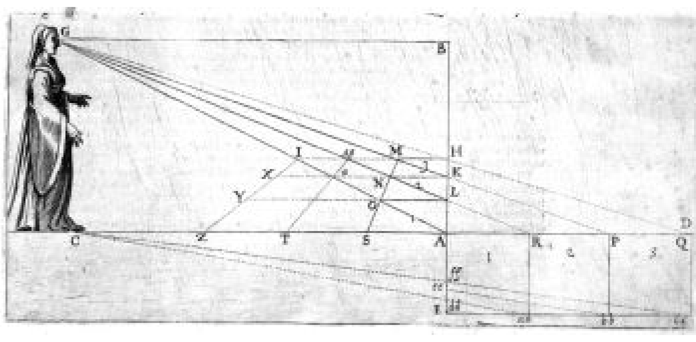


Figure 6.3. First rule of perspective (AE, picture plane in plan; AB, picture plane in elevation). Woodcut by Giacomo Barozzi da Vignola. (Vignola 1583.)

14 See Dürer 1983, IV figure 59–61; Serlio 1545, II 19; Vignola 1583, 98ff.

15 Vasari, *Vita di Filippo Brunelleschi*, in Vasari 1906, II 332: "*egli trovò da sè un modo ... che fu il levarla* [la prospettiva] *con la pianta e profilo e per via della intersecazione*."

a problem of solid geometry—the intersection of a cone by a plane—it was necessary to adopt a procedure up to then used mainly by architects, the only one that allowed a three-dimensional object to be visualized in a two-dimensional drawing. The first clear description of this procedure was given by Piero della Francesca who, while acknowledging the greater operational difficulties involved, considered it "easier to demonstrate and to understand …"[16] The two projections in plan and elevation, in fact, made the basic geometric model—the section of the visual cone—increasingly evident. Only this procedure could, moreover, guarantee a correct representation of the most complex objects such as vaults, capitals, *mazzocchi* and even the human head, which Piero geometricized with meridians and parallels, almost as if it were the terrestrial globe. The operational difficulties lay not only in preparing the preliminary plan and elevation drawings, but also in transposing the intersections to the painting, an operation, which Piero suggested could be facilitated by using simple "instruments': a hair from a horse's tail to trace the intersections in orthogonal projection with maximum precision, and various strips of paper to measure their "latitude" and "longitude" to be transferred to the base and to the vertical edges of the painting, respectively.

The concept that the perspective position of a point could be measured by two coordinates, just as cartographers defined the topographic position of a place, was very clearly expressed in the invention of the first instruments for perspective drawing. If the grid of Leon Battista Alberti derives from the method of *quadrettatura* employed by painters to enlarge their drawings, the so-called "sportello," or window, of Albrecht Dürer seems to show a more direct link with the cartographers' grid.[17] A frame with two orthogonal wires used to transfer to a drawing the coordinates of places from the Ptolemaic tables was described in the *Cosmographia* by Petrus Apianus (1524). It shows features similar to those of Dürer's instrument.[18] The "sportello," described in the fourth book of the *Underweysung der Messung* (1525), represents a constant reference point in the literature on perspective of the sixteenth and seventeenth centuries. Egnazio Danti cited it in his commentaries on Vignola expressly to explain "in what consists the foundation of perspective," since with it "we will see distinctly both the visual cone and the plane that cuts it" (figure 6.4).[19] The line of sight is in fact visualized by a thread tied to a nail, which serves as the eye. The point at which this thread traverses the frame, which marks the pictorial sur-

16 Piero della Francesca 1984, III 128ff.

17 In *De pictura* (Alberti 1973, III), II.31, Alberti describes the use of a transparent "veil" woven of thicker threads, which formed a reference grid used for drawing from life. An object seen through this grid could be easily traced on a drawing sheet with a *quadrettatura* of the same size. Dürer describes this instrument in the enlarged edition of the *Underweysung der Messung* (1538), noting its usefulness in enlarging drawings. Each square in the grid could in fact correspond to a sheet of the desired size. In the first edition of the treatise (1525), the German painter had proposed the use of two different instruments: a simple pane of glass on which the artist could draw directly the scene appearing through it, and a more refined frame fitted with a shutter, which allowed an object to be drawn by measuring the lines of sight "by means of three threads." One of the threads traversed the frame simulating the line of sight, while the other two defined with their intersection the picture plane and contemporaneously fixed the point of intersection between the plane and the line of sight.

18 See Apianus 1524, I 60.

19 Vignola 1583, I.3 55–56.

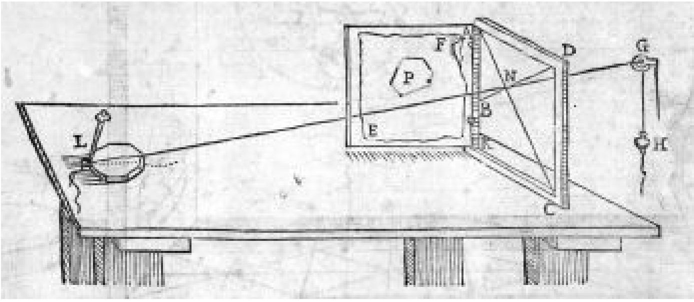


Figure 6.4. Albrecht Dürer's "sportello." Woodcut by Egnazio Danti. (Vignola 1583.)

face, is identified by the two orthogonal threads, which measure its "longitude" and "latitude." The intersection of the threads is then simply transferred onto the drawing sheet once the "sportello" has been closed.

The idea of materially representing the line of sight with a thread or cord had already been proposed by Francesco di Giorgio Martini in his treatise on practical geometry.[20] The context and the evidence of the drawing, which illustrates the perspective instrument, testify to the close relationship between the geometry of painters and that of surveyors (figure 6.5). This relationship also transpires in writings by Leonardo and Luca Pacioli, making it highly probable that Brunelleschi's invention sprang from the elaboration of a perspective procedure well known to surveyors and already taught in the abacus schools.[21] The concept of intersection of the visual pyramid was in fact basic to the methods used to measure distances by sight. An instrument described in the late fourteenth century by the abacist Grazia de' Castellani to measure where "the eye went with the line of sight," is entirely similar to the one illustrated by Francesco di Giorgio, and the geometric laws that could be applied to measure the

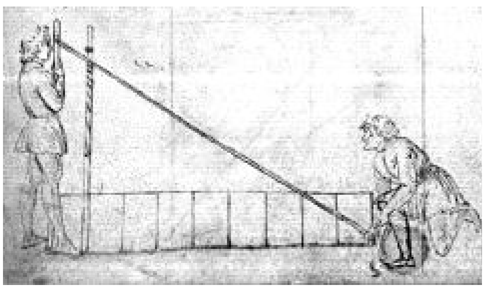


Figure 6.5. Practical demonstration of linear perspective. Drawing by Francesco di Giorgio Martini. (*La praticha di gieometria,* Florence, Biblioteca Medicea Laurenziana, Ashb. ms 361, c. 32^{v}.)

20 Francesco di Giorgio Martini 1967, I 139–140 (c. 32^{v}).

distance of a point were the same as the ones that could be used to represent its perspective image.[22]

On the basis of this common appurtenance to the principles of Euclidean geometry, perspective underwent significant development precisely in the field of measurements. The process of codification having been launched with the fundamental contributions of Dürer, Serlio, Daniele Barbaro, Vignola, and Guidobaldo del Monte, the use of instruments for the practical, fast execution of any type of perspective, from that of a single object to an urban or landscape veduta, spread rapidly.[23] These instruments proved especially useful in the military field, where the need to represent correctly fortresses and territories could not be satisfied by directly surveying the place to be drawn. This military function of linear perspective emerges from a first testimonial by Egnazio Danti, who dedicates his edition of Vignola's treatise to the captain of the papal army Giacomo Boncompagni, explaining that perspective

> also offers great advantages in attacking and defending fortresses, since it is possible with the instruments of this Art to draw any site without approaching it, and to have not only the plan, but also the elevation with every detail, and the measurement of its parts in proportion to the distance lying between our eye and the thing we wish to draw.[24]

Many were the instruments devised for this perspective-topographic scope: the *distanziometro* of Baldassarre Lanci, for example, a refined invention by one of the most active military engineers of Cosimo I; the *gnomone* of Bernardo Puccini, another Medicean engineer; the *proteo militare* of Bartolomeo Romano; and even the *sportello* of Dürer, utilized by Pietro Accolti to explain how it is possible "from a certain perspective drawing, carried out with the said Instrument, to investigate, and represent its Geometrical Plan, and the quantity of each of its parts."[25] Accolti explains that, once the perspective drawing of a fortress has been executed, and the positions of the vanishing point and distance point established, it is possible to reconstruct the layout of the building by applying the perspective procedure inversely (figure 6.6). A point, which, in the perspective view, is identified by the intersection of two straight

21 In the *Codex Atlanticus* (Leonardo 1973–1980) are various sheets containing sketches and notes dedicated to perspective and methods of measuring by sight: 103^{br} (36^{vb}), 119^{r} (42^{rc}), 339^{r} (122^{vb}), 361^{r}, 400^{v} (148^{vb}), 672^{v} (248^{va}), (265^{rb}). On sheet 103^{br}, for example, Leonardo writes: "Quella linia dov'è segnata in testa a, si chiama l'occhio. Quella dov'è segnato b, si chiama parete, cioè dove si tagliano tutte le linie che vengano all'occhio," and notes at the center of the drawing "Da misurare ogni distanza o altezza che tu vuoi." Luca Pacioli deals with the problem of perspective in his *Summa de aritmetica* (see Pacioli 1494): *Trattato geometrico, Distinctio octava, Capitulum secundum,* cc. 296^{r-v}. On the connection between measuring methods and Brunelleschi's perspective, see Krautheimer 1956, 238; Kemp 1978; Baxandall 1972, 86–108.

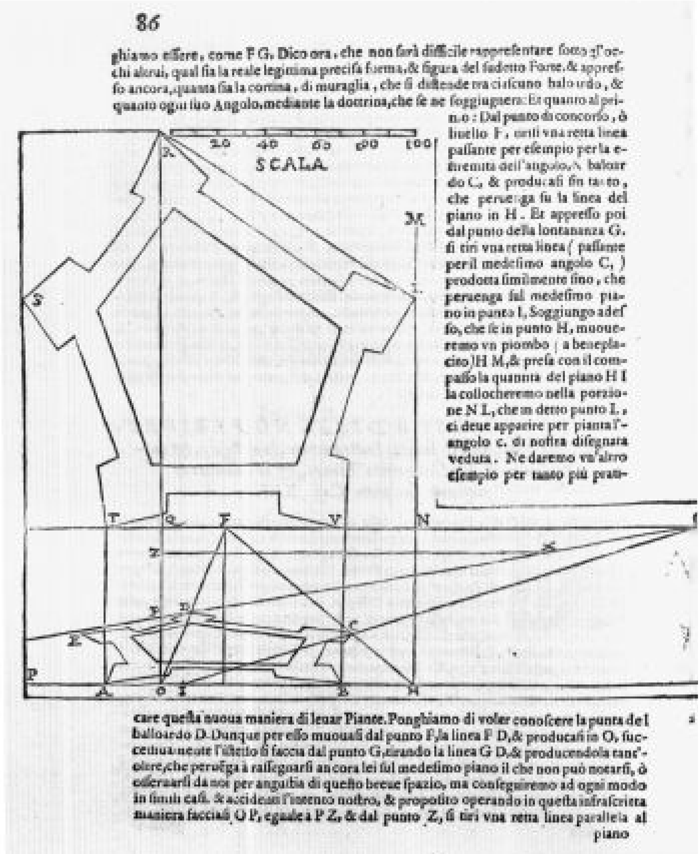
22 See Arrighi 1967, last paragraph of the "*quarta distinzione che tratta del modo di misurare chol'ochio, cioè chon strumenti*" (cc. 407^{v}–412^{v}).

23 In addition to the previously mentioned treatises by Dürer, Serlio, and Vignola, see Barbaro 1568 and Del Monte 1600.

24 Vignola 1583, dedication.

25 Baldassarre Lanci's *distanziometro*, now in the Florence Museum of the History of Science, inv. n° 152, 3165, is described by both Daniele Barbaro (Barbaro 1568, IX, IV) and Egnazio Danti (Vignola 1583, I.3 61). The *gnomone* is illustrated in detail by Bernardo Puccini in a treatise entitled *Modo di misurar con la vista* (1570–71), ms., Florence, Biblioteca Nazionale, Fondo Nazionale II–282, file. 15 (see Lamberini 1990, 351–403). Bartolomeo Romano also dedicates an entire treatise to his singular measuring "dagger" (Romano 1595). For the topographic operation of Dürer's *sportello*, see Accolti 1625, XVI.

lines, converging at the vanishing point and at the distance point respectively, will be identified in the plan by the intersection of a straight line perpendicular to the picture plane and one oriented at 45°. This procedure is a precursor of today's photogrammetry, conferring on perspective a leading role even among the most refined measurement methods of the Renaissance.



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ghiamo essere, come F G. Dico ora, che non sarà difficile rappresentare sotto gl'occhi altrui, qual sia la reale legitima precisa forma, & figura del sudetto Forte, & appresso ancora, quanta sia la cortina, di muraglia, che si distende tra ciascuno baloardo, & quanto ogni suo Angolo, mediante la dottrina, che se ne soggiugnerà: Et quanto al primo: Dal punto di concorso, ò livello F, tirisi vna retta linea passante per esempio per la estremità dell'angolo A baloardo C, & producasi fin tanto, che peruenga su la linea del piano in H. Et appresso poi dal punto della lontananza G. si tiri vna retta linea (passante per il medesimo angolo C,) prodotta similmente fino, che peruenga sul medesimo piano in punto I, Soggiungo adesso, che se in punto H, muoueremo vn piombo (a beneplacito) H M, & presa con il compasso la quantità del piano H I la collocheremo nella porzione N L, che in detto punto L, ci deue apparire per pianta l'angolo C. di nostra disegnata veduta. Ne daremo vn'altro esempio per tanto più praticare questa nuova maniera di leuar Piante. Ponghiamo di voler conoscere la punta del balloardo D. Dunque per esso muouasi dal punto F, la linea F D, & producasi in O, successiuamente l'istesso si faccia dal punto G, tirando la linea G D, & producendola tant'oltre, che peruenga à rassegnarsi ancora lei sul medesimo piano il che non può notarsi, ò osseruarsi da noi per angustia di questo breue spazio, ma conseguiremo ad ogni modo in simili casi, & accidenti l'intento nostro, & proposito operando in questa infrascritta maniera facciasi, O P, eguale à P Z, & dal punto Z, si tiri vna retta linea parallela al piano

Figure 6.6. Method for working out the plan of a fortress from its perspective drawing. Woodcut by Pietro Accolti. (Accolti 1625.)

2. PERSPECTIVE FOR SOLDIERS AND INSTRUMENT MAKERS: FIRST THOUGHTS ON ORTHOGRAPHIC PROJECTION

The inverse procedure that could be used to determine the layout of a building by means of perspective testifies to the efforts exerted in the military field to render measurable even that which traditionally was considered false and deceptive. Throughout the sixteenth century the need to have a drawing that was both "pictorial" and measurable had favored the dissemination of a practical, rapid system of "perspective" through which the geometric characteristics of the object represented could be retained unaltered. Girolamo Maggi, who published the treatise *Della fortificatione della Città* in 1564, incorporating in it the writings of Jacopo Castriotto, explicitly mentions the use of this type of representation as specific to military architecture.

> Let no one think to see in these drawings of mine either methods or rules for perspective, firstly because, not being a soldier's technique, I would not know how to do it, and then because, due to the foreshortening, too much of the plan would be lost. These drawings serve instead expressly to show the plans, and this is called *soldierly perspective* (figure 6.7).[26]

The drawing proposed by Maggi is what is now called "military axonometry," retaining a term, which indicates its origin, or at least its predominant usage, in fortified architecture. Bonaiuto Lorini calls it "the most common perspective," Bartolomeo Romano "spherical perspective," Giovanni Battista Bellucci "perspective that serves for practical uses," and again in the nineteenth century, the first codifier of this method, William Farish, was to call it "isometric perspective."[27]

This kind of perspective without foreshortening, widespread since antiquity, had been throughout the Middle Ages the *prospettiva* of mathematicians. In treatises on practical geometry, as in the earlier land-surveying codes, geometric solids were represented in precisely this manner, according to a practice that was to remain in vogue throughout the Renaissance, even among the greatest theoreticians of linear perspective. Piero della Francesca, for instance, utilized it for the drawings in the *Trattato d'Abaco* and the *Libellus de quinque corporibus regularibus*, to lay emphasis on the geometric characteristics of bodies rather than on their apparent image.[28] Leonardo employed it superbly to represent machines and mechanical elements (figure 6.8), and the greatest mathematicians of the sixteenth century continued to use it, at least up to Federico Commandino and Guidobaldo del Monte, who were the first to introduce the perspective drawing into strictly scientific contexts. This change was hailed with enthusiasm by Bernardino Baldi, who praised the work of Commandino

> for the clarity of the language and the diligence of the figures, in which the employment of the art of perspective avoids that crudeness in which those who follow and have followed depraved usage and barbarous custom incur and have incurred in the past.[29]

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26 Maggi and Castriotto 1564, II.3 40 (my italics). On this subject, see Scolari 1984.
27 See Lorini 1597, 32–33; Romano 1595, last plate. Sanmarino 1598, 1–6. The isometric drawing was to be definitively codified starting with the work of Farish 1822.
28 Piero della Francesca 1995.
29 Baldi 1998, 518.

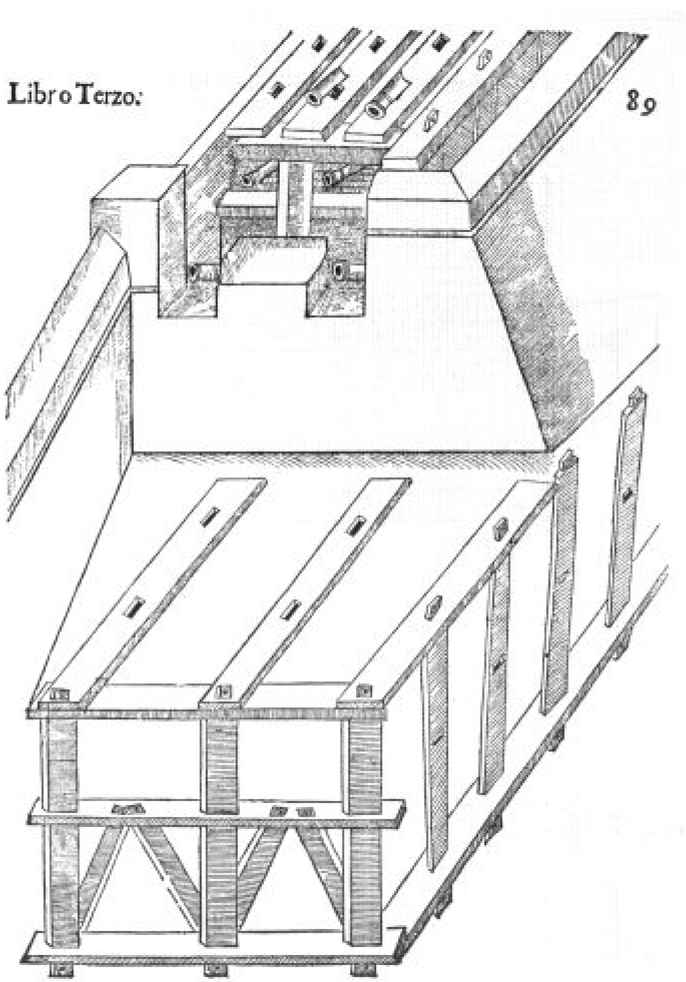

Figure 6.7. A bastion in "soldierly perspective." Woodcut by Girolamo Maggi and Jacopo Castriotto. (Maggi and Castriotto 1564.)

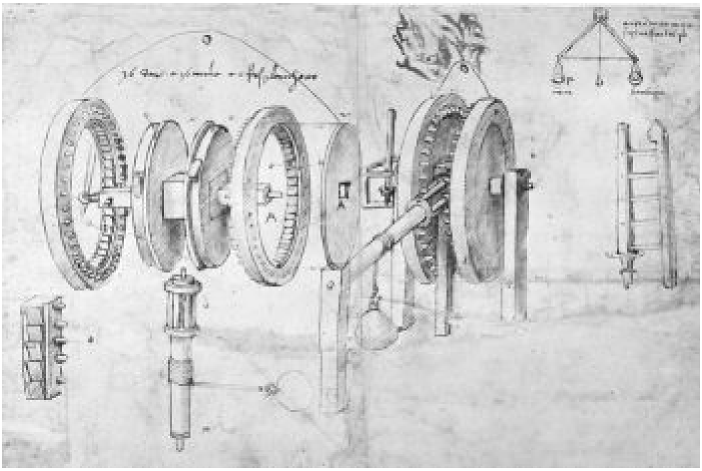

Figure 6.8. Isometric view of a winch with two wheels. Drawing by Leonardo da Vinci. (*Codex Atlanticus,* Milan, Biblioteca Ambrosiana, c. 30^{v}.)

The plates illustrating Commandino's edition of Ptolemy's *De analemmate* were considered a demonstration of this. So were those, at a later date, of Guidobaldo del Monte's *Planisphaeriorum universalium theoricae*, where the celestial sphere is drawn strictly in perspective with the circles deformed into ellipses, more or less flattened depending on their orientation in respect to the eye of the observer.[30] Despite such a systematic use of perspective drawing, it was precisely within this context that the foundations were laid for parallel projection, typical of the so-called "soldierly perspective."

The context was that of theoretical discussions of the plane representation of the celestial sphere. In his comment to Ptolemy's *Planisfero*, Federico Commandino had sufficiently demonstrated that the projection of the celestial sphere on the plane was an operation of linear perspective.[31] The boreal hemisphere was represented on the equatorial plane as if seen by an observer standing at the South Pole and, as demonstrated by Daniele Barbaro, the problem had become one of those discussed in treatises on pictorial perspective.[32] In his *De astrolabo catholico*, Reiner Gemma Frisius had sustained that the universal planisphere also derived from perspective, since the eye was situated at one of the equinoxes and the circles were projected onto the sol-

30 See Commandino 1562, 80; and Del Monte 1579.
31 See Commandino 1558.
32 See Barbaro, 1568, VI.2.

stitial colure, according to the projection procedure of Ptolemy, which Aguilonius was later to call "stereographic projection."[33] The universal planisphere described by Gemma Frisius had been widely known since the Middle Ages under the name of "saphaea azarchielis" (Azarchiel's plate) and, unlike the Ptolemaic planisphere, it allowed the utilization of only one planispheric plate for all latitudes. The circles in the sphere were reduced to arcs of a circle, and the entire planisphere could be easily drawn with a compass.

From the mid-sixteenth century on, a new universal planisphere based on the assumption that the distance of the eye could be infinite found increasing use (figure 6.9). It had been invented by Juan de Rojas Sarmiento, a pupil of Frisius, who maintained that this procedure, too, derived from perspective, on the condition, as Gemma Frisius was to explain, that one could imagine placing the eye at an infinite distance along the straight line passing through the equinoxes.[34] The points of the sphere were in fact transferred onto the plane according to what Aguilonius was to call "orthographic projection," i.e., by means of an array of parallel rectilinear lines used to represent both the parallels and the ecliptic by straight lines. Although perspective and orthographic projection belonged conceptually to the same projection principle, it was still difficult to imagine the eye as placed at an infinite distance. For Guidobaldo del Monte, for instance, placing the eye at an infinite distance meant "putting it in no place," a concept that "is abhorrent to perspective itself."[35] Nonetheless, apart from whether or not the parallel lines should converge at a point—a crucial problem in Postulate V of Euclid's *Elements*—Guidobaldo points out an obvious drawing error in Rojas' construction. He notes that the Spanish astronomer sustained that the meridians should be drawn as arcs of a circle, and that Gemma Frisius, while acknowledging that these were curves of another kind, did not know their geometric nature, so that he suggested drawing them by points. Guidobaldo explains that the curves of the meridians were none other than ellipses, adopting a demonstration already given by Federico Commandino in his edition of Ptolemy's *De analemmate*

33 Gemma Frisius 1556, I 4: "Astrolabum nostrum [universale] Spahera item plana est, ex visu defluxu similiter ut praecedens descripta. Verum eo solum differì, quod oculus non in polo, sed in Aequinoctiali constitutur, atque ita oppositum oculo hemisphaerium in planum perpendiculum obiectum visu describitur …" See also Aguilonius 1613, 503: In dealing with question of perspective representation, Gemma Frisius suggests that the planisphere could also be drawn mechanically, by depicting an armillary sphere with an instrument such as Dürer's window (I 2–3). This method had in fact already been applied by Leonardo, as suggested by a drawing in the *Codex Atlanticus* (f. 1^{r-a}) which illustrates a painter using a pane of glass to draw an armillary sphere. Presumably Dürer himself used a pane of glass for this purpose. The invention of the instrument dates from the years in which the German painter was working as a cartographer (1514–15), i.e., when he drew the splendid terrestrial globe in perspective for Johann Stabius and the representation of the Earth according to the third Ptolomaic method, which his friend Willibald Pirckheimer was to publish in his edition of the *Geografia* (Ptolemy 1525, VII).

34 Rojas Sarmiento 1550. On developments in orthographic projection, see the chapter by Lefèvre in this volume; see also Dupré 2001, 10–20.

35 Del Monte 1579, 141: "Da ciò appare quanto riduttive siano le loro parole per spiegare la sua origine. Giovanni De Rojas infatti omise del tutto dove bisognava collocare l'occhio. Gemma Frisio invece stabilisce che l'occhio (ove possibile) venga collocato a distanza infinita, cosa che senz'altro corrisponde a non collocarlo in nessun luogo. A quale condizione è infatti possibile che qualcosa nasca dalla prospettiva se l'occhio si allontana a distanza infinita? Senza dubbio ciò ripugna alla stessa prospettiva …" See Guipaud 1998, 224–232.

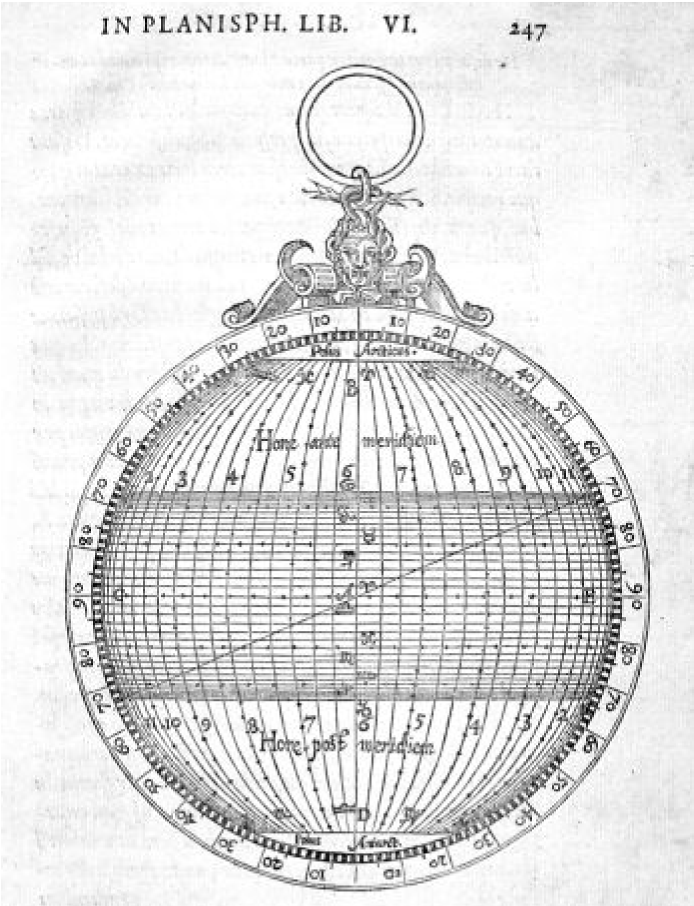

Figure 6.9. Orthographic projection of the celestial sphere (the "point of view" is at infinity). Woodcut by Juan de Rojas Sarmiento. (Rojas Sarmiento 1550.)

of 1563.[36] Considering the difficulty of tracing as many ellipses as there are visible meridians, all different from one another, Guidobaldo proposed the use of an ellipsograph of his invention, which he presumably also used to draw the extraordinary

perspective representations of the celestial sphere, which illustrate his text.[37] Within this debate on orthographic projection may also be included the only perspective instrument of the Renaissance designed to obtain a perfect "isometric perspective," the one described in Hans Lencker's *Perspectiva* (1571), which materializes the direction parallel to the lines of projection by means of a metal stylus, always orthogonal to the drawing sheet (figure 6.10).[38]

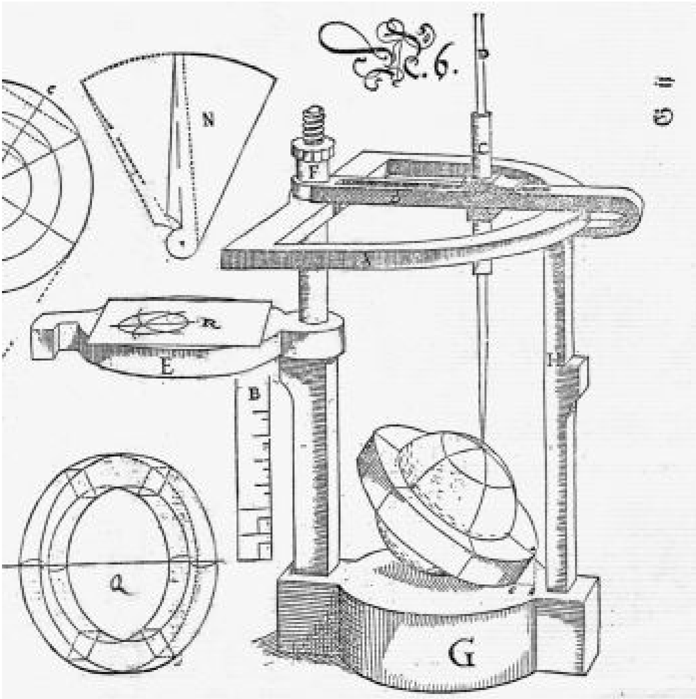

Figure 6.10. Instrument for orthographic projections. Drawing by Hans Lencker. (Lencker 1571.)

36 Del Monte 1579, 167: "Egli [il Rojas] credeva, non bisogna dimenticarlo tuttavia, che noi dovessimo disegnare archi di cerchio nel tracciare sul planisfero i meridiani." Then he quotes Gemma Frisius's words: "gli stessi meridiani, poiché non si conosce bene [quale sia] il loro tracciato, vengono ricavati per punti secondo un percorso disuguale; e non essendo questa operazione alla portata di un artefice qualsiasi, avviene che spesso si giunge ad un discorso privo di senso sia nel disegno sia anche nell'uso" (186). See Gemma Frisius 1556, I 4: "Meridiani verò lineis cursus anomalis, quae neque circoli sunt, tantum per puncta adsignata manu diligenti traductae."

37 Del Monte 1579, II 105.

38 See Lencker 1571, 21.

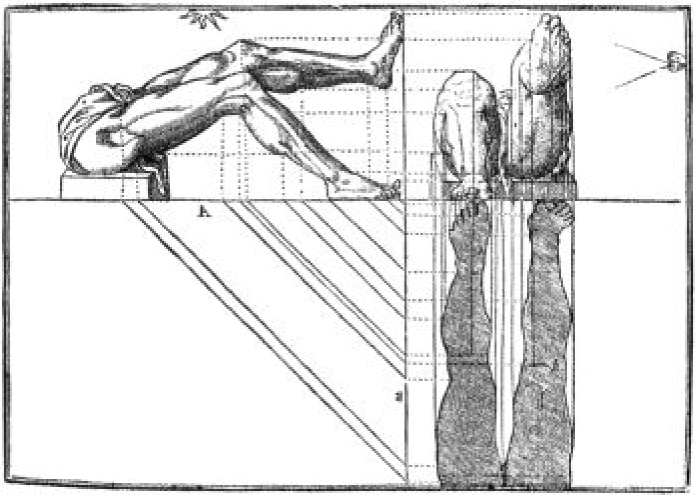

Figure 6.11. Orthographic projection of a human figure. Woodcut by Jean Cousin. (Cousin 1571.)

Apart from the doubts expressed by Guidobaldo del Monte, orthographic projection was generally considered to be a perspective projection with the point of view placed at an infinite distance. Dürer and Jean Cousin showed how it could be used to draw the foreshortened human figure (figure 6.11). Galileo applied it to demonstrate the existence of sunspots; and Pietro Accolti, accepting the demonstrations of Galileo, conferred a physical image on the ideal point of orthographic projection, identifying it as "the eye of the Sun" (figure 6.12).[39] Accolti mentions it within the context of a discussion on the projection of shadows, comparing two representations of the same cube, one seen by the eye of man and thus drawn in perspective—with the foreshortened sides converging toward a vanishing point—the other seen by the eye of the sun, and consequently drawn in orthographic projection, with the foreshortened sides parallel. "It is thus we consider that the aforesaid drawing should be, for repre-

39 See Dürer 1996, and Cousin 1571. Galileo employed parallel projection to explain the appearance of sun spots in the telescope (*Istoria e dimostrazioni intorno alle macchie solari e loro accidenti*, Roma 1613, 34; in Galilei 1890–1909, V 213–215). See Accolti 1625, III XXVIII, *Altra invenzione per conseguire la naturale incidenza de lumi, et dell'ombre sopra diversi piani ove vanno a cadere*: "Et insegnandoci il testimonio del senso visivo … che il Sole … manda l'ombre sue, parallele sul piano …; così restiamo capaci potersi all'occhio nostro, et in disegno far rappresentazione di quella precisa veduta di qualsivoglia dato corpo, esposto all'occhio (per così dire) del Sole, quale ad esso Sole gli si rappresenta in veduta: … così intendiamo dover essere il suddetto disegno, per rappresentazione di veduta del Sole terminato con linee, et lati paralleli, et non occorrenti a punto alcuno in Prospettiva."

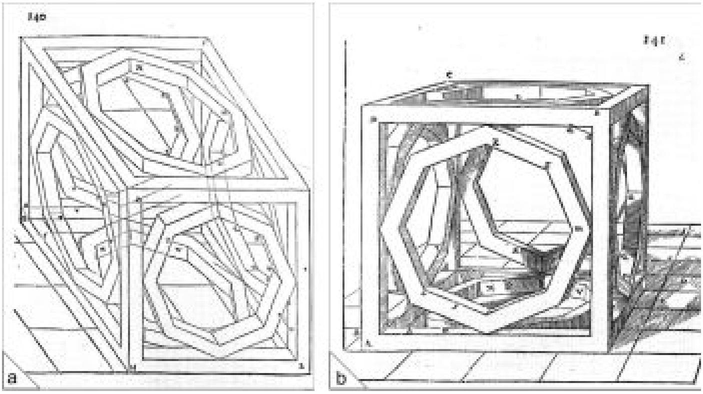

Figure 6.12. Two cubes in perspective: one viewed by a hypothetical eye situated in the sun (left), the other viewed by a human eye. Woodcuts by Pietro Accolti (Accolti 1625).

sentation of the viewpoint from the Sun, terminated with lines, and parallel sides, not concurring at any point in Perspective."[40] Although the author's objective was that of describing an alternative mode of representing shadows in perspective, employing a preliminary drawing, which he called *ombrifero*, or generator of shadows, this preliminary drawing contained all of the elements, which, with Girard Desargues, were to lead to definition of the ideal point in projective geometry. The cube appears to us as if we were looking at it from the position of the sun, that is, from a viewpoint at infinite distance. Among its many other merits, the dissemination of Galileo's telescope should probably also be credited with having favored the acceptance of this concept, which was still for Guidobaldo del Monte foreign to the physical reality of perception.

3. THE GEOMETRY OF LIGHT: TOWARDS A THEORY OF SHADOW PROJECTION

Pietro Accolti's *ombrifero* drawing is a "singular," though hardly practical, attempt to solve a problem of crucial importance to painters but not yet defined on the level of projection: that of *sbattimenti*, or shadow projection. According to Accolti, up to then the authors concerned with perspective had gone "adventuring," concentrating mainly on the case of a punctiform light source, which produces "rays that are pyramidal, and concurrent," and ignoring the case, which would instead have been more useful to painters, that of the exposure of objects to daylight, which produces "paral-

40 Accolti 1625, 134.

lel rays, as the Sun does." The isometric cube was for Accolti the image of an object seen by the eye of the sun; but since the sun is at the same time both seeing eye and source of light, the cube appears free of shadows, or rather the shadows are concealed by the parts of the object, which produce them, since the lines of sight coincide with the rays of light. In order to see them, the eye would have to be shifted to another position, as in the cube drawn in perspective, where the shadows are produced by simply transferring from the *ombrifero* drawing the points where that which lies in front is superimposed on that which lies behind.

The considerations made by Accolti are an immediate reflection of Galileo's arguments in favor of his interpretation of the orographic nature of the moon. As we know, Galileo sustained that the moon had a mountainous surface similar to that of the Earth. The craggy profile of the terminator, he believed, was the clearest proof of this, even though the mountain peaks could not be seen where it would be most reasonable to expect them, along the circumference of the moon. According to Galileo, this depended on two main factors: the overly narrow breadth of the optical angle that could embrace the height of the mountains, and the fact that along the circumference—given the grazing direction of the lines of sight—only the lighted part could be seen. As any painter well knew, in the absence of shadow the perception of relief is lost.[41] If in fact we imagine an irregular surface lit up by the sun, explains Galileo,

> the sun, or one who might stand in its place, absolutely would not see any of the shadowy parts, but only those that are lit up; because in this case the lines of sight and of illumination proceed along the same straight paths, nor could there be shadow there where the ray of light arrives, so that none of the dark parts could be seen.[42]

To see the shadows it would be necessary for "the line of sight to rise above the aforesaid surface more than the solar ray," i.e., for the eye to look at the object from another point of view, just as Accolti was to propose.

The definition of two types of *sbattimenti*, or shadow projection, those produced by the light of a torch and those produced by sunlight, had appeared in treatises on pictorial perspective since Alberti's *De pictura*. Alberti, however, did not venture beyond the simple statement that "the light of the stars casts a shadow the same size as the body, but firelight makes them larger," although he suggested that a practical, rapid mode of drawing the perspective of a circle with precision could be that of projecting the shadow after having "positioned it, with reason, in its place."[43] Piero della

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41 See for example, Cigoli 1992, f. 82^{v}: "Gl'oggetti veduti dalla parte luminosa, per la scarsità dell'ombra non hanno rilievo." The question had been brought up by Galileo himself in a letter to Cigoli on the superiority of the arts in which, claiming the supremacy of painting over sculpture, he concluded: "Et avvertasi, per prova di ciò, che delle tre dimensioni, due sole sono sottoposte all'occhio, cioè lunghezza e larghezza … Conosciamo dunque la profondità, non come oggetto della vista per sé et assolutamente, ma per accidente e rispetto al chiaro et allo scuro …" (letter dated 26 June 1612, in Galilei 1890–1909, XI 340–343).

42 Letter dated 1 September 1611 to Christopher Grienberger, in Galilei 1890–1909, XI 178–203, specifically, 185.

43 See *De pictura* (Alberti 1973, III), I.11. The manner of drawing an ellipse as the projection of the shadow of a circle is mentioned in II.34; Alberti does not mention the type of light source but it is probable that he had in mind a torch collocated in place of the viewpoint. For a history of the theory of shadows, see Da Costa Kauffmann 1975, 258–287; *idem* 1993; see also De Rosa 1997.

Francesca deliberately chose not to deal with color, that third part of painting, which was supposed to comprise what Alberti called the "receiving of lights." But near the end of his treatise Piero describes a perspective procedure based not so much on the concept of *intersection* as on that of *projection*. An object placed between the eye and the painting is in fact projected by the lines of sight exactly as would be done by the rays of light from a punctiform light source.[44] The coincidence between the shadow of the object and its perspective image is well visualized in a drawing by Leonardo in which the effects of the projection of a sphere produced by an eye are compared with those produced by a candle.[45] To Leonardo we owe the studies that delve deepest into the question of shadow projection. The idea of dedicating an entire treatise to the problem of light and shadow had led him to investigate the phenomena of lighting effects with the great acuity characteristic of his theoretical reflections. In his writings we find important considerations on the difference between shadow proper (*ombra congiunta*) and cast shadow (*ombra separata*); on the separating line, or rather on that "means," which separates the shadowy part from the lighted one; on the *lume universale*, or diffused lighting, which produces half-light and sfumato; and on secondary light, the light reflected from any opaque body, which lightens the shadows of nearby objects.[46] Alberti too had theorized on *raggi flessi*, reflected rays, noting that they bring with them "the color which they find on the surface" of the object that reflects them: "Notice that one who walks in the meadows under the sun has a greenish tinge to his face."[47]

In spite of these keen observations, no geometric rule for the correct representation of shadows was established. Leonardo seems convinced, in fact, that shadows cannot be described geometrically, while

> the features can be made to gleam with veils, or panes of glass interposed between the eye and the thing that you wish to make shine, shadows are not included under this rule, due to the inconsistency of their terms, which are for the most part confused, as is demonstrated in the book on shadows and lights.[48]

The innovative practice of representing shadows geometrically was introduced by Albrecht Dürer, who in the four famous plates of the *Underweysung der Messung* demonstrated their principles of projection with crystalline clarity.[49] In the first panel the artist drew the shadow of a cube in plan and elevation, precisely localizing the source of punctiform light (figure 6.13a). In the second plate the cube and its shadow are drawn in perspective through the so-called "laborious procedure," i.e., Brunelleschi's legitimate construction (figure 6.13b). The third panel illustrates the method of creating shadow through the so-called "shorter way," Alberti's method, which is confused here with construction using the distance point (figure 6.13c). Lastly, the fourth

44 Piero della Francesca 1984, 210–215, LXXVIII–LXXX.
45 Leonardo da Vinci 1986, *Manoscritto C*, fol. 9.
46 Leonardo's reflections on the projection of shadows, the subject of a lost book "of shadows and lights," are found scattered through various manuscripts. They were collected by his pupil Fancesco Melzi in the fifth part of the *Libro di pittura*; see Leonardo da Vinci 1995.
47 See *De pictura* (Alberti 1973, III), I.11.
48 Leonardo da Vinci 1995, II 304, § 413.
49 Dürer 1983, IV: text to figure 52–58.

plate shows the finished drawing, without construction lines and with a pictorial touch in the gradation of the cast shadow and the representation of the sun as source of light (figure 6.13d). On the one hand, this last plate reveals Dürer's sensitivity to the phenomenon of diffraction that causes lightening of the shadow in the parts nearest to the light. On the other hand there is, as has been noted by critics, an obvious incompatibility between this drawing and the previous plates, where the light source is clearly punctiform and at a finite distance.[50] In that position, in fact, the sun would have cast a longer shadow, since its horizontal projection would necessarily have been on the line of the horizon. This "oversight" seems to proclaim a basic problem in understanding the difference between punctiform light source and light source at

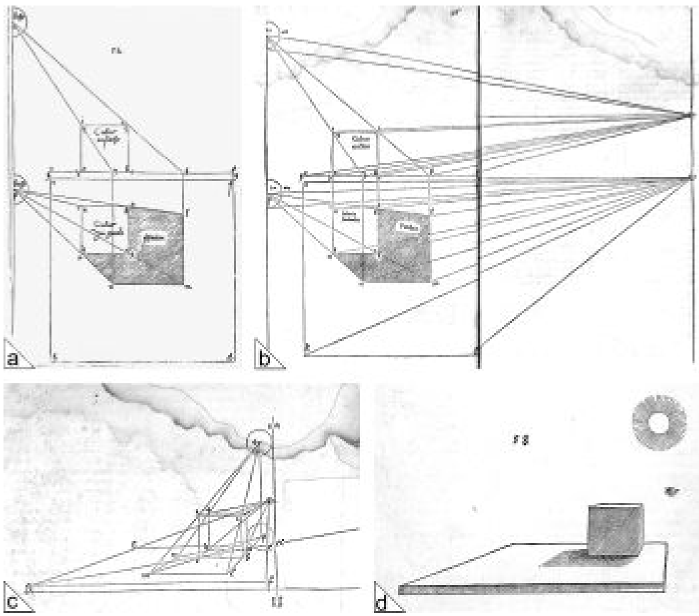

Figure 6.13. The perspective of shadows: a) plan and elevation of a cube with a light source and cast shadow; b) the cube and its shadow drawn in perspective according to the "laborious procedure" (intersection of the visual pyramid in plan and elevation); c) the cube and its shadow drawn in perspective according to the "shorter way" (use of the distance point); d) final drawing of the cube and its shadow in perspective. Woodcuts by Albrecht Dürer. (Dürer 1983.)

50 See Da Costa Kauffmann 1993, 49–78.

an infinite distance. The problem, however, was to be ignored throughout the century. As noted by Pietro Accolti, the treatise authors had focused exclusively on the representation of shadows projected by a torch or a candle. Dürer's scheme was to be proposed again, without substantial modifications, at least up to François d'Aguillon's *Opticorum libri sex* (1613), where we find it specified that

> those things lighted by lamps … should be expressed *scaenographicae*, that is in perspective, that are exposed to the direct rays of the sun should be presented *orthograpicae*, that is in orthographic projection.[51]

Aguilonius, however, does not indicate how this orthographic projection should be represented in a perspective drawing, and critics have often noted that no solution was to be given before the crucially important studies of Girard Desargues.[52]

A correct solution for representing shadows projected by the sun was, in reality, given by Ludovico Cigoli, one of the painters closest to Galileo. The fact that his manuscript on *Prospettiva pratica* remained unpublished certainly prevented this rule from undergoing further development, and it even seems to have escaped the notice of those theoreticians who presumably had access to the text—Pietro Accolti and Matteo Zaccolini.[53] In the section entitled *Degli sbattimenti*, Cigoli describes three types of shadows: those larger than the illuminated body, produced by a punctiform light source, for which he adopts Dürer's scheme; those smaller than the illuminated body, as with an object placed before a window, where he gives instructions for producing the gleam (*barlume*); and those the same size as the illuminated body, which are the so-called *sbattimenti del Sole*, or "shadows projected by the Sun." Since *sbattimenti* of this kind are "contained by parallel lines" lying on the horizontal plane, Cigoli was concerned with finding their convergence point on the horizon, adopting the theory of the so-called "points of convergence" elaborated by Guidobaldo del Monte in his *Perspectivae libri sex*.[54] Guidobaldo had explained that, beyond the vanishing point and the distance point, on the horizon there could be determined as many vanishing points, or points "of convergence," as were the directions of the straight lines of the object to be represented. After having drawn the object, Cigoli then establishes the direction of the solar rays, their representation by means of a straight line drawn in plan and elevation, and finds the point of convergence of that straight line "and its parallels."

Cigoli notes that this rule was necessary since "the Sun being too big and too far away, we cannot draw its plan and profile." In the case of a punctiform light source, in dealing with a point in space at a close distance, it was both possible and necessary to determine its exact position by means of the plan and elevation views. Cigoli continues to use the terms "plan" and "height" even as concerns perspective construction with a distance point, indicating that these two spatial references—the horizontal

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51 Aguilonius 1613, 683.

52 Bosse 1647, 177–178.

53 See Cigoli 1992, 79^{v}–82^{v}. Left unpublished, as it still is, was also the important manuscript by Matteo Zaccolini, *Della Descritione dell'ombre prodotte de' corpi opachi rettilinei* (see Florence; Biblioteca Medicea Laurenziana, Ashburnham ms. 1212).

54 Del Monte 1600, II VIff.

plane and the vertical one—should still be considered measuring loci even in their apparent deformation. As is clearly apparent in the plates by Guidobaldo del Monte as well, the representation of shadow was determined by the contemporary presence of these two planes: the horizontal one that received the shadow of the object, and the vertical one that visualized the projection of the cone of light. This favored, or rather demanded, that the method of representation commonly used in architecture—projection in plan and elevation—should become the necessary condition for any problem relevant to the geometric description of space: "Since the reason for the appearance of any object whatsoever derives from proceeding with strict reason in imitating it," wrote Cigoli with exemplary clarity, "it is necessary to know its shape and position, something which is obtained by means of two figures known as plan and profile, which reveal to us the three dimensions: length, width and depth."[55]

4. THE SPECIFICITY OF ARCHITECTURAL DRAWING: THE DOUBLE PROJECTION

Theoretical reflections on orthographic projection and the dissemination of rules for perspective had contributed to codifying a method of representation that had for centuries been specific to architecture. The need to define the shape and size of the building to be constructed called for a linear, measurable drawing that clearly expressed the architect's idea with no concessions to the pleasure of the eye. "Between the graphic work of the painter and that of the architect," wrote Leon Battista Alberti in *De re aedificatoria*,

> there is this difference: the former endeavors to portray objects in relief on the panel through shading and the shortening of lines and angles; the architect instead, avoiding shading, represents reliefs through a plane drawing, and represents in other drawings the form and extension of each facade and each side utilizing real angles and non-variable lines, as one who desires that his work not be judged on the basis of illusory appearance, but evaluated exactly on the basis of controllable measurements.[56]

Although the numerous drawings of Renaissance architecture show that forms of mixed representation survived for years, with shaded perspectives or combinations of perspective and orthogonal projections, the general intention on the theoretical level was that of conferring on the architectural drawing a specific nature of its own, able to describe "the acuteness of the conception, not the accuracy of the execution."[57]

On the difference between the painter's drawing and that of the architect, Raphael too was highly explicit when in his famous letter to Leon X he described precisely the characteristics of the architectural drawing:

> The drawing of buildings pertaining to the architect is divided into three parts, of which the first is the layout, that is, the plan, the second is the outer wall with its ornaments, and the third is the inner wall, also with its ornaments …[58]

55 Cigoli 1992, 17^{r}.
56 Alberti 1975, II.1. On the subject, see Thoenes 1998 and Di Teodoro 2002.
57 Alberti 1975, II.1. The latter quotation refers to the use of the models, which Alberti preferred simple and linear rather than too elaborate and colorful.

The three parts clearly recall the three "species" of Vitruvius, with the difference that to *ichnographia* and *orthographia*, Raphael adds "the inner wall," i.e., the section. The need for this third part was to lead Daniele Barbaro, alone among Vitruvius' commentators, to interpret as "profile" the third Vitruvian species, that of *scaenographia,* which was instead traditionally synonymous with perspective.[59] Raphael himself interprets the third Vitruvian species as *prospectiva*,[60] but in the letter to Leo X this kind of drawing is considered a pictorial method to be utilized only during the design stage.

> And although this mode of drawing in perspective is proper to the painter, it is nonetheless convenient to the architect as well because, just as it is good for the painter to know architecture in order to draw the ornaments well measured and with their proportions, so the architect is required to know perspective, through which exercise he can better imagine the building with its ornaments.

The architectural concept is instead to be conveyed entirely by orthogonal projections.

> And in such drawings there is no diminishing ... Because the architect cannot take any correct measurement from a diminished line, as is necessary to that artifice which requires all of the measurements to be perfectly made and drawn with parallel lines, not with those that seem so but are not …

Drawing in orthogonal projection had been applied in architecture since antiquity. The extraordinary working drawings in the temple of Apollo at Didime, for example, show a refined ability to visualize three-dimensional shape through the two projections in plan and elevation.[61] Good examples of the correct use of these projections appear again in the drawings on parchment from the Gothic period, but in these, as in the ancient drawings, it is by no means taken for granted that the two projections should be aligned, nor that they should be drawn to the same scale. The famous drawing of the dome on St. Maria del Fiore, done by Giovanni di Gherardo da Prato as an illustration for the public discussion of a building problem, shows how even in the early fifteenth century, and within a context anything but marginal, the custom of representing plan and elevation views on the same sheet in arbitrary positions and scales was still widespread.[62] This situation began to change with the correct use of orthogonal projections, thanks to the dissemination of linear perspective. To represent the perspective image of a point it was in fact necessary to define its exact position in space, and this could be done only by first drawing the projection in plan and elevation according to a determined reduction ratio.

The drawings of Piero della Francesca are the first and finest examples of this mode of representation. In Theorem XIII of *De prospectiva pingendi* the three kinds of drawing indicated by Raphael, plan, elevation, and section, appear not only drawn

58 On the letter to Leo X, see Di Teodoro 1994.
59 See Vitruvio 1567, I.2.2 30.
60 See Vitruvio 1975, I.2.2.
61 See Haselberger 1980 and 1983.
62 The drawing is now in the Archivio di Stato di Firenze, Exhibition Inv. n° 158.

to the same scale but also superimposed one above the other in the same drawing (figure 6.14). Undoubtedly, Piero thus intended to explicitly show the coincidence between viewpoint and vanishing point, demonstrating that he had perfectly mastered the problem of overturning planes. And this is not an isolated example. Francesco di Giorgio Martini demonstrates in the same way the coincidence between "center," i.e., "point and end of all lines" (the vanishing point) and "counter-center," i.e., "the eye which looks at the point," by overturning on the picture plane the orthogonal plane passing through the eye of the observer, and thus offering the first clear representation of the distance point perspective method.[63] A decisive contribution to the dissemination of double representation (plan and elevation) and triple orthogonal projection (plan, elevation and section, or lateral elevation) was made by Albrecht Dürer.[64] The first of his treatises, *Underweysung der Messung* (1525), is designed to furnish the necessary geometrical instructions not only to painters but also to "goldsmiths, sculptors, stonemasons, woodworkers, and others who base their art on the correctness of the drawing."[65] The themes dealt with range from conic sections to the drawing of letters, from the proportioning of architectural elements to the design of solar clocks, from measurement techniques to perspective drawing. Each subject is illustrated with drawings of exemplary clarity, whose potential value to architectural drawing is fully demonstrated by the refined illustrations to the subsequent treatise on fortifications, *Etliche underricht, zu befestigung der Stett, Schlosz, und flecken* (1527).

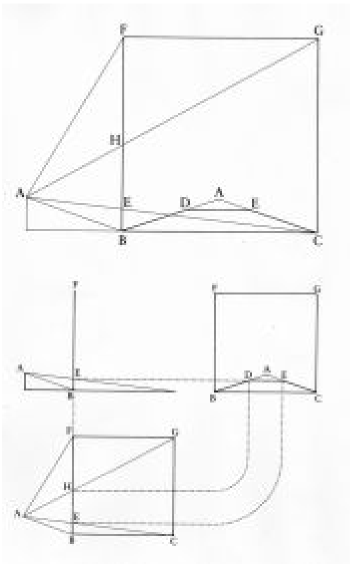

Figure 6.14. Perspective construction of a square horizontal plane. Drawing by the author, based on Piero della Francesca *De prospectiva pingendi,* c. 1475, I, XIII.

But the most extraordinary applications of orthogonal projection are found in the treatise, published posthumously, on the proportions of the human body: *Hieirin sind begriffen vier Bücher von menschlicher Proportion* (1528). Here Dürer's refinement is expressed through a series of multiple projections designed to provide geometric

63 Francesco di Giorgio Martini 1967, I 139–140 (c. 32^{v}). On Piero della Francesca, see Di Teodoro 2001.
64 See Lefèvre in this volume.
65 Dürer 1983, preface.

control even over the various positions of the human body (figure 6.15). Piero della Francesca's attempt to demonstrate that it was possible to geometricize even a geometrically undefined body such as the human head was developed by Dürer into a methodical system, subjecting the infinite variations of nature to the control of geometry. In these drawings appear orthogonal projections of bodies not only in frontal but

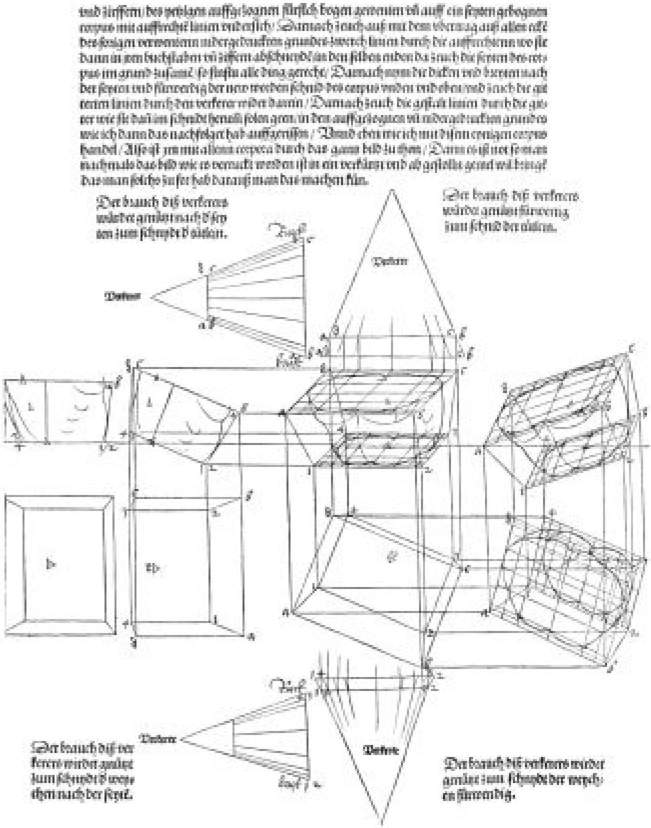

Figure 6.15. Multiple projections of a human torso. Woodcut by Albrecht Dürer. (Dürer 1996.)

also in oblique view, seen from below or above: real stereometries of human torsos, limbs and heads. Educated in a family of goldsmiths, Dürer possessed the expertise to dominate almost entirely the field of geometric constructions, but he himself stated that this type of drawing originated in a precise professional context, that of the stonemasons. In his dedication to Willibald Pirckheimer, he in fact writes that whoever wishes to confront the study of proportions must have

> well assimilated the manner of measuring and must have understood how all things must be extracted from the plan and elevation, according to the method daily practiced by the stonemasons.[66]

5. STONEMASONS' GEOMETRY: ROTATION, OVERTURNING, AND DEVELOPMENT

According to Dürer, construction sites were the places where drawing was practiced at its highest level. On construction sites, the transition from the project drawing to actual construction was made through two basic elements: the wooden model—which visualized the building in three dimensions, furnishing all of the information necessary for carrying out the work, from construction system to decoration—and the working drawing on the scale of 1:1, which furnished the exact measurements of each individual architectural element. The working drawings were carried out on the site progressively as construction proceeded, so that each new element to be worked, in wood or in stone, could be perfectly adapted to the part already constructed. These drawings were made, under the supervision of the architect, by the carpenters or stonemasons themselves, who then had to shape each architectural element from a formless beam or a rough-hewn block of stone. The previously mentioned drawings of the Temple of Apollo at Didime show exactly this process of detailed definition of dimensions and shapes. The drawings, engraved in stone, would have furnished the necessary information to all of the stonemasons who succeeded one another during the long period of construction of the building. Similar graphic documents are found in some Gothic cathedrals, as for example on the terrace of the choir in the Cathedral of Clermont-Ferrand, where a drawing on the scale of 1:1 of one of the portals on the facade has survived.[67]

The working drawing had to be extremely precise, detailed but also essential, free from any sign that had no structural function. The essential nature of such drawings is clearly indicated by the manuals of Matthäus Roriczer and Hans Schmuttermayer, who divulge for the first time the "secrets" of the worksite drawing, revealing how knowledge of the method applied to extract the elevation from the plan had favored a stratagem that today would be called ergonomic. By utilizing a particular geometric

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66 Dürer 1996, letter of dedication to Willibald Pirckheimer: "Darumb thut einem yglichen der sich diser kunst vndersteen will not / das er zuuor der messung wol vnderricht sey / vnd einen verstand vberkome / wie alle ding in grund gelegt / vnd auffgezogen sollen werden / wie dann die kunstlichen Steinmetzen in teglichem geprauch habenn."

67 The drawing is an elevation of the northern portal of the Cathedral of Clermont-Ferrand; see Sakarovitch 1998, 126.

figure—a square inscribed in another square rotated 45°—it was possible to deduct from the plan drawing alone the widths of all of the other points in the piece to be worked.[68] A particularly vivid picture of the techniques employed in medieval worksites appears in the famous notebook of Villard de Honnecourt, which describes various graphic procedures for proportioning architectural elements and the human figure, enlarging the size of the design drawing to that of the working drawing, measuring heights and distances, tracing pointed arches and, above all, for designing complex structural features such as a stone arch over an oblique passageway or in a circular wall.[69]

Building this type of arch called for great technical skill that implicated an uncommon knowledge of static and geometric factors. The oblique vault was generated from a cylinder that traversed a straight parallelepiped, i.e., the wall thickness, at an angle. For the arch to fulfill its static function, the joints between the ashlars had to lie on planes perpendicular to the wall sector. Due to the oblique nature of the passage, the intrados surfaces of the various ashlars were not uniformly curved but twisted to varying degrees, and the joint lines appeared as elliptical segments. In all likelihood the problem was solved through a method now termed "by squaring," which consisted of obtaining the oblique piece from the cubic block by the orthogonal projection of its shape onto the faces of the cube. The ensemble of the six orthogonal projections, which make up a cube is precisely the system adopted by Dürer in representing the stereometry of the human torso, and it is presumably this method to which the German painter was referring in the passage quoted above.

In his drawings Villard illustrates how these problems of stereotomy could be solved on the worksite by the appropriate use of a square or ruler. The use of these instruments, however, seems to implicate the knowledge of a more refined system of geometric projections. In addition to the square and the ruler, Villard traces signs, which indicate the number of ashlars of which the arch is composed and the angles of the different cuts necessary to transform the cubic block of a single ashlar into an oblique parallelepiped. The problem was probably solved with the aid of scale models, but the signs left by Villard hint at the application of a graphic procedure not unlike the one codified later in subsequent treatises on stereotomy. The first description of the drawing methods employed by stonecutters was given by Philibert De L'Orme, whose *Premier tome de l'architecture* (1567) constitutes today, under this aspect, an exception in the panorama of Renaissance architectural treatises.[70] Books III and IV of this important treatise are entirely dedicated to "the art of stone cutting," which is presented here essentially as the "art of geometric drawing," indicating that the problem of cutting stones lay above all in the correct execution of the drawing. The importance attributed by De L'Orme to this aspect of architecture emerges already in the title-page, where eight "stereotomic icons" illustrate the graphic resolution of some of the most significant cases of stereotomy: the hemispheric vault, the

68 Roritzer 1486; Schmuttermayer 1489.
69 See Bechman 1993.
70 De L'Orme 1567. On the subject of stereotomy in the work of De L'Orme, see Pérouse de Montclos 2000; Potié 1996; Sakarovitch 1998.

Figure 6.16. Title-page with stereotomic diagrams. Woodcut by Philibert De L'Orme (*Le premier tome de l'architecture,* Paris 1567).

three-dimensional arch over a cylindrical wall, the so-called “vis de Saint-Gilles,” i.e., the barrel vault over a helicoid staircase, the “trompe de Montpellier,” i.e., the conical vault supporting a cylindrical volume, and the so-called “trompe quarré,” the conical vault supporting a square volume (figure 6.16). As proven by the names still used to define some of these vaults, they are architectural types coming from the Romanesque and Gothic tradition of Southern France. The so-called “appareilleurs” (stonemasons) were those who guaranteed the correct execution of these works by superintending the cutting and laying of the stones, an extremely delicate operation that required technical expertise and a thorough understanding of the geometric development of bodies.

On the level of geometry, the problem to be solved was that of the intersection between two or more volumes. The so-called “trompe quarré,” for instance, originated from the intersection of a horizontal half-cone and a vertical square volume. The ashlars of which it is composed have faces belonging to both of these figures and could be worked only after their exact geometric development had been determined. The stonemason first had to define the “slope line” in real size, a line that varied for each ashlar, and then to develop the intrados surface in order to design the so-called “panels,” the templates used to cut the blocks of stone. The “par panneaux” construction method, described by De L’Orme, differs from the one called “par équarrissement” in that it could be used to design beforehand the exact shape of each individual face of the oblique block, thanks to a refined system of projections, rotations and overturning (figure 6.17). De L’Orme notes the innovative aspect of this method and claims credit for its invention, well aware of the fact that he was the first to write on

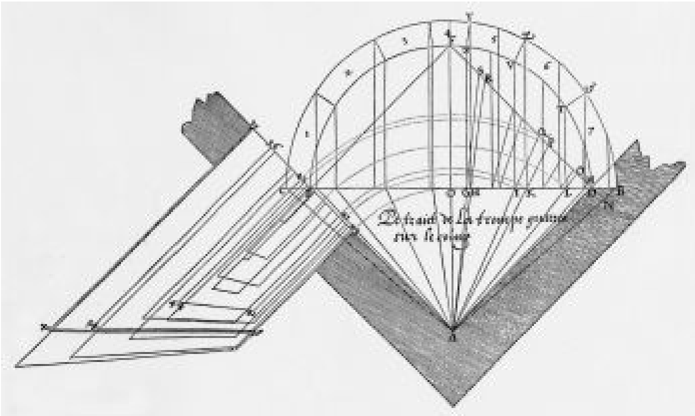

Figure 6.17. Constructive drawing for the “trompe quarrée.” Woodcut by Philibert De L’Orme. (De L’Orme 1567.)

the subject. The French architect proclaims his ambition to raise “the art of geometric drawing” from its status of worksite practice to that of architectural theory, proposing to “review Euclid and accommodate his theory to the practice of our architecture.”[71] This ambition, however, was to find no adequate realization. De L’Orme avoids facing the problem on the mathematical level and offers no definition of his refined procedures, which he explains instead in a practical and verbose manner.

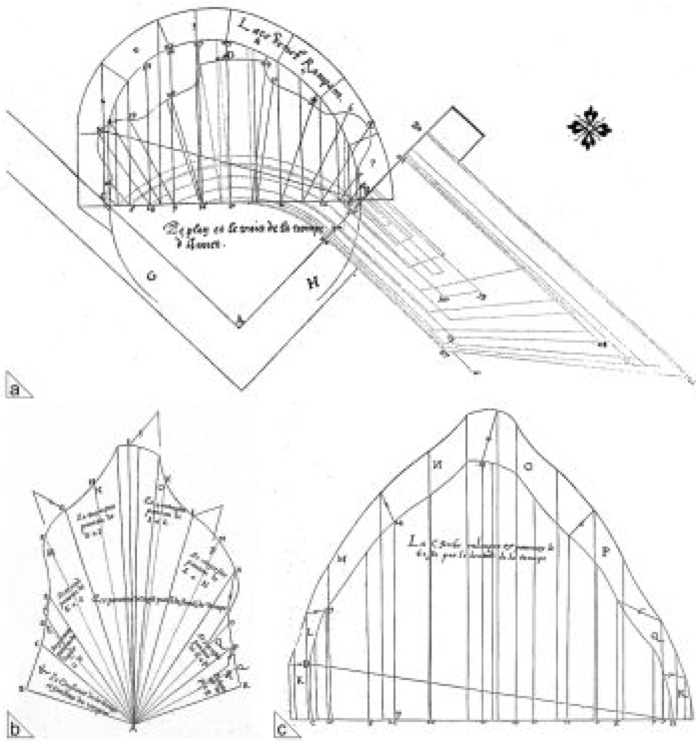

Figure 6.18. The “trompe d’Anet”: overturning of different sections for determing the “slope line” of the ashlars (a), development of the vault’s intrados (b), and development of the frontal edge of the ashlars (c). Woodcuts by Philibert De L’Orme. (De L’Orme 1567.)

71 De L’Orme 1567, III f. 62^{r}.

The design of the "panels" can be compared to the development of the faces of the polyhedrons and the sphere illustrated by Albrecht Dürer, in his treatise on geometry designed to teach how to construct those geometric solids materially.[72] But while the faces of the polyhedrons are all regular geometric figures—triangles, squares, pentagons, hexagons, and lunes in the case of the sphere—the faces of an oblique ashlar are irregular figures, also delimited by curved lines. In the absence of earlier graphic testimony, we must believe that De L'Orme's method consisted exclusively of total mastery of drawing; that is, of the application of a general projection principle, which can be used to resolve not merely a particular shape but any invention the architect is capable of imagining. Not by chance a large portion of the two books on stonecutting is dedicated to the detailed explanation and illustration of a new "trompe," of which De L'Orme writes, "I am certain that no worker in this kingdom has ever heard tell of it before."[73] This was the "trompe surbaissée, bias et rampante," a variation of the "trompe de Montpellier" distinguished by an elliptical shape, a rampant arch, of the generatrix of the cone. The invention of this "trompe" dates from 1536, when De L'Orme, upon returning from his voyage to Rome, built the first example of it in the Hôtel Bullioud at Lyon. About ten years later he created an even more refined variation in the Castle of Anet, which represents his masterpiece of stereotomy (figure 6.18). While the "trompe" of Lyon supported a simple cylindrical volume, that of Anet bore up a mixtilinear cylinder, which conferred on the line of intersection with the elliptical cone an undulating profile of bold audacity, both technically and formally: "which I wanted to make of a form so strange as to render the trompe of the vault more difficult, and beautiful to behold."[74]

De L'Orme evidently pursued a stereotomic virtuosity that emerged as a stylistic sign, and sought a new aesthetics based on geometry rather than on ornamentation. And it was in the name of this principle, in fact, that he criticized the father of Renaissance classicism, Donato Bramante, whose helicoid staircase in the Vatican Belvedere he considered an exemplary case of geometric incongruity. "If the architect who built it had known the geometric method of which I speak," wrote De L'Orme, he would have operated so that each element followed the geometric nature of helicoid development, even the bases and the capitals of the columns, which instead, in keeping with Vitruvius' rules, Bramante had drawn as if they supported "a straight, horizontal portico."[75] This geometric incongruity is also criticized by Juan Caramuel de Lobkowitz who in the next century was to base the principles of "oblique architecture" expressly on stereotomic geometry.[76] The solution suggested by De L'Orme for Bramante's staircase is the one employed in the tribune of the Cathedral of Saint-Étienne-du-Mont in Paris, which also appears in a drawing in the Louvre traditionally attributed to the French architect.[77] Of the helicoid staircase built in the Castle of

72 See Dürer 1983, IV figure 29–34.
73 De L'Orme 1567, IV 91^r^.
74 *Idem*, IV.2 89.
75 *Idem,* IV.19 124.
76 Caramuel de Lobkowitz 1678, tratado VI, articulo XII.
77 Paris, Cabinet des Dessins du Louvre, n. 11114; see Blunt 1958, figure 139.

Fontainbleau, De L'Orme notes the "great artifice and the amazing difficulty" presented by the intersection of the three cylinders, which define, respectively, the perimeter of the staircase, the horizontal vaults of the flying buttresses and the oblique vault of the helicoid staircase.[78] His *esprit de géometrie* reaches perhaps its greatest aesthetic heights in the chapel in the Castle of Anet, which is a true architectural expression of the art of "extracting the elevation from the plan" (figure 6.19). The de-

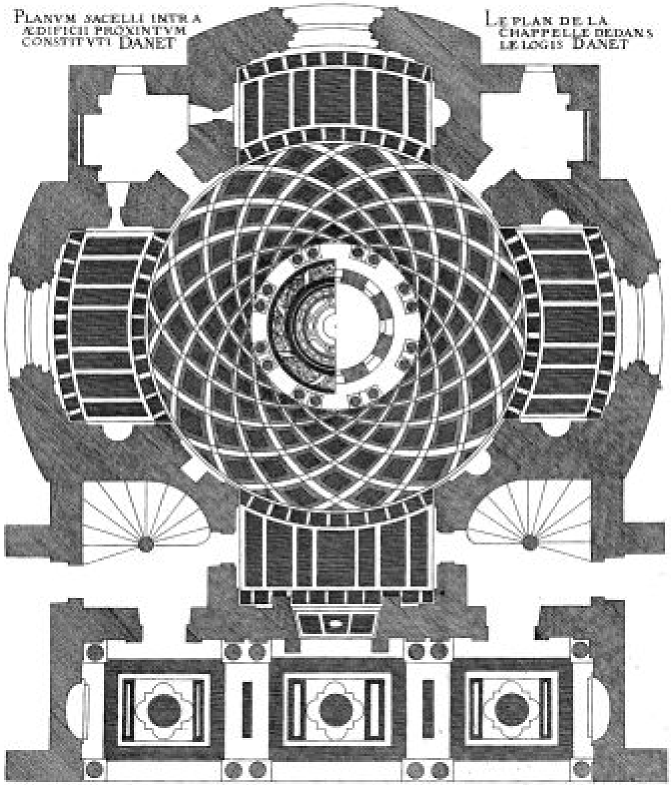

Figure 6.19. Plan of the chapel in the Castle of Anet. Engraving by Androuet du Cerceau. (du Cerceau *Les plus excellents bastiments de France,* Paris 1579.)

78 De L'Orme 1567, IV.19 125.

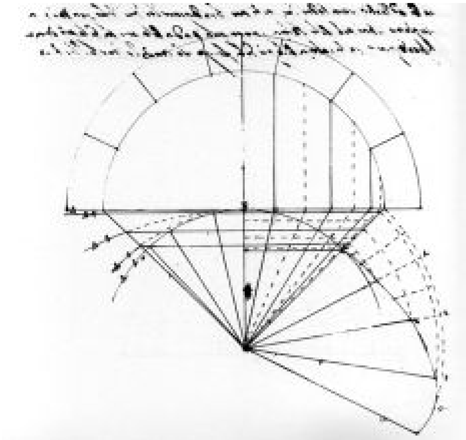

Figure 6.20. Constructive drawing for the "trompa de Monpeller." Drawing by Alonso de Vandelvira. (Vandelvira *Libro de traças de piedras,* ms., c. 1575.)

sign of the floor is in fact a perfect orthogonal projection of the three-dimensional arches opening onto the cylindrical perimeter and of the amazing coffered hemispherical vault. It is as if the real-size drawing made for executing the "panels" for cutting the stones had been transformed from a worksite document into a decorative motif.

The natural ease with which the stonemasons mastered drawing for stonecutting is also demonstrated by the only other treatise on stereotomy from the sixteenth century: the *Libro de traças de cortes de piedras* by Alonso de Valdelvira.[79] The Spanish architect composed this work between 1575 and 1590; although presumably familiar with De L'Orme's writings, he developed the theme in an entirely independent manner. The *Libro de traças* is in fact the first specialized treatise on stereotomy: a refined compendium of worksite techniques, which is even more eloquent than the work of De L'Orme as regards the stonemasons' knowledge of geometry. The introduction echoes that of many treatises on practical geometry and perspective:

> The beginning of the drawing is the point, and from it proceeds the line, and from lines the area and the surface, and from the surfaces the bodies on which all operations are carried out, and thus the species of the basis of this art are three: the lines which are those that surround the surfaces; the surfaces which are those that delimit the bodies, and the body which is the corporeal substance of the construction; (the species of the forms of these bodies are infinite).[80]

79 Valdelvira 1977.

As to the individual stone ashlar a description is given of each of its six faces, the method for determining its shape, and the instrument used to work it and to verify its installation: a mobile square called *baivel* (*biveau* in French). In the construction of the "trompa de Monpeller," Vandelvira applies a method slightly different from that of De L'Orme, which allows less superimposition of the graphic indications and thus easier reading of the drawing (figure 6.20). While De L'Orme draws the generatrix of the cone on a secant plane, passing through the points at which the cylindrical volume meets the rectilinear walls of the building, Vandelvira plots it on a tangent plane, displaying the intersection between the volumes much more clearly. Furthermore, the Spanish architect does not effect overturning to find the "slope line," but immediately constructs the development of the cone, which visualizes the true shape of the intrados "panels."

Vandelvira's method also shows how the problem to be solved was that of determining the transformation of geometric figures: a plane arch that is transformed into a three-dimensional arch for example, or a circle that is transformed into an ellipse. The concept of projection is implicit but not necessarily present in all of these operations. Only with the work of Girard Desargues did it become certain that this geometry of transformations was knowledgeably based on universally valid principles of projection. "This manner of carrying out drawings for the cutting of stones," writes Desargues in the *Pratique du trait* (1643), "is the same as the method of practicing perspective without employing any third point of distance, or of any other type, that lies outside of the painting …" And further on he reiterates:

> … so that there is no difference between the manner of depicting, reducing or representing anything in perspective, and the manner of depicting, reducing or representing in a geometrical drawing, so that geometrical drawing and perspective are no other than two species of the same gender, which can be enunciated or demonstrated together, using the same words …[81]

Viewed in the light of hindsight, this truth emerged throughout the Renaissance. Albrecht Dürer, in particular, was the artist who came closest to a general formulation of "descriptive geometry." He put together the technical and geometric knowledge of many professional categories—painters, architects, cartographers, stonemasons, astronomers and instrument makers—demonstrating how the *dottrina* of drawing lodged in the workshops of craftsmen more than in the minds of geometers. He himself invented new graphic and mechanical procedures for geometric drawing: multiple orthogonal projections, the projection of shadows, the famous perspective instruments, and the extraordinary compasses for drawing ellipses, spirals, and new curves not yet defined on the geometric level. His numerous technical and geometric inventions, often developed and experimented within the workshops, and subsequently disseminated through the press, are a striking example of how ingenuity progressively favored the evolution of drawing toward a doctrine that was later to be reinforced by the geometric concepts of infinity, projection, and homology.

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80 *Idem*, introduction.
81 See Desargues 1640; in Desargues 1864, I 305–362 (in particular, 305ff.).

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THE EMERGENCE OF COMBINED ORTHOGRAPHIC PROJECTIONS

WOLFGANG LEFÈVRE

INTRODUCTION

Modern engineering employs all kinds of drawings—from sketches, over several orthographic plans and combinations of such plans, through a variety of schematics and diagrams, up to all sorts of illustration drawings in perspective.[1] Plans, and in particular combined plans, constitute the core of this abundance of engineering drawings, at least in the process of designing and manufacturing.[2] It is therefore surprising that plans are rather the exception than the rule among the drawings traced and used by engineers of the Renaissance.[3] From Taccola's well-known illustrated manuscripts of the first half of the fifteenth century up to the famous *Theatres of Machines* of the late sixteenth century, perspective drawings dominate the picture. And of the few plans known from this age, hardly any could be called a blueprint in the present understanding of the term.

Destruction or interrupted transmission can be ruled out as the main causes of this remarkable absence of plans,[4] which thus indicates a practice of engineering that, as a rule, designed and manufactured intricate devices without plans. A reconstruction of this praxis must focus on issues like the organization and division of labor in projects where engineers were involved, the prevalence of adaptations of traditional solutions to local circumstances, the almost complete lack of manufacturing the different parts of a mill, a water lifting device or a machine at different locations, the role of models and templates, the education of the craftsmen, and so on.

However, such a reconstruction of a practice of engineering without plans can easily be mistaken as an explanation of the absence of plans that argues: Since these engineers could do without plans, they didn't take pains to construct ones. But this conclusion is only valid if one can presuppose that Renaissance engineers would have been able to construct plans if they had wanted to. Are there reasons to assume that they had sufficient command of the techniques necessary for drawing orthographic plans? Now the fact that almost all of them also worked as architects appears to be a

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1 See, for instance, Rising and Amfeldt 1964.
2 I disregard here the sweeping changes that the computer and specific electronic applications for engineering drawing brought about in the design process during the last two decades.
3 See Lefèvre 2003.
4 By their very nature, plans that actually served as a means of designing and manufacturing can be expected only either in the private files of the engineers themselves and of private customers, or in the archives of public commissioners. And it is rather probable that many of these documents are indeed lost. However, the few cases where such materials survived and are known confirm the overall picture of the marginality of plans in the realm of Renaissance engineering. The Württemberg engineer and architect Heinrich Schickhardt (1558–1635), whose private files are preserved and held in the Hauptstaatsarchiv Stuttgart, is probably a good case in point—see Popplow 1999.

very strong indication. Rather, what seems to require explanation is the strange fact that people who were familiar with orthographic plans when constructing buildings very rarely took advantage of these means when functioning as engineers. But again, this involves an unproven presupposition. This time, one would assume that all basic techniques of constructing orthographic plans were known and employed in the realm of Renaissance architecture. But is this true?

In the literature on Renaissance engineering, architecture, or perspective painting, it often is taken for granted that the engineers and architects of this age had basically the same techniques of constructing plans at their disposal as their modern colleagues. Since the drawing of orthogonal plans is considered a rather trivial task in contrast to perspective depiction, the technique of constructing plans is usually supposed to be given where plans occur: for instance, in ancient Mesopotamia or ancient Egypt. The technique of constructing plans, thus, has the appearance of a cultural achievement without a really historic development: its origin is hidden in the dawn of the earliest cultural developments, and its history seems to be that of a mere technical refinement of a procedure, the essence of which was given with its first invention and remained the same up to the present day.

The period under scrutiny here proves that this picture is wrong. For the Renaissance was the time of a decisive turn in the history of plan construction. The double-faced character of such periods of change is clearly discernible. On the one hand, one encounters techniques and conventions of producing plans that differ distinctively from present ones, not only in some secondary aspects, but in substance. (And one may ask whether the plans from the Renaissance convey an impression of how the technique of constructing plans might have looked in earlier ages, not only in the Gothic period, which is essentially the same with respect to plans, but also in classic Antiquity.) On the other hand, the modern techniques of constructing plans were actually invented—albeit rarely employed—in just this period. This chapter, therefore, has two goals. It tries to show that Renaissance (and Gothic) plan construction was not based on a technique that is essential for present-day plan construction, namely the combined views technique. Furthermore, it tries to show that this technique was invented at the turn of the fifteenth century and introduced to architecture by Antonio da Sangallo the Younger and Albrecht Dürer in the second decade of the sixteenth century, and that it took the entire sixteenth century before it became a standard technique in architecture.

Before coming to how the chapter proceeds, a brief clarification of terms like plan, elevation, and combined views may be useful. In this chapter, the term “plan” denotes any kind of orthographic projection, that is, a projection where each line that connects a point of the depicted object with its representation on the drawing meets the drawing plane at right angles.[5] In this way, it is warranted that plans depict true angles and true distances, or, in the case of scaled plans, true proportions among distances. It is this fidelity with regard to angles and distances that gives plans their spe-

5 More precisely, I use the term “plan” to denote all kinds of multiplanar orthographic projections. For a contemporary classification of projections, see, for instance, French 1947, 91.

cial value for architecture and engineering and distinguishes them from perspective pictures. The latter systematically distort certain angles and certain distances in order to depict the object as it appears to the eye. Perspective drawings, therefore, have the advantage of conveying an impression of the object as a solid and of its being located in three-dimensional space. In contrast, plans can only offer flat views of their objects. Thus, in order to depict a three-dimensional object by orthographic projection, at least two plans from different points of view have to be constructed.[6]

This is the principal reason why in architecture and engineering different kinds of plans are employed: ground plans, elevations, and sections, to name just the most common. Usually, the drawing planes of different plans that depict the same object from different points of view are perpendicular to each other, elevations or sections to the ground plan, and different elevations to each other (with the exception of the parallel drawing planes of elevations from exactly opposite points of view). Presupposed, furthermore, that these plans are drawn to the same scale, they can be unified in an unambiguous manner. The technical term "combined views" denotes a particular kind of such combinations of plans, namely the combination of a ground plan with one or two elevations, the planes of which cut one another at right angles (figure 7.1).[7] This combination layout is not just unambiguous but furthermore allows, in the case of three views, the position of a point in one of the plans to be deduced if this point is represented on the two others. Thus combined views are not only a means of repre-

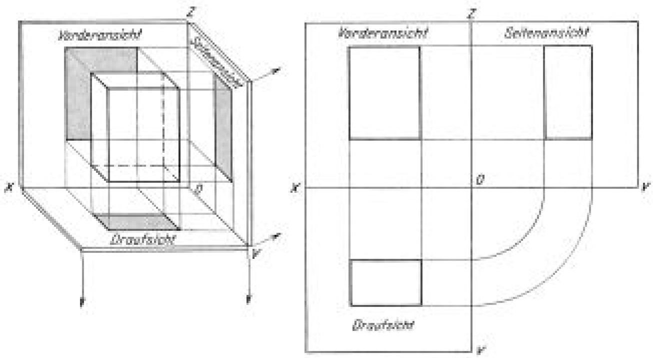

Figure 7.1. Combined views. (Bachmann and Forberg 1959, 168.)

6 For the purposes of this chapter, it is not necessary to discuss the different kinds of axonometric projections—isometric projection, dimetric projection, trimetric projection—which try to find a compromise between the advantages of both perspective drawings and plans. It is also not necessary to go into the different kinds of oblique projections, the best known of which is probably the "cavalier projection." For the problems of such projections, see French 1947, 459ff. and 468f.

7 Such combinations of a ground plan and one elevation are usually called "two-view drawings" or "double views" and those of a ground plan with two elevations "three-view drawings" or "triple views."

senting an object, but at the same time a means of constructing orthographic plans. To my knowledge, triple views—a ground plan with two elevations—became the standard form probably not before the nineteenth century. What is usually found before this century, beginning with a few exceptional cases around 1520, are double views—a ground plan and only one elevation—which are, nevertheless, suitable as a means for the construction of plans.

There is one kind of plan in particular that needs to be constructed by means of combined plans, namely, elevations of objects with surfaces that are not parallel to the drawing plane. Elevations, thus, are indicative of the presence or absence of the combined views technique. They can be read as symptoms. That is the reason why elevations from the period before 1500 are in the focus of the *first part* of this chapter, which comes to the conclusion that combined views were not in use before the sixteenth century. This means that, before this century, architects had command of a technique of constructing plans that differs significantly from our technique, not only in range but also in its inner structure.

The *second part* of the chapter gives some context of this technique by providing a concise survey of the role plans played in building practice up to the sixteenth century. The main result of this survey is that no urgent needs of the construction practice stimulated the invention of a general method of deriving elevations from ground plans. Actually, there was no such need. The architects of the age had at their disposal all means needed for their purposes.

The *third part* then deals with the emergence of the method of constructing elevations of any kind by means of combined views at the beginning of the sixteenth century. However, no narrative of the emergence of this method is attempted, merely a discussion of some of its aspects, background, and context without any claim to completeness or systematicity. I focus on Albrecht Dürer's achievement and only occasionally point to that of Antonio da Sangallo to draw attention to striking parallels or interesting contrasts.

1. THE PROBLEM OF ELEVATIONS

At first glance, the claim that the technique of constructing elevations was not fully developed before the first decades of the sixteenth century, not before the time of Antonio da Sangallo (1484–1546) and Albrecht Dürer (1471–1528), seems untenable. Too many pieces of counter-evidence can be adduced. Texts may come to mind immediately that testify to the contrary.[8] For instance, in a well known paragraph of his *De Re Aedificatoria*, Leon Battista Alberti (1404–1472) not only mentioned explicitly elevations of front and side façades of a building, but also stressed the difference between the perspective drawings of painters and the orthographic plans of architects.[9] The famous *Lettera a Leone X* from the beginning of the sixteenth century, which was formerly attributed to Raphael and later to Castiglione, is another

8 For these texts, see also part 4 of the chapter by Filippo Camerota.
9 See Alberti 1966 book II, chapter 1.

example that could be quoted.[10] This text, too, stressed the difference between pictorial drawings and plans and admonished architects to refrain from shortening in the manner of perspective drawings when constructing elevations. The circumstantial and sometimes even awkward way in which the letter tries to make this point is, however, rather indicative of how little ingrained the construction of orthographic elevations still was at that time.[11]

However, written texts alone cannot settle the question, as the case of Vitruvius' distinction between *Ichnographia* (ground plan), *Orthographia* (elevation), and *Scaenographia* (a kind of perspective) shows:[12] Because hardly any examples of such drawings are actually extant, one can only speculate how these different kinds of projections may have looked and how they were constructed and applied by Roman architects. With regard to the Renaissance and even to the Middle Ages, we are in a better situation. Even among the architectural drawings of the thirteenth and fourteenth century, that is, among the oldest known architectural plans *(Baurisse)* of the Occident, not only ground plans can be found but also elevations. For the mediaeval countries north of the Alps, one can point to the Reims palimpsest from the beginning of the thirteenth century[13] or the elevations of the façade of the Strasbourg cathedral from the same century,[14] and for Italy, to the elevations for the façade of the Orvieto *Duomo* from the beginning of the fourteenth century.[15] These elevations already display such a high standard of draughtmanship that it was taken as a extremely strong indication, if not almost proof, that the beginnings of architectural plans in the Occident must be dated to a time that long preceded this early evidence.[16]

Obviously, our claim needs qualification. For this, a closer look has to be taken at the architectural elevations from before the sixteenth century. There are sometimes elevations of complete façades of cathedrals, in particular the splendid west façades, or of church towers, mostly, however, elevations of parts of such façades, of walls with windows or arcades, of rose windows, and the like. In other words, what these elevations depict are those sides of buildings or of certain parts of a building whose planes are exactly parallel to the drawing plane. The same holds for elevations of the period, which belonged to the realm of carpenters and goldsmiths, even though only conditionally with respect to the latter. It goes without saying that it is much easier to construct the elevation of such a parallel surface than that of an object with oblique sides. It would, therefore, be premature to conclude from such façade elevations

10 Text in Bruschi et al. (ed.) 1978, 469ff.
11 See Germann 1993, 106; see also Thoenes 1993, 566f.
12 See Vitruvius I.2 (Rose 11:23ff.).
13 See Branner 1958.
14 See Anonymous 1912.
15 See Degenhart et al. (ed.) 1968 vol. I.3, table 25 and 27. As the rendering of the portals shows, these elevations are not purely orthogonal, but also employ techniques of shortening in the manner of perspective drawings. This combination of different drawing techniques is characteristic of Italian architectural elevations and sections before the sixteenth century, and even during this century as will be shown.
16 See Booz 1956, 67. Whether or not one is ready to accept Booz's conclusion, there is in any case indisputable evidence for a highly developed architectural draughtmanship that precedes the first known plans, namely the *"Ritzzeichnungen,"* that is, architectural drawings scratched in stone on the floor or on walls of mediaeval cathedrals—see Schoeller 1980 and 1989.

whether or not their creators had techniques at their disposal that enabled them to construct elevations of objects regardless of their position to the drawing plane. In other words, it is not clear whether these architects had command of a general technique of constructing elevations, or only of a partial one that was restricted to the case of parallel surfaces. This is the decisive question in our context. What I am claiming exactly is that such a general technique was not part of the professional skills of architects before the sixteenth century.

The preserved elevations themselves offer the strongest testimony in favor of this claim. Due to the very nature of the depicted objects, it was not always possible to avoid oblique sides when constructing elevations of the parallel surfaces of a building. And in such cases the construction of these oblique parts is always "wrong" according to our present standards. The "errors" that Villard de Honnecourt (fl. 1225–

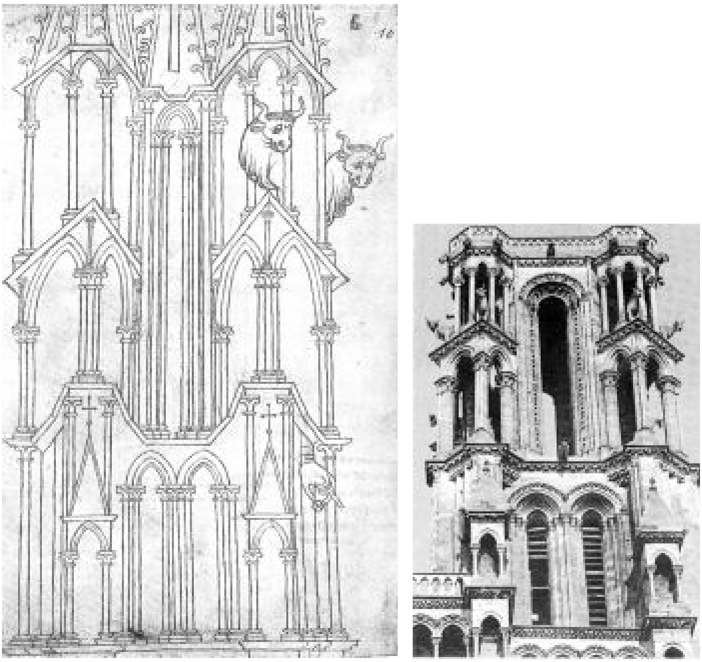

Figure 7.2. Villard de Honnecourt's elevation of Laon Cathedral. (Paris, Bibliothèque nationale de France, ms.fr 19093, fol. 19; courtesy Bibliothèque nationale de France.)

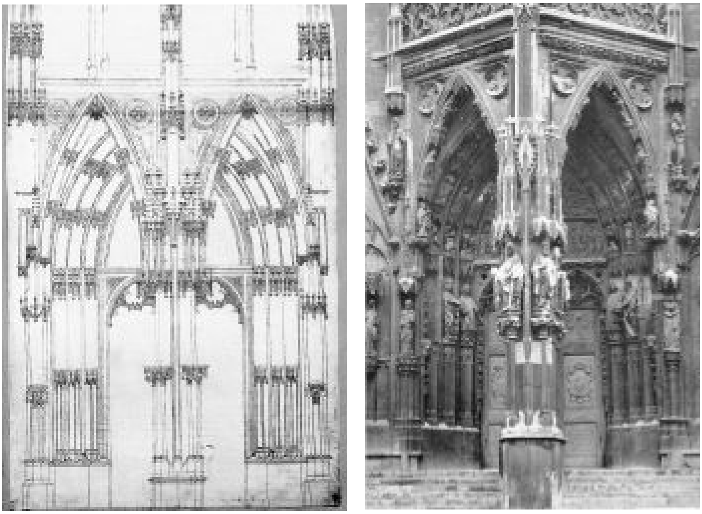

Figure 7.3. West portal of Regensburg Cathedral, c. 1500. (Vienna, Kupferstichkabinett der Akademie der Künste, Inv. Nr. 16.871; courtesy Kupferstichkabinett der Akademie der Künste.)

1250) committed in this respect are famous (figure 7.2). But, if they are considered as plans at all, they may prove nothing more than that Villard was not "an accurate draftsman."[17] If, however, such "errors" also occur in elevations of outstanding mastery, which, furthermore, were drawn centuries after Villard (figure 7.3), one can no longer resort to individual faults but must acknowledge a general pattern. Focusing on this pattern, Peter Pause found out that the "errors" on elevations from the countries north of the Alps follow certain rules such that one can make out a small set of conventions, which have nothing to do with orthographic projection or with linear perspective.[18] One of these conventions, for instance, consisted in a simple cut-off rule: By cutting off a vertical strip of a window or arcade, the draftsman indicated that they belonged to an oblique wall (figure 7.4). From the age of Villard[19] until the sixteenth century, architects followed these conventions when rendering the shortening of oblique sides. This practice would be absolutely unintelligible if these architects had been able to construct such oblique surfaces in a correct orthographic manner.

17 See Branner 1963, 137.
18 See Pause 1973, 57ff.
19 For Villard, this cut-off rule is described in Hahnloser 1972, 32.

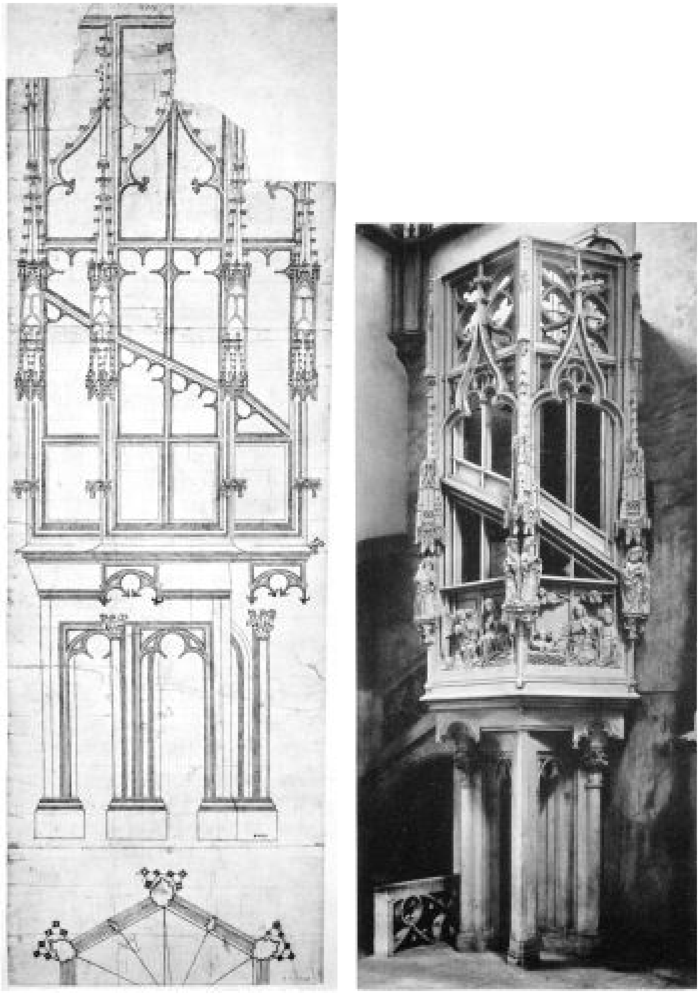

Figure 7.4. “Schnegg” of Konstanz Cathedral. (Vienna, Kupferstichkabinett der Akademie der Künste, Inv. Nr. 17.028; courtesy Kupferstichkabinett der Akademie der Künste.)

The absence of a general technique for constructing elevations of objects, whatever their position to the drawing plane, could also be shown by pointing out a different practice that was then common in Italy. Italian architects did not follow these conventions of their northern colleagues, but used linear perspective when rendering such oblique parts of buildings. As Wolfgang Lotz could show for sections, one encounters no purely orthographic sections in late fifteenth- and early sixteenth-century Italy. Rather, up to the time of Antonio da Sangallo, architectural sections always combined orthographic with perspective rendering.[20] This is all the more remarkable since these sections prove that a highly developed skill in constructing perspective drawings obviously was already remarkably widespread among Italian architects at the turn of the fifteenth century (figure 7.5). Thus it has the appearance that, at least in Italy, architects were able to construct perspective renderings in an unrestricted manner earlier than orthographic elevations. (I will come back to the relations of these two techniques in the last part of this chapter.)

It seems, perhaps, quite unbelievable that men capable of constructing correct perspective drawings[21] would not be able to construct all kinds of orthographic elevations. A possible reason why this appears astonishing might be that one underestimates the difficulties associated with the task of constructing elevations. And it is indeed probable that this underestimation is a result of the cultural fact that, among all of the modern techniques of geometric projection that emerged in the Renaissance, only linear perspective has enjoyed the undivided and continuing attention of a broad audience ranging from art historians over historians of culture, mathematics, and science up to semiotic experts, philosophers, and psychologists. The modern projection techniques of cartographers, astronomers, engineers, and architects, on the other hand, the foundation of which was also laid in this age, are taken to be either trivial or only of technical interest for certain experts.

In order to make the difficulties accompanying the construction of an elevation a bit more discernible, I should stress that drawing an elevation does not mean portraying something, but constructing a representation—a flat model, as Nelson Goodman might say.[22] This is obvious in the case of elevations for a planned building drawn in the design process. But even when the task is to record an already existing building by means of elevations, such elevations have to be constructed point by point in conformity with distances and angles taken from the real building using appropriate instruments. This architectural recording *(Bauaufnahme)* much more closely resembles the mapping of surveyors than the depicting of painters. Vision is suspended in

20 See Lotz 1977. As mentioned above, this also holds for the earliest known Italian elevations, that is, the elevations of the façade of the *duomo* in Orvieto from the beginning of the fourteenth century, and the famous elevation of the *campanile* of *Santa Maria del Fiore* in Florence from the middle of this century, which is thought to be a copy of an original drawing by Giotto. See for the latter Degenhart et al. (ed.) 1968, vol. I.3, table 66 and vol. I.1 89ff. See also Evans 1997, 166f.

21 Apart from a few rare exceptions, we do not actually know how perspective drawings were produced in the fifteenth and in the early sixteenth century. It thus seems well advised in our context to recall an easily neglected difference between the drawing of perspective pictures and elevations: Whereas the former could be produced without construction by means of a variety of mechanical-optical devices then available, there was (and is) no escape from constructing in the case of the latter.

22 See Goodman 1976, 172f.

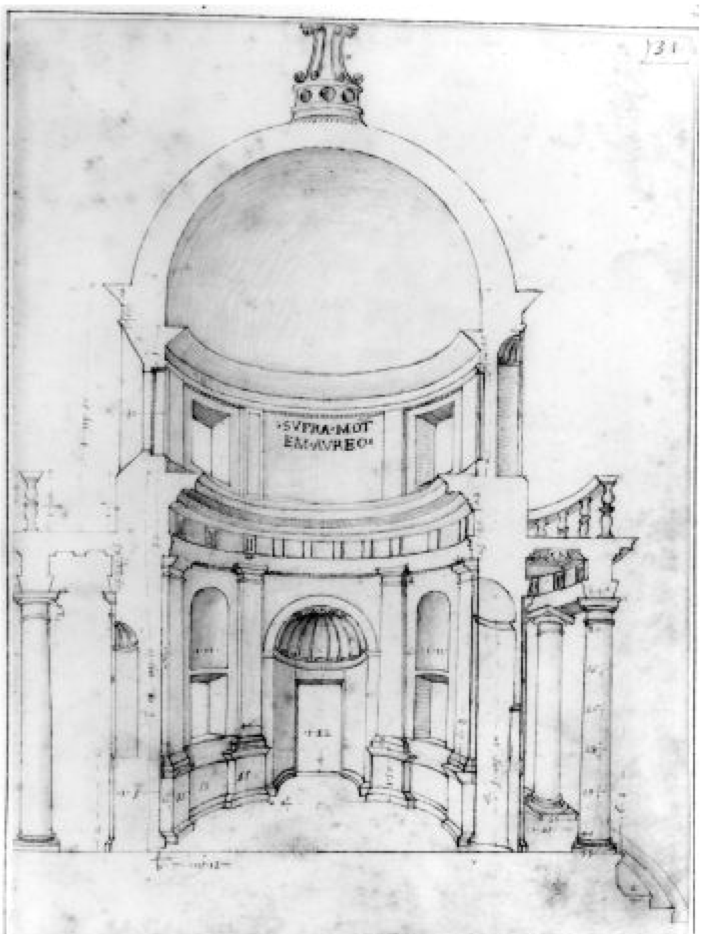

Figure 7.5. San Pietro in Montorio. Section from 16th-century Italy. (London, Sir John Soane's Museum, Codex Coner, fol. 34; courtesy Sir John Soane's Museum.

favor of construction, that is, in favor of a complex process of measuring, recording measurements taken, processing these data, and, based on them, tracing the graphic representation according to geometrical rules.

As a means of the designing process, the tracing of elevations seems to be less complex, except for the difficulties of non-trivial geometrical constructions and accurate tracing in general, which I ignore here. The desired measurements—for instance, the height and width of the windows or the breadth of the wall between them—can be translated directly into appropriate geometrical lines and shapes on the plan. But this is true if, and only if, the plane of the wall or window represented is thought to be parallel to the drawing plane. In all other cases, one has to derive the correct translation from the ground plan. And this derivation is exactly the point where the combined views come into play. From a ground plan drawn to the same scale as the elevation and attached to it in the right way, one can "deduce" the correct positions of these problematic parts by means of ruler and compass. Moreover, given a list with the intended heights of all parts in question, one can develop the entire elevation from the ground plan. On the other hand, without this technique of combined plans or any substitute for it, it is simply impossible to determine the exact position and accurate size of the representations of those parts that are thought to lie in an oblique plane. An octagon like the Florentine baptistery is a case in point (figure 7.6). There is no other way to determine the distances between the vertical edges of such a simple and regular geometric body on the elevation than that of deriving them from the ground plan.

The extant elevations from before 1500 show, thus, that, apart from two or three exceptions,[23] which confirm rather than challenge the rule and will be discussed below in the third part, combined views were not in use before the sixteenth century. This is also confirmed by the telling fact that the famous treatises on architecture of the fifteenth century—besides the treatise by Alberti and the letter to Pope Leo X mentioned above, the *Trattato di Architettura* by Antonio Averlino, known as Filarete (1400–1469);[24] the treatises *Architettura ingegneria e arte militare*[25] and *Architettura civile e militare*[26] by Francesco di Giorgio Martini (1439–1501); and the drafts on architecture of Leonardo da Vinci (1452–1519)[27]—are absolutely silent on this technique.[28] Thus one can assume that combined views were actually unknown in this age. Not even Cesariano's Vitruvius edition of 1521 gives any indication of a change in this respect.[29] But if combined views had not yet been discovered as a decisive means of constructing plans, it is no longer strange that one does not encounter correct elevations of objects with oblique sides before Dürer and Antonio da Sangallo.

23 See Sakarovitch 1998, chap. 1.
24 Filarete 1972, English: Filarete 1965.
25 Francesco di Giorgio Martini 1967, vol. I.
26 Francesco di Giorgio Martini 1967, vol. II.
27 Edited in Bruschi et al. (ed.) 1978, 277ff.
28 See Sakarovitch 1998, 55ff. With respect to Alberti's treatise on architecture, see also Thoenes 1993, 569.
29 Note that, in Cesariano's edition, the plate to book I, chapter 2, which displays example drawings for Vitruvius' *orthographia*, also includes drawings that are not orthographic elevations but rather perspective drawings.

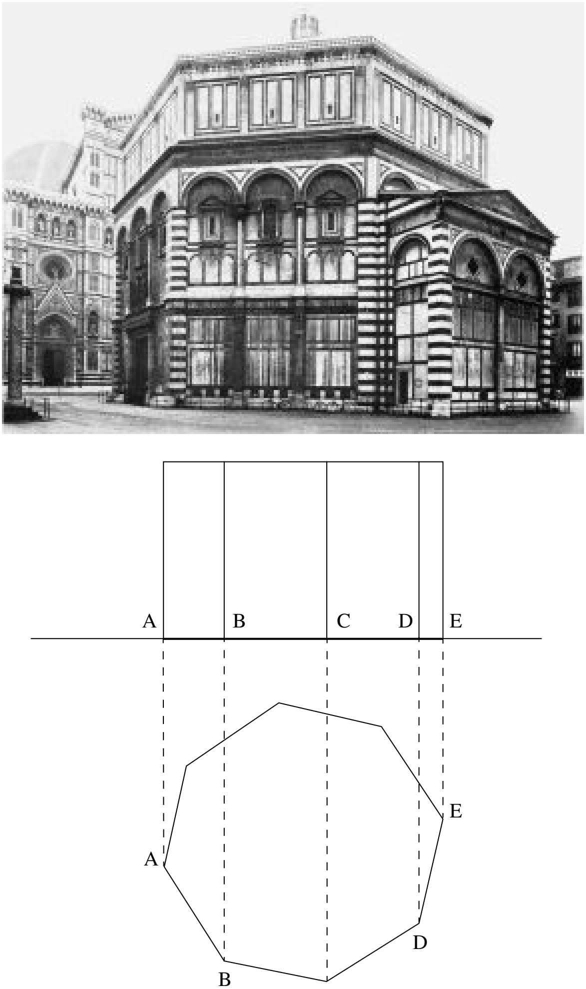

Figure 7.6. Deriving the elevation of an octagon from its ground plan. Drawing by the author.

On the contrary, it would be a miracle if the architects of the age had been able to construct correct elevations despite the lack of this technique.

Summing up this first part of this chapter, it can be stated that, in the realm of architecture, the technique of combined views does not occur in fully developed form until Antonio da Sangallo and Albrecht Dürer, that is, not before the first decades of the sixteenth century. Before going into these achievements and their context and prerequisites, it might be useful to take a quick glance at the role that plans and drawings played in the process of designing and constructing buildings until the end of the fifteenth century.

2. PLANS IN ARCHITECTURE UNTIL 1500

Regarding the role of plans for the architectural practice of the Middle Ages and the early Renaissance, the most striking feature of this practice is probably what can be called its improvisatorial character. As a rule,[30] the architectural features of a planned building were not fixed in all their aspects and details in advance. Commissioner and architect confined themselves to appoint only main features when contracting. Above all two reasons seem to be responsible for this practice. First was the custom of postponing decisions on certain questions to a time when they could be made in light of the growing building. Second, and probably more important, was the fact that many features needed no explicit agreement because they were obvious within the given tradition of construction. All that had to be done was to adapt the canonical rules and conventions of this tradition to local circumstances. And this adaptation was tacitly included in the overall design of the building upon which commissioner and architect had agreed.

The famous controversial conferences of the Milan cathedral council between 1389 and 1401, which are remarkable in so many respects,[31] provide a good example of the custom of postponing decisions. On the one hand, these conferences testify to ongoing revisions of former decisions, that is, to a thorough planning process. On the other hand, however, they testify to a lack of planning in advance that must be inconceivable for a contemporary architect. The elementary features of the foundations of walls and piles had not only been designed, but actually laid and constructed when the question of the height of the naves ignited the controversy. The essential question of the height of the cathedral was obviously not fixed along with the ground plan. *Santa Maria del Fiore*, the Florence cathedral, is another case in point. After the original foundations had been dismissed and replaced by significantly larger ones, an endless process of decisions and revisions of decisions took place with regard to the overall shape of the cathedral, which the *Operai di duomo* finally cut off with a somewhat iconoclastic act, namely by destroying the competing models.[32] However, what remained unsettled by this finality and engendered the subsequent "drama"[33] of the

30 See for the following Müller 1990, 14ff. where the most important literature is discussed.
31 See Ackerman 1949.
32 See Lepik 1994, 35.
33 Saalman 1980, 11.

cupola—not just a decorative spire on top of the crossing, but a weighty, important part of the cathedral, structurally as well as aesthetically—was the fundamental question of whether or not the huge octagonal crossing, which was already built could be vaulted with a cupola of the appointed shape. A similar drama was played out a century later in Rome when the new *San Pietro* church was built.[34] These three prominent examples may represent the innumerable cases up to the beginning of the sixteenth century where the definite design of essential parts of an building was not appointed in advance along with the ground plan, but left to later decisions.

In the light of this improvisatorial building practice, it becomes intelligible why one encounters only ground plans and elevations or sections of certain parts of a building among the preserved architectural plans of the age but, to my knowledge, never a set of plans that determines all main features of a building. The basis of this astonishingly flexible practice was a traditional framework of rules and conventions for how to build a sacral or profane building—a basis, however, which always became perilous when, as in the prominent cases mentioned, the designed buildings went beyond the limits of this tradition. The fact that these rules and conventions[35] had guiding functions for architects and craftsmen of the age equivalent to those plans have today explains why so many features of a building were not fixed in advance by means of plans. In Lorenz Lechler's *Unterweisungen* of 1516,[36] for instance, one of the few preserved texts that document such rules and conventions,[37] one finds several rules, including alternative ones that connect the measures of the essential parts of a church, such as the width of the main nave with its height, the width of the arcades, the thickness of piles, walls, buttresses, the measures for the adjacent chorus, and so on. The actual constructed churches were, of course, not mere executions of a set of rigid rules and conventions. Rather, these rules formed so flexible a system that decisions were necessary in any case as to which of the countless possibilities was to be realized in the situation at hand. And the architect often furthermore had to accommodate wishes of his commissioner that were at odds with the rules of the art.[38] On the basis of this system of rules, however, such decisions and accommodations could be appointed without a comprehensive set of plans. A ground plan and elevations of some critical parts usually sufficed for this purpose.

It is known that, in the countries north of the Alps, plans were part of the contracts between architects and clerical or secular commissioners. Their function was the same as that of the wooden models in Italy, namely to document the design the con-

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34 See Bredekamp 2000.
35 See for the following Müller 1990, 78ff.
36 A diplomatic copy of the three preserved manuscripts of the booklet can be found in Coenen 1990, 174–266.
37 Apart from Lechler's booklet, there are five more such practitioners' texts written by architects for architects: 1)*Von des Chores Maß und Gerechtigkeit* of about 1500, of which only an incomplete nineteenth-century edition exists but neither the original nor contemporary copies (see Coenen 1990, 25); 2) the so-called *Wiener Werkmeisterbuch* from the fifteenth century; 3) Matthäus Roriczer's *Büchlein von der Fialen Gerechtigkeit* of 1486, and 4) his *Geometria Deutsch*, written slightly later; 5) Hans Schmuttermayer's *Fialenbüchlein* of the 1480s. Coenen 1990 provides diplomatic copies of all of these texts. Shelby 1977 provides an modernized edition and English translation of the two booklets by Roriczer and the one by Schmuttermayer.
38 See, for instance, Lechler's complaints: Coenen 1990, pp. 179, 233, 262.

tracting party had agreed upon.[39] But it is not known how sketchy or sophisticated these plans were. Some scholars regard the bulk of the preserved plans, and in particular the splendid elevations, as illustration drawings addressed to commissioners.[40] The reason for this ascription is mainly that these plans look too elaborated for drafts and too inaccurate for working drawings *(Werkrisse)*, that is, plans that were used in the construction process.[41] This leads to the question of exactly what functions plans had in the process of designing on the one hand, and for the construction process on the other, given the flexible character of the architectural practice outlined above.

Beginning with the function of plans in the architectural design process of the Middle Ages and the early Renaissance, there is little accord among the experts. Since no single plan is known that without any doubt served the purpose of design, and the same holds for sketches before Leonardo, one can only speculate about the graphic means that the architects of the age used when designing a sacral or profane building. It thus comes as no surprise that one meets the whole spectrum of possible assumptions in the literature, ranging from designing without any kind of plans[42] over sophisticated geometrical design procedures[43] up to designing with all kinds of orthographic plans.[44] One point, however, seems to be clear: Plans and other graphic representations were no means of reflection or thought experiments regarding the static of the planned building. Leonardo's sketches in the *Codex Madrid* are absolute exceptions. Another point seems to be at least very likely, namely the use of grids for designing ground plans. The famous plan of the Abbey of St. Gall from the ninth century obviously uses this grid technique, which can be traced back to ancient Egypt. There is no reason to assume that this suitable graphic technique went lost in the course of the Middle Ages.

With respect to the function of plans in the construction process, one should distinguish between reduced plans that are scaled down and full-size plans that are constructed to full scale (1:1). It goes without saying that, in contrast to full-size plans, reduced plans do not yield any information that can be picked up mechanically by means of compasses or comparable instruments. Rather, one first has to process the information of these plans through both arithmetical and geometrical procedures before measures and shapes can be transferred to the building materials. According to the literature, it is still an open question whether or not the few reduced plans we have can be taken to be accurately scaled plans, and therefore to be real working drawings.[45] But even if one regards some or all of them as working drawings, it has the appearance that their use was not widespread,[46] probably above all because the

39 See, for instance, Kletzl 1939, 4.
40 See, for instance, Saalman 1959, 103; see, on the other hand, Lepik 1994, 15.
41 For the term "working drawing," see French 1947, 307, for *Werkrisse*, see Kletzl 1939, 4ff.
42 See, for instance, Booz 1956, 68f.
43 Speculations about geometrical design procedures for the entire building, called *Proportionierungssysteme*, go back to the *Neugothik* of the nineteenth century; for a principal critique see Hecht 1997; see also Müller 1990, 39ff.
44 See, for instance, Lepik 1994, 15.
45 See Booz 1956, 75ff., Hecht 1997, 361ff., Lepik 1994, 15f. I do not raise here the related question of reduced plans with dimensions and measures marked on them.
46 See for the following Müller 1990, 15ff.

craftsmen were not trained to use them. Furthermore, ground plans usually also did not function as real working drawings. The standard procedure for setting out the lines of foundations was not to transfer step by step the lines of a ground plan to the building ground, but to construct the design immediately on this ground with pegs and strings, whereby a plan, if one was used at all, served as a means of orientation rather than as a blueprint.

In contrast to reduced plans, full-size plans seem to have been an indispensable means of the construction process since the appearance of the Gothic style. As already mentioned, the oldest of such 1:1 constructions, that is, the *Ritzzeichnungen* scratched on walls or stone floors, precede the oldest known plans traced on parchment or wooden drawing tables. These plans usually concerned parts of the building with complicated shapes and curvature—e.g., arcades, windows, rose windows, and

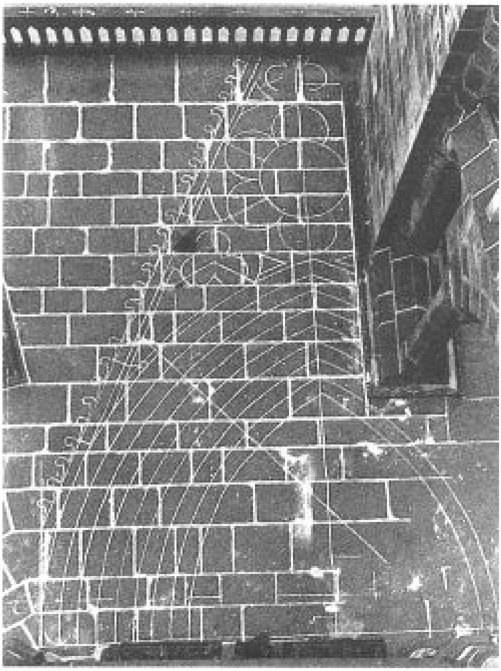

Figure 7.7. Full-size drawing scratched in the terrace floor of Clermont-Ferrand Cathedral. (Photo: Robert Berger, Clermont-Ferrand.)

so on.[47] They often more resemble geometric constructions from which the form of a certain part of, say, a window can be taken mechanically, than they do plans of this window (figure 7.7). These constructions are thus closely related to other means employed in building practice for the creation of complicated shapes, ranging from special kinds of diagrams for the derivation of the rib systems of complicated vaults *(Bogenaustragung)*[48] to templates, which came into use in mediaeval architecture along with a standardization of stone-dressing at the beginning of the thirteenth century.[49] These geometric aids of the construction process, most of the full-scale *Ritzzeichnungen* included, probably do not immediately come to mind when plans are mentioned and, indeed, can hardly be called architectural plans in the usual sense of the term. They are, however, reminiscent of the fact that such plans, if used in the construction process, are just one of many geometric means that support the creation of the desired shapes of a building. They point furthermore to geometrical practices of craftsmen that may be of interest with respect to the roots of the combined views technique as developed by Dürer.

Yet, before coming to the emergence of this technique, this concise survey of the use of plans in architecture before 1500 should be summed up by stressing that the architects and craftsmen of the age could obviously do very well without a general method of deriving elevations from ground plans. As Schofield put it for Italy:

> There can be little doubt that the orthogonal was used in the fifteenth century in the same way for the same type of architectural features, i.e., flat exteriors and flat interiors and details, but would not have been used for general views of interiors or exteriors where there were large-scale features that projected or receded from the front plane. Indeed, in the case of large-scale architectural units, it is to be doubted that a reasonably accurate, or even moderately inaccurate elevation drawing in perspective accompanied by dimensions (either drawn-on or on list) was any less useful than a fully marked-up orthogonal. The reason for thinking this is simply that a careful perspective drawing, especially one with dimensions marked on it, was better able than an orthogonal to provide precise information about details, large or small, that receded or projected from the frontal plane. This is confirmed by the practice of architects drawing in the early sixteenth century.[50]

Whether actually “better able” or only equally well, the architects of the age had in any case all means at their disposal needed to fulfill their tasks. Thus, in this investigation of the emergence of the combined view technique, one has to start from the fact that, obviously, no urgent needs of the construction practice stimulated Dürer or da Sangallo to create this new technique. Given this situation, it also comes as no surprise that this new method was only hesitantly adopted by sixteenth-century architects and became a standard technique of the art not before the seventeenth century.

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47 For the full-scale *Ritzzeichnungen*, see Schoeller 1989 and 1980, as well as Davis 2002.
48 For such plans, see Müller 1990, 152ff.
49 See Müller 1990, 126ff. and Kimpel 1983. See also Shelby 1971.
50 Schofield 1991, 129.

3. THE GENERAL METHOD OF CONSTRUCTING ELEVATIONS

From the perspective of the history of mathematics, the new method in question is nothing but descriptive geometry in an early—actually in its first—but nevertheless manifest stage of development. However, in this first stage, this branch of mathematics looks not at all like what one expects from a scientific field. First, Dürer and da Sangallo were no mathematicians, notwithstanding the fact that they had obviously some mathematical knowledge and were in touch with men who can be regarded as mathematicians.[51] Second and more importantly, they established no theory. Da Sangallo did not publish a single treatise,[52] and Dürer's treatises do not argue for the new method of constructing plans but give, among many other things, practical step-by-step instructions on how to derive certain elevations from ground plans. Particularly his *Underweysung der Messung* from 1525,[53] the most important text in this context, looks like a book of recipes rather than a theoretical text. It is, thus, not sufficient to state that the beginnings of descriptive geometry were imbedded in the practice of artists. Rather, one has to realize that descriptive geometry was one of the techniques employed in this practice at the time and nothing else. Such practitioners' knowledge took on a scientific character only later, beginning at the turn of the sixteenth century, when men with scientific interests discovered its importance and reshaped it in theoretical frameworks which, at the same time, detached it from its origins.[54] The subject matter here is not descriptive geometry as it emerged through this appropriation and transformation by savants, but descriptive geometry as it was in its beginnings, that is, as practitioners' knowledge.[55]

In Dürer's treatises, one encounters for the first time a construction of an elevation by combined views in the first book of the *Underweysung der Messung*. The topic of this first book is techniques of constructing several shapes and curves by means of ruler and compasses. After a few pages with more or less trivial drawing tasks leading to the construction of several spirals (*Schnecken*—snails), Dürer almost casually goes on to elevations of helices, that is, of three-dimensional spirals. It is important to note that he himself does not use terms like projection or elevation in this passage as

51 For Dürer see, for instance, Steck 1948, 6 and 85ff. (note 8); for da Sangallo, see, for instance, Frommel 1994b, 3.

52 It is known, however, that da Sangallo considered publishing a commented edition of Vitruvius—see Frommel 1994b, 36.

53 Albrecht Dürer, *Underweysung der Messung / mit dem zirckel undrichtscheyt / in Linien ebnen unnd gantzen corporen / durch Albrecht Dürer zusamen getzogen / und zu nutz allen kunstlieb habenden mit zu gehörigen figuren / in truck gebracht / im jar. MDXXV,* published in Nuremberg. For a modern facsimile edition see, for instance, Dürer 1983. There exists no modern German edition of this treatise that would comply with present standards of critical editions. A facsimile edition with an English translation was edited by Walter L. Strauss—see Dürer 1977. Jeanne Peiffer edited a French edition along with an excellent introduction and valuable appendixes—see Dürer 1995.

54 For Dürer, see Peiffer 1995, 123.

55 This is the reason why Dürer and da Sangallo are not considered mathematicians in this chapter, which may appear strange in several respects. This categorization rests neither on a disdain of their mathematical abilities nor on their social standing—which contemporary mathematician enjoyed a higher standing than da Sangallo? The decisive point in our context is that these men developed and practised the new projective geometry as a technique of craftsmen and not as a doctrine. Furthermore, this technique cannot be taken to be an applied science. Rather, the later science of projective geometry emerged from this technique.

Strauss' English translation suggests.[56] Rather, he employs a practitioners' language in the fashion of the booklets of Roriczer or Schmuttermayer when describing the procedure: "to draw it [sc. the snail] up from below over itself" ("sie von unden ubersich ziehen") or "the snail drawn up out of the ground" ("der schneck auß dem grund auf gezogen"). This plastic language emphasizes the essential feature of the technique, namely that the helix has to be developed out of the ground plan. He also stresses explicitly that there is no other technique of constructing the elevation of the helix than this development out of the ground plan. This said, he goes on to teach the technique by giving minute instructions on how to proceed step by step.

The first helix in book I has the particular feature that it becomes steeper and steeper as it winds up (figure 7.8). Dürer mentions an architectural application of this helix that is not entirely clear[57] and assures generally that this helix is of manifold use. Next he displays a second helix that winds up in equal degrees (figure 7.9). Again and more intelligibly than in the first case, he stresses the architectural benefit of this helix, namely for masons in charge of constructing a spiral staircase. His wording[58] actually indicates that such a helix was already known and used by masons. No matter what masons' plans Dürer had in mind,[59] it is interesting to see that Dürer was fully aware that the practical character of such designs had two dimensions: They were set down for the benefit of several crafts, on the one hand, and they were rooted in practitioners' knowledge, on the other. In what follows, I will deal with these two dimensions of the new method of constructing elevations—with its applications as an important context and with its prerequisite knowledge as an important background.

Beginning with the applications of the new method, architecture, on which this chapter focuses and which, surprisingly, was mentioned as the first such field of application by Dürer, naturally comes to mind with respect to da Sangallo. The latter is the first architect known who systematically constructed elevations by combined views when designing buildings (figure 7.10). He also used this technique for recording existing buildings *(Bauaufnahmen)*, in particular, edifices or ruins of Antiquity, which were zealously investigated by the members of the Bramante and Sangallo circle in Rome (figure 7.11).[60] For Dürer, too, the benefit of the new method for architecture was obviously more than a mere side effect of a painter's achievement. He dedicated the first half of the third book of the *Underweysung* to architectural issues and employed the new method there where appropriate.[61] Moreover, Dürer, himself not active as an architect but ambitious in engineering like so many of the great artists of

56 See Dürer 1977, 61. Unfortunately, Strauss' translation is not always reliable and often fails at exactly those passages that are decisive for understanding.
57 "[…] schnecken steyg / in ein durn dach […]": spiral staircase in a spire? See Dürer 1977, 61 and Dürer 1995, 153.
58 "[…] schneckenlini […] die auch die steyn metzen zu den stygen gebrauchen […]"—snail line […] which is also used by masons for stairs. See Dürer 1977, 67 and Dürer 1995, 155.
59 Ground plans of staircases can be found, for example, in the sketch-book of master WG (fl. about 1500) which is in the collection of the Städelsche Kunstinstitut Frankfurt am Main—see Bucher 1979, plates WG 19, 26, and 35.
60 See Frommel 1994b, 9f.
61 See book III, figures 1, 2, 5, 6, 8, and 12;—in Dürer 1977, pp. 186, 188, 200, 202, 208, 222; in Dürer 1995, pp. 232, 234, 246f., 249, 258.

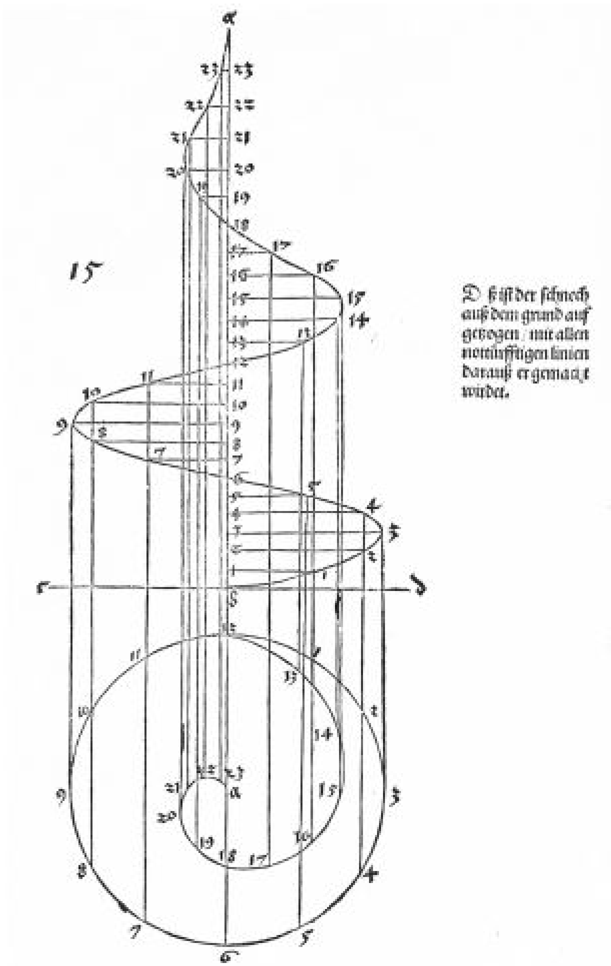

Figure 7.8. Elevation of a helix. (Dürer 1983, figure I.15.)

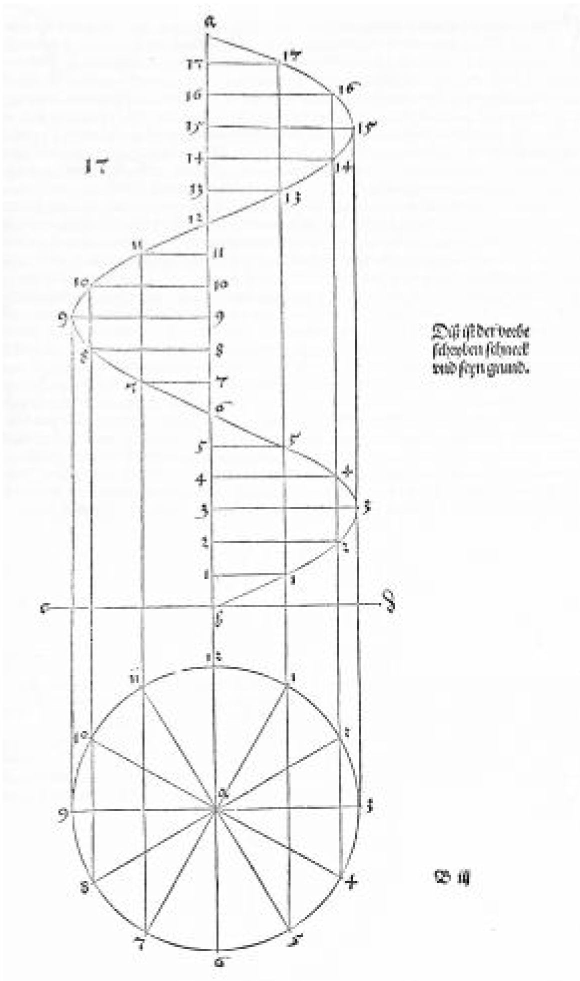

Figure 7.9. Another elevation of a helix. (Dürer 1983, figure I.17.)

the age, published a treatise on fortification that contained architectural plans including combined views.[62] These are the first architectural plans of this kind ever published in print. However, as contended above, sixteenth-century architects adopted this

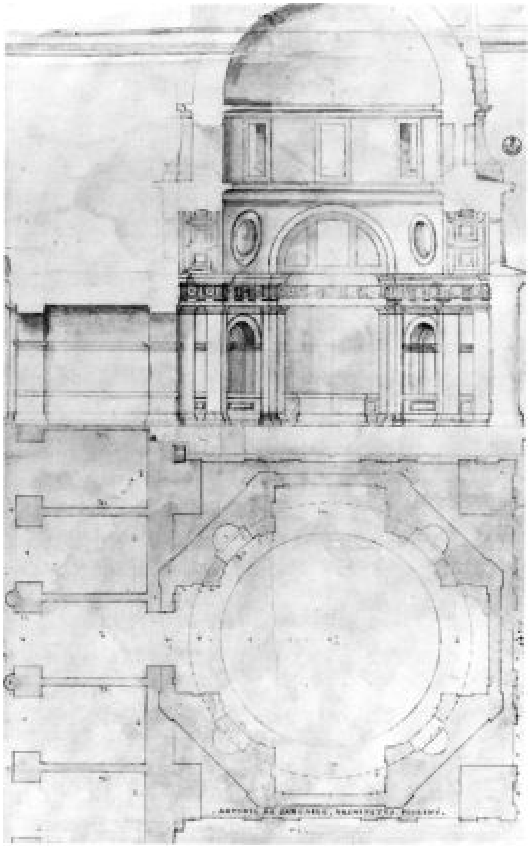

Figure 7.10. Project for a chapel. Drawing by Antonio da Sangallo the Younger, c. 1535. (Florence, Gabinetto Disegni e Stampe, U172A^r.)

62 Albrecht Dürer, *Etliche underricht / zu befestigung der Stett / Schloß / flecken*, published 1527 in Nuremberg. For a modern facsimile edition, see Dürer 1969.

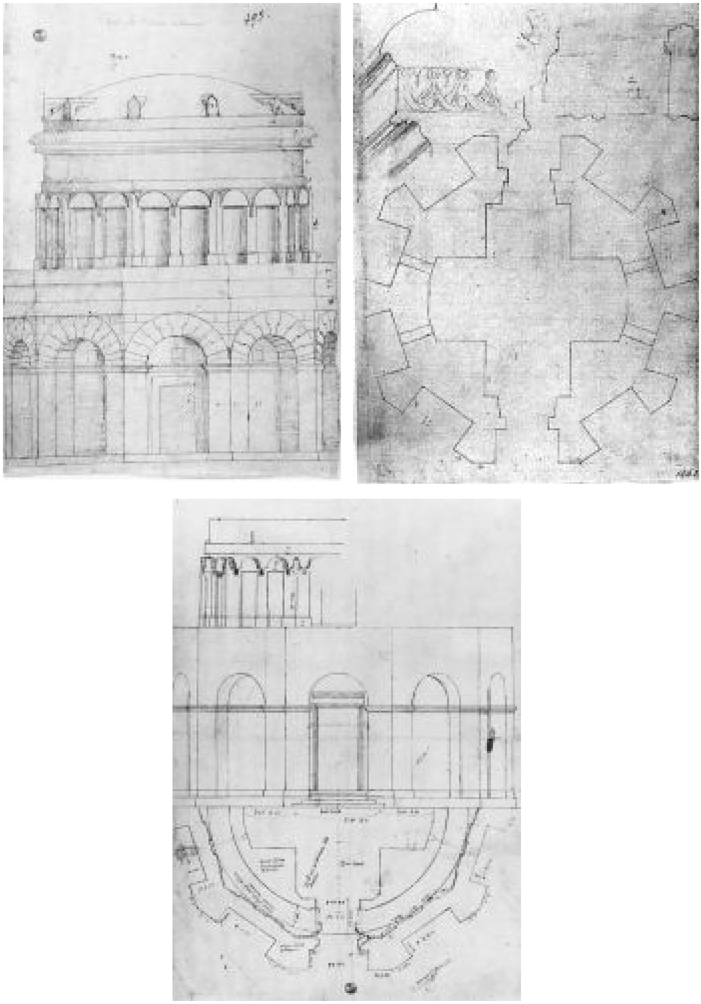

Figure 7.11. Tomb of Theodoric recorded by Antonio da Sangallo. (Florence, Gabinetto Disegni e Stampe, U1536A^{r} and 1406A^{r}.)

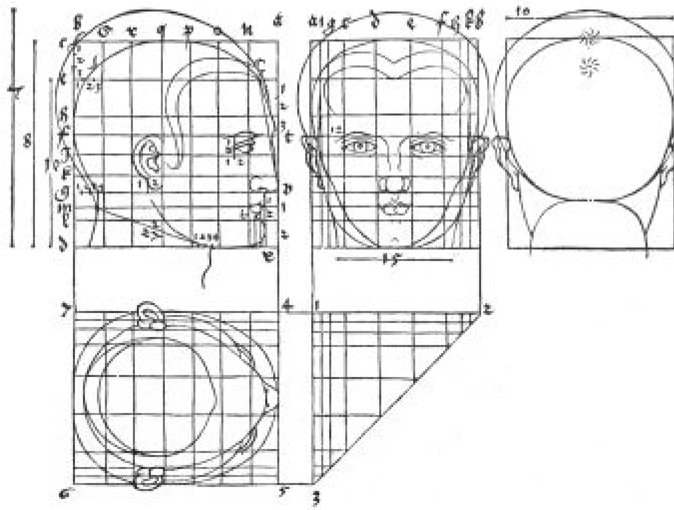

Figure 7.12. Combined plans. (Dürer 1996.)

new method only very reluctantly since they could still do very well without this innovation. Only in exceptionally complex cases, like the new *San Pietro* in Rome, did the familiar methods of designing demonstrate their limitations. This state of affairs did not change until the emergence of the baroque style in architecture.[63]

The field of application of the new method that Dürer doubtless had in mind primarily is that of art, and in particular that of painting—the term "painting" being used here to comprise drawing, engraving, etching, calligraphy, and so on. The most developed examples of the technique of combined views one can find in Dürer's writings occur in his treatise on the proportions of the human body,[64] one of the famous Renaissance treatises on painting. Here one even encounters the arrangement of three-view drawings (figure 7.12). Of particular interest is the invention of the triangle, which allows the connection of those lines in the ground plan and the front view that cannot be connected directly by straight lines. Dürer called this triangle "der ubertrag," the transfer, and gave step-by-step instructions on how to construct it. But,

63 As in the case of the Gothic style four hundred years previously, baroque architecture required new drawing methods, probably less because of its constructional problems and dimensions than because of its ornamental features, which included shapes of unheard-of complexity.

64 Albrecht Dürer, *Hierinn sind begriffen vier bücher von menschlicher Proportion*, published 1528 in Nuremberg. For a modern facsimile edition, see Dürer 1996.

as usual, he stuck to the manner of practitioners' booklets and refrained from giving geometrical explanations of such issues as, say, why the distances among the connecting lines are preserved by the "ubertrag."

The importance of painting as a context of the new method is indicated not only by the fact that, in Dürer's work, its applications in connection with painting prevail over those in any other field, but above all by the intimate relation of the new method to the perhaps most important technical achievement of Renaissance painting, namely to linear perspective. Actually, the new method was first developed in the context of linear perspective.

When the development of perspective depiction entered its first manifest stage in the fifteenth century, it was accompanied by treatises, with or without diagrams, that tried to establish rules on how to construct perspective pictures.[65] Furthermore, the first of those treatises, Alberti's *De pictura* (c. 1435), already offered reflections on such rules. These essays implicitly employed combinations of plans in the manner and in continuation of those diagrammatic representations of the visual cone that had been used in optics since antiquity, and also in mediaeval treatises on surveying. These diagrams are implicit in the arguments about how to construct perspective pictures even when, as in the case of Alberti's treatise, no figures are attached.[66] Being introduced and handled in this new context, it seems to have been only a question of time until these diagrams, which are essentially ground plans and elevations, were taken to be such plans, and until the potential of combined plans for constructing perspective pictures was realized by one of the artists.

Indeed, only about forty or fifty years after Alberti's treatise, this potential of plans was discovered, developed, and set down by Piero della Francesca (c. 1420–1492). In his treatise *De prospectiva pingendi,* written about 1480, Piero employed the technique of combined plans not only for the construction of the points where the visual rays cut the drawing plane,[67] but also for the establishment of the so-called *construzione legittima* of perspective depiction.[68] Furthermore, he apparently was fully aware of the fact that the correctness of rules for perspective constructions can only be assessed and demonstrated by means of combined plans.[69] What is more important in the context of this chapter, Piero also used this technique to construct orthogonal projections of the objects that were to be depicted in perspective. Aiming, for the sake of generality, at demonstrations that hold for every object, regardless of how its surfaces might be situated with respect to the drawing plane, he demonstrated how to transform the orthogonal depiction of an object in such a way that it appears

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65 Note that I do not deal here with the more or less unknown rules that painters of the Renaissance actually followed when producing a perspective picture, but only with stated rules documented in texts.
66 See §§ 14–20 of Alberti 2000. For this background of Alberti's treatment of perspective depicting, see part 1 of the chapter by Filippo Camerota.
67 See, for instance, figure XLVI in Piero della Francesca 1984.
68 See Panofsky 1955, 249f.; see also Kemp 1990, 27ff. The term *construzione legittima* was first coined at the beginning of the seventeenth century by Pietro da Fabbrizio Accolti who ascribed this method to Alberti—see Field 1997, 30.
69 See Andersen 1992, 15.

to be turned around, tilted, shifted, and so on. In doing so, he unfolded in a masterly manner the possibilities of deriving plans from plans.[70]

One thus encounters the first mature instance of the combined views technique not in the domain of architecture, but in that of painting.[71] Although this achievement appears almost natural in the context of fifteenth-century perspective depiction and contemporary essays to establish its rules, it is perhaps not by chance that it was eventually attained by a man of outstanding mathematical education.[72] In the form developed by Piero, this technique went far beyond both the needs and the knowledge of an artist trained to the usual standards of the age. Thus it can be doubted that this method would have spread among artists even if Piero's treatise had been published.[73] It seems rather likely that his treatise would have been eclipsed by treatises such as Viator's *De artificiali perspectiva* (1505),[74] which avoided complex constructions and provided, instead, handy rules and examples appropriate to the needs and the grasp of the artists. This conjecture is also backed by the fate of Dürer's exposition of perspective construction in his *Underweysung*, which is almost as complex as Piero's.[75] This specific exposition's career in the sixteenth century rested on the Latin translation of the book,[76] that is, on its separation from the sphere of common artists.[77] The famous question of whether or not Dürer had knowledge of some of Piero's achievements[78] can, therefore, be put in this way: How was Dürer able to capitalize on Piero, if he actually did so,[79] or, if he did not, to invent the technique of combined views independently?

As already mentioned above, Dürer was interested in mathematical questions[80] and had command of mathematical knowledge that, though certainly not comparable with that of Piero, was at least uncommon among his colleagues.[81] In the fourth book

70 See, for instance, figures LII–LIV in Piero della Francesca 1984. In not yet published notes of lectures taught at the University of Lüneburg, Diethelm Stoller reconstructed by means of diagrams the transformation steps that Piero only described (Piero della Francesca 1984, 145ff.) but did not depict.

71 In Piero's treatise, one finds the technique of combined views also employed for rendering buildings or parts of them—see Piero della Francesca 1984, figures XLI, XLII, or LIX and LX. But his purpose was, of course, the depiction of such buildings and not their construction.

72 Piero wrote two books on mathematical topics *(Trattato d'abaco* and *De corporibus regolaribus)* and was the teacher of Luca Pacioli. Several of the achievements that were formerly ascribed to the latter are owed to Piero—see, for instance, Mancini 1916.

73 *De prospectiva pingendi* was not published until 1899, his other writings subsequently in the twentieth century. Before his rediscovery in the last century, Piero was almost forgotten even as painter.

74 A facsimile edition of this booklet can be found in Ivins 1973.

75 This exposition can be found at the end of the fourth book—see Dürer 1977, 364–389 and Dürer 1995, 335–351.

76 According to Steck 1948, 106f., two different Latin editions were published in 1532 and, apparently, a third one in 1534, all of them in Paris.

77 For the rather ambiguous impact of Dürer's exposition on contemporary artists, see Peiffer (in this volume), Kemp 1990, 61f., Panofsky 1955, 253 and 257. In his usual attention to practical applicability, Dürer added two mechanical devices for perspective drawing (see Dürer 1977, 387ff. and Dürer 1995, 351ff.) which can be considered derivatives of Alberti's *velum*.

78 See, for instance, Panofsky 1955, 251f. Despite some striking parallels, it seems very unlikely that Dürer ever had access to Piero's perspective treatise itself. Even if he actually obtained knowledge of the *construzione legittima* through an unknown intermediary, it still remains an open question whether he received the "original" Piero or only something derived from him.

79 See Peiffer 1995, 100.

80 Fortunately, among the few preserved letters of Dürer, there is one from 1522 addressed to the imperial architect Johann Tscherte, which testifies that the two men exchanged thoughts about the solution of geometric problems—see letter no. 40 in Rupprich 1956–69, I 94f.

of his *Underweysung*, for instance, he is dealing with the Delian problem,[82] that is, a highly theoretical problem of classic geometry, notwithstanding the fact that Dürer, as always, saw possible practical applications of its solution. Another example of his mathematical interests is closer to the topic of this chapter. In the first book of the *Underweysung*, he dealt with conic sections or, more precisely, with the problem of how to draw the curves of these sections.[83] And here one encounters a further field where the technique of combined views was applied, for Dürer constructed the ellipse, the parabola, and the hyperbola by means of combined plans (figure 7.13).

Strikingly, ellipse constructions by means of combined plans also occur in the papers of da Sangallo (figure 7.14).[84] It is sheets like these that confirm unambiguously that da Sangallo used combined orthogonal plans as a general technique and not only in the context of architectural plans. Scrutinizing the sheet, however, it is not entirely clear whether da Sangallo actually tried to produce the ellipse in the same way as Dürer, or whether he considered the ellipse to be a distorted image of a circle seen from an oblique point of view, and therefore constructed, by means of combined views, the perspective setting in which a circle would appear as an ellipse.[85] In any case, the amazing resemblance of da Sangallo's arrangement of the plans in this construction to Piero's usual arrangement raises the question of whether the former had knowledge of the latter's treatise on perspective.[86] Given the fact that the members of the Bramante and Sangallo circle obviously had command of advanced methods for constructing perspective drawings, it might well be the case that even the architect da Sangallo first acquired the combined views technique in the context of linear perspective.

Linear perspective thus appears not only as an important field of application for the combined views technique, but also as one of its roots. However, not the handy perspective technique accessible for and used by the common artist of this age, but only a geometrically modelled perspective could be exploited as such a source by men of some learning. Therefore it is not only the practical, but at the same time the learned background of the new method that one encounters in linear perspective. Did Dürer's technique of combined views also originate from genuine practitioners' knowledge?

As discussed above, the technique of combined views is essentially a general graphic technique of deriving plans from plans, and, in particular, elevations from

81 The classic work on Dürer and mathematics is Staigmüller 1891. Steck 1948 provides a still very useful collection of all kinds of biographical materials relating to Dürer and mathematics.
82 Geometrically phrased, the problem of finding the side of a cube with double the volume of a given cube—see Dürer 1977, 347–363 and Dürer 1995, 324–334.
83 On contemporary endeavours to find convenient techniques or tools for drawing these curves, see Rose 1970.
84 See U 830A recto in Frommel 1994a, 332.
85 In this chapter, I cannot go into the obvious problems of this construction. There are traces that the transformation of a circle into a ellipse through oblique projection was already discussed in Antiquity—see theorems 34 and 35 in Euclid's *Optics* and proposition 53 in book VI of Pappus' *Mathematical Collections*.
86 This question is also suggested by da Sangallo's two constructions of the famous *mazzocchio*—U 831 A recto and U 832 A recto (Frommel 1994c, 333)—which strongly resemble those of Piero—see Piero della Francesca 1984, figure L. For the two constructions of da Sangallo, see Frommel 1994c, 150.

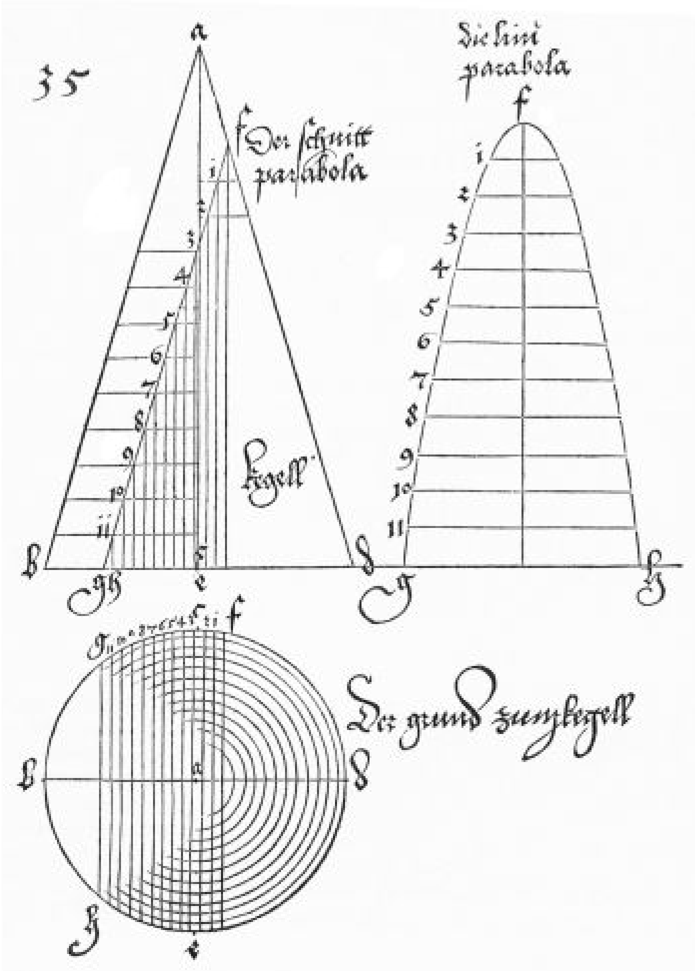

Figure 7.13. Construction of a parabola. (Dürer 1983, figure I.35.)

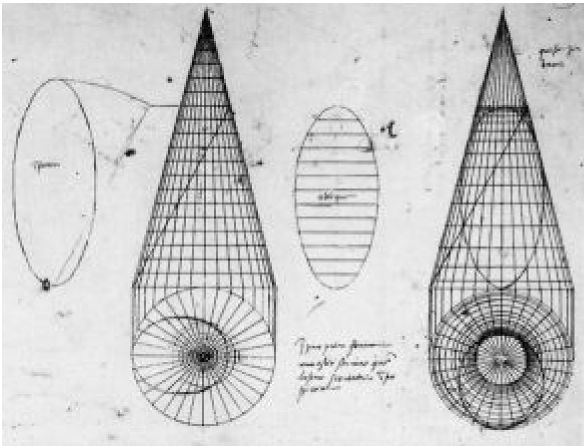

Figure 7.14. Construction of an ellipse. Drawing by Antonio da Sangallo the Younger. (Florence, Gabinetto Disegni e Stampe, U830A^{r}.)

ground plans. If one abstracts from the learned background of geometric construction and representation that shaped the development of this technique in the context of linear perspective, an elementary practical feature of this technique seems to be decisive, namely the treatment of two different plans, the ground plan and the elevation, as a unit. For the graphic derivation of the elevation from the ground plan in a way suggests itself if, but only if, both plans are treated as a unity. A systematic coordination of plans that allows their treatment as a unity seems, thus, to be the practical key for the combined views technique. Does one encounter such a coordination of plans, presupposing plans of the same scale and with perpendicular planes, in the realm of the practitioners, and if so, in which craft?

Considering the conclusions of our survey of the use of plans in the building trade of the age, the contemporary architectural practice of plan drawing seems to be a very unlikely candidate. Instances of ground plans and elevations, or sections for that matter, of one and the same part of a building are generally very rare before the sixteenth century. And in the few plan sets of this kind preserved, ground plan and elevation (section) appear almost never to be drawn to the same scale, least of all unambiguously coordinated (if at all), and are in the most cases even traced on different sheets.[87] The few combined plans mentioned but not discussed above appear all the more interesting: Among the preserved architectural drawings produced prior to Dürer and da Sangallo, there are indeed two or three plans that are, or might be considered to be, instances of such a coordination.[88]

The most beautiful example and at the same time the only one that can unambiguously be taken to be such a coordination of plans is the ground plan and elevation of the tower of the *Münster* in Freiburg (Breisgau), preserved in Vienna (figure 7.15).[89] It goes without saying that this plan, which is dated to sometime after 1400,[90] cannot be taken as testimony to the drawing practice of common architects of the age. On the contrary, it is one of a very small number of exceptional plans[91] that pose exactly the same questions regarding their roots and background as does Dürer's technique of combined views. One particular feature of the set of plans, however, hints at the direction in which one should seek the origins of this technique. On closer inspection, the ground plan turns out to be a whole set of several superposed ground plans, which represent horizontal sections through different levels of the tower (figure 7.16). Such a superposition of plans, which may appear strange from the perspective of later architectural plan conventions, was not only a wide-spread feature of ground plans of towers at that time,[92] but also points to a chief means of designing common among stonemasons and stonecutters.[93] Rather than fixing one's attention on architectural plans when looking for roots of the combined views technique, one should follow this hint and look at the practice of stonemasons.

Interestingly, Dürer himself related the coordination of ground plan and elevation not to architectural drawings but to the art of stonemasons:

> It is, therefore, necessary for anybody who wants to venture this art to be well acquainted in advance with measurement and to obtain an understanding of how ground plan and elevation of all things need to be drawn [literally: how all things need to be laid into ground and drawn up] as practised by the skilled stonemasons every day.[94]

It seems noteworthy that Dürer does not address the combination of ground plan and elevation as a new technique just developed by the stonemasons of his day. On the contrary, he actually seems to express the view that this technique had always pertained to the art of stone-dressing. Indeed, when stonemasons carve outlines on two

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87 The same holds for the drawings of goldsmiths with which the young Dürer had become familiar in the workshop of his father. The famous collection of such design drawings *(Goldschmiederisse)* held by the Basel Kupferstichkabinett displays those patterns that were characteristic for architectural plans: plenty of "wrong" elevations, though also a remarkable amount of "right" ones, cases of resorting to perspective rendering, separation of ground plans and elevations, no single instance of combined views, and so on. See Falk 1979, figures 371–702.

88 See, for instance, Sakarovitch 1998, 43. Sakarovitch's second example, i.e., Antonio di Vicenzo's famous plan for the Milan cathedral from 1389, which inserts sections of three naves into the ground plan, is a rather problematic instance—see the discussion of this plan in Hecht 1997, 158ff.

89 Kupferstichkabinett der Akademie der bildenden Künste Wien, Inv.-no. 16874; reproduced, for instance, in Koepf 1969, figure 78 and in Recht 1989, 413. Sakarovitch 1998, 42 reproduces a different plan of this tower held in the Germanische Nationalmuseum Nürnberg. For this plan, see Broda 1996, 34ff. Another drawing that can, to some extent, be regarded as such a combined view, is the plan of the east façade of Clermont cathedral held in Clermont-Ferrand—see Recht 1989, 420.

90 See Koepf 1969, 16.

91 Some of which are only copies or derivations of this plan—see Koepf, *ibid*.

92 See, for instance, Koepf 1969, figure 12, 15, 16, 65, etc.

93 See Shelby 1977, 74ff.

94 "Darumb thut einem yglichen der sich diser kunst understeen will not / das er zuvor der messung wol underricht sey / und einen verstand uberkome / wie alle ding in grund gelegt / und auffgezogen sollen werden / wie dann die kunstlichen Steinmetzen in teglichem geprauch habenn […]." Dürer 1996, dedication to Pirckheimer, my translation.

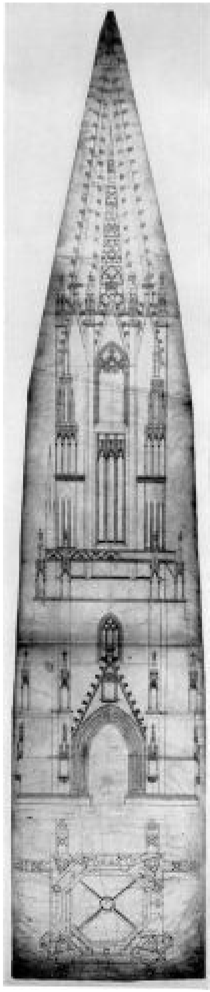

Figure 7.15. Ground plan and elevation of Freiburg Münster tower. (Vienna, Kupferstichkabinett der Akademie der Künste, Inv. Nr. 16.874; courtesy Kupferstichkabinett der Akademie der Künste.)

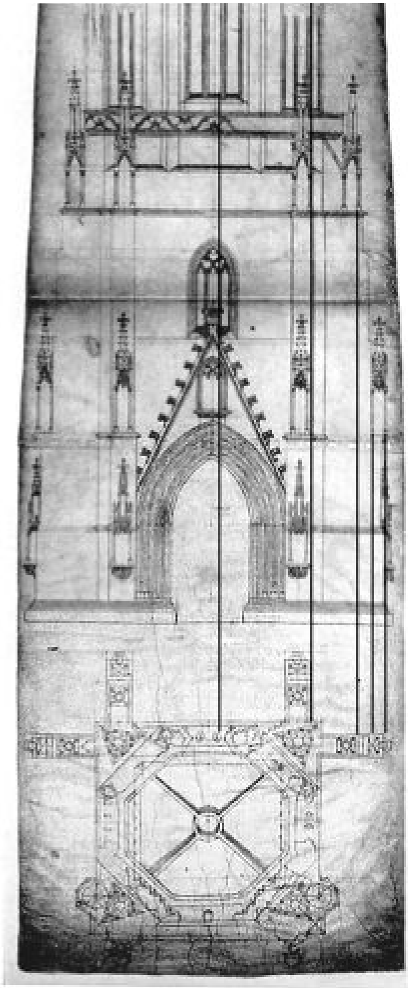

Figure 7.16. Ground plan with superposed levels. Detail from figure 7.15.

or three adjacent sides of a square hewn stone, these sides appear—at least in hindsight—as different views of the intended finished stone, which are attached to one another in such a way that simply folding them into the same plane would suffice to make them veritable combined views. It seems therefore very likely that techniques equivalent to that of interconnected plans were used by stonemasons from time immemorial. Indications of such a practice can be found in ancient Egypt.[95] Unfortunately, for want of documents, one does not know very much about the details of this practice for the epochs prior to the sixteenth century, when it became the highly developed art of stereometry; even later, it was transformed into a branch of theoretical geometry.[96] No wonder that the one and only document of this art from the Middle Ages, which can be found in the aforementioned sketch-book of Villard de Honnecourt from the first half of the thirteenth century,[97] attracted the attention of many scholars. Although this document poses still insurmountable obstacles for our understanding,[98] some general features of this technique became comprehensible. Drawing a general conclusion of his investigations into Villard's sketch-book, Lon Shelby characterized the geometry of mediaeval masons as

> constructive geometry, by means of which technical problems of design and building were solved through the construction and physical manipulation of simple geometrical forms: triangles, squares, polygons, and circles.[99]

This telling characterization is confirmed by two documents from the late fifteenth century, which give insight into a technique of developing geometrical forms in the designing process of special parts of Gothic churches. This technique, which is closely related to the construction of ground plans and elevations, is taught in two of the practitioners' booklets mentioned above, namely the booklets on designing pinnacles of Matthäus Roriczer (fl. 1460–1490) and of Hans Schmuttermayer (fl. 1480–1520).[100] This stonemasons' design technique is not the invention of these two men but goes back, as Roriczer himself assumes, to the "iungkeh(er)rn von prage,"[101] that is, the Parlers in the second half of the fourteenth century, and may be even older as regards its principal features.

What is taught in the booklets consists mainly in sophisticated rules for developing a pinnacle's horizontal forms and dimensions at different levels by turning and transforming a square in certain ways, rules which fit Shelby's characterization perfectly. The result of these manipulations of a square is a diagram (figure 7.17) that provides the measures of essential parts of the pinnacle and can at the same time be

95 See, for instance, the elevations of capitals from Abu Fodah (Heisel 1993, figure Ä30 and Ä31) in connection with the famous elevations of a shrine from Ghorab (*ibid.*, figure Ä19 and Ä20); see also Sakarovitch 1998, 24ff.
96 See chap. 2 and 3 of Sakarovitch 1998. See also part 4 of the chapter by Filippo Camerota.
97 See Hahnloser 1972, tables 39, 40, and 41.
98 See *ibid.*, 104ff.; see also Shelby 1971 and 1972; see furthermore Bechmann 1993, chap. 5 and 6.
99 Shelby 1972, 409.
100 Coenen 1990 offers diplomatic copies of these booklets, Shelby 1977 an modernized edition and English translation. See for the following, for instance, Müller 1990, 59ff. and the introduction of Shelby 1977.
101 Coenen 1990, 312.

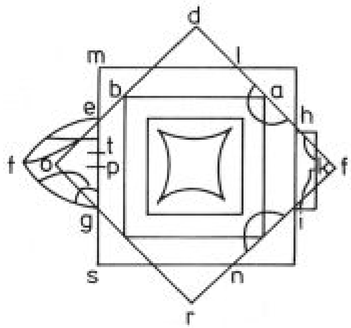

Figure 7.17. Scheme of horizontal sections of a pinnacle. From Matthäus Roriczer *Büchlein von der Fialen Gerechtigkeit,* 1486. (Coenen 1990, 337.)

taken as a ground plan of the pinnacle that consists of superposed horizontal sections at different levels. Taking the presumably old age of this stonemasons' design technique into account, it seems very likely that one encounters here the roots of the striking convention of superposed ground plans observed in the case of the plan set of the Freiburg *Münster* tower.

In Shelby's "constructive geometry" of the masons, one thus apparently encounters a practical geometry that constituted the background of the architectural plan construction of the age. True, this geometry does not aim firstly at the construction of plans but at that of diagrams, templates, and the like as means of determining and manufacturing the shapes of stones or composed parts of a building. But it makes little sense to assume that the master masons did not rely on the resources given by this constructive geometry when they had to construct plans. Moreover, as remarked above, plans that were used in the construction process—e.g., the full-scale plans scratched on the floor or walls of the building *(Ritzzeichnungen)*—were just one of many geometric means that supported the creation of the desired shapes of a building. They are products of the same practical geometry by which the masons constructed templates or diagrams like that in the booklets of Roriczer and Schmuttermayer. At least with respect to the countries north of the Alps, it may be well advised, therefore, to look at architectural plans of this age from the perspective of this constructive geometry rather than of our projective geometry.

The booklets also teach how to derive the "body" ("leib") of the pinnacle, that is, its vertical shapes, from these superposed horizontal sections.[102] The method taught is not that of a graphic derivation of an elevation by treating the two drawings as a unit, but that of determining the measures of the "body" by processing those of the ground plan in such a way that certain proportions between breadth and heights were met.[103] This is in accordance with the purpose of the two booklets, which do not aim at constructing plans, but at determining measures and forms. Nevertheless, if one abstracts from the development of the vertical measures of the "body" and focus on the horizontal ones, one encounters procedures of how to take distances from the ground plan using a compass and how to transfer them to the diagram of the "body" to construct its elevation.[104] In other words, the booklets give testimony to geometri-

102 See, for Roriczer, Coenen 1990, 316ff. and Shelby 1977, 90ff., for Schmuttermayer, Coenen 1990, 355ff. and Shelby 1977, 129ff.
103 See Shelby, 70f.

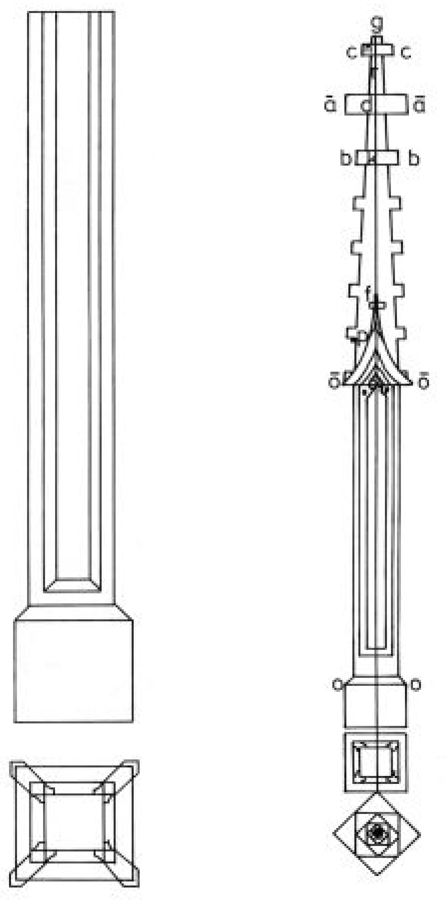

Figure 7.18. Ground plan and elevation coordinated. From Matthäus Roriczer *Büchlein von der Fialen Gerechtigkeit,* 1486 (left), and Hans Schmuttermeyer *Fialenbüchlein,* c. 1485 (right). (Coenen 1990, 337 and 365.)

104 Additionally, the advantage of superposed ground plans becomes visible if it is, as in the case of pinnacles, about the determination of the vertical shapes of building parts that narrows towards the top.

cal methods of determining the vertical shapes of building parts based on the ground plan, which are of immediate bearing for the construction of elevations.

With respect to Dürer's method of combined views, two points of the booklets' elevation construction are of special interest. The one is of linguistic nature. Roriczer uses the words "den grunt auszczihen" and "ausczug"[105] to designate the action and the result, respectively, of the construction of an elevation, and also indicates that these denominations are the established terms of the craft.[106] These are, however, exactly the terms Dürer uses for the construction of elevations as quoted above: "sie [die Schnecke] von unden ubersich ziehen" or "der schneck auß dem grund auf gezogen."[107] The language that Dürer uses in the *Underweysung* is obviously closely related to, if not dependent on, that connected with the practical geometry of masons as documented in the contemporary masons' booklets. This is one of the reasons why Dürer scholars regard it an established fact since Staigmüller's essay from 1891 that the knowledge of the Gothic masons and goldsmiths form a important background for his practical geometry.[108]

The second interesting point concerns the arrangement of the figures in the masons' booklets. As already said, the booklets teach a method of deriving the measures of the "body" from those of the ground plan that is geometric, but does not consist in a graphic derivation of the elevation from the ground plan by arranging the two plans in such a way that they can treated as a unit. Accordingly, the booklets' diagrams that represent ground plans and elevations are usually separated from each other and drawn to different scales. But, in each of the booklets, there also is one or two drawings where a ground plan is coordinated with an elevation in the manner of combined views (figure 7.18). It is, of course, not possible to determine whether this arrangement of ground plan and elevation can be taken as a trace of a more graphic method of deriving the one plan from the other and, thus, whether such a method was in use in the realm of the stonemasons' practical geometry, but is not dealt with explicitly by Roriczer and Schmuttermayer. But these drawings show in any case that it was obviously not an unfamiliar feature of this practical geometry to coordinate plans of the same scale and with perpendicular planes, a coordination, which can be regarded as the practical key for the combined views technique. It therefore has the appearance that this geometry constitutes the practitioners' knowledge sought as a root and background of Dürer's technique of combined views.

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105 Shelby (*ibid*., 90) translates with "extrapolate the base plan" and "extrapolation," respectively. It is essential to be aware of the close relation between the German "ziehen," from which the verb "ausziehen" and the noun "Auszug" are derivatives, and "zeichnen:" "eine Linie ziehen" means "to draw a line;" thus, "draw out of the ground" may be a more adequate translation since "to draw" means both "to pull" and "to trace."

106 "... merck der wirt gehayssen ..." (... note, it is called ...)—Coenen 1990, 316 and Shelby 1977, 90.

107 "[…] to draw it [sc. the snail] up from below over itself"—"the snail drawn up out of the ground." See text and caption to figure 15 of the first book of Dürer 1983.

108 See Günther 1887, 358, Staigmüller 1891, 50ff., Steck 1948, 83f. (note 6), and Peiffer 1995, 51ff.

8

PROJECTIONS EMBODIED IN TECHNICAL DRAWINGS: DÜRER AND HIS FOLLOWERS

JEANNE PEIFFER

INTRODUCTION

The aim of this chapter is to focus on technical drawings as mediators between practical and theoretical knowledge. Albrecht Dürer's *Underweysung der Messung* (Nuremberg 1525) offers some interesting examples of drawings embodying workshop techniques and aspects of theoretical knowledge. My reading of these drawings will make apparent a hitherto unnoticed aspect of Dürer's notion of *Messung*, namely construction in a visual space. In the second section, I look at the changes that occur in the understanding of that notion in its use by the immediate followers of Dürer, mainly in Nuremberg. Special attention is paid to the use of plans and instruments in the representation practices developed from Dürer's methods. The last section offers an outlook into the early seventeenth century, where on the one hand one encounters investigations of a more mathematical character with men like Egnazio Danti, Guidobaldo del Monte, and especially Girard Desargues, and on the other, some of their methods are discussed, criticized, and rejected by practitioners like the Parisian stonecutters. The chapter closes with a short depiction of the debate between Abraham Bosse and the French Academy, a debate in which the notion of representing things as they are perceived, and not as they are, is violently rejected.

1. CONSTRUCTIVE GEOMETRY IN A VISUAL SPACE: ALBRECHT DÜRER

When Dürer calls the geometry, the instructions of which he is assembling, *Messung* (from *messen*, to measure), he is referring to a geometry that owes a lot to the Euclidean *Elements*, but is far from demonstrative geometry. It is a constructive, concrete and material geometry that Dürer has in mind, one which has to do with real objects and artifacts that the artist can place before him ("vornemen"). He only once uses the Latin "geometria," which clearly refers to Euclid. Thus *gemessen linien* are curves constructed pointwise by ruler and compass. The term *gemessen* may also evoke three-dimensionality as in *gemessen leng* (II.30),[1] or the materiality of a line, as opposed to the abstract lines of Euclidean geometry. In his *Underweysung der Messung*, Dürer aims to offer his companions a range of geometrical forms, regularly constructed by ruler and compass, be the construction exact or approximate. I will concentrate in what follows on another aspect of his *Messung*, an aspect which has to do with vision. Some of Dürer's drawings take into account the fact that the objects

1 See Dürer 1983, II.30. I adopt this condensed form to refer to Dürer's *Underweysung der Messung*: book II, figure 30 or the text preceding the figure.

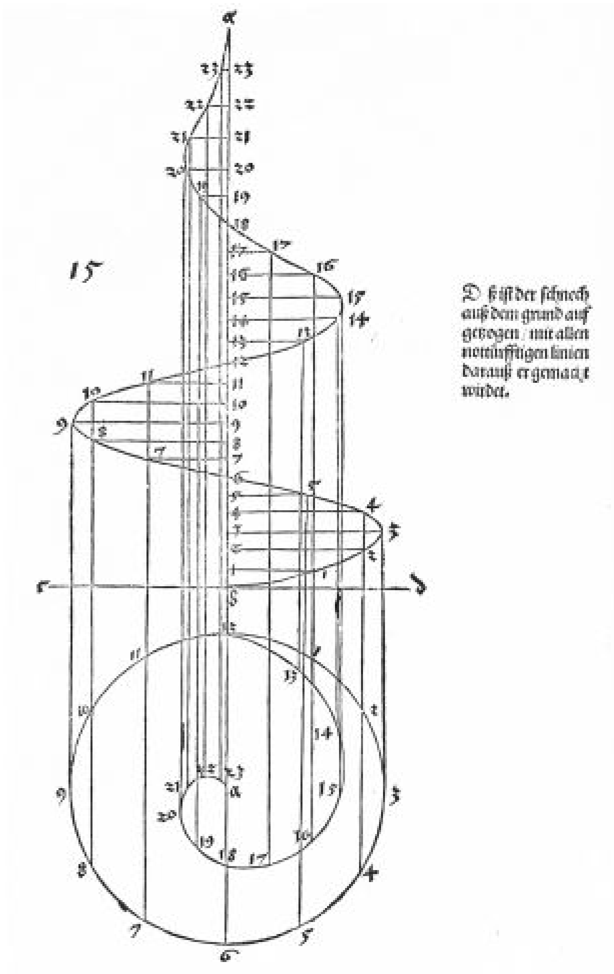

Figure 8.1. Plan and elevation of an helix. (Dürer 1983 figure I.15.)

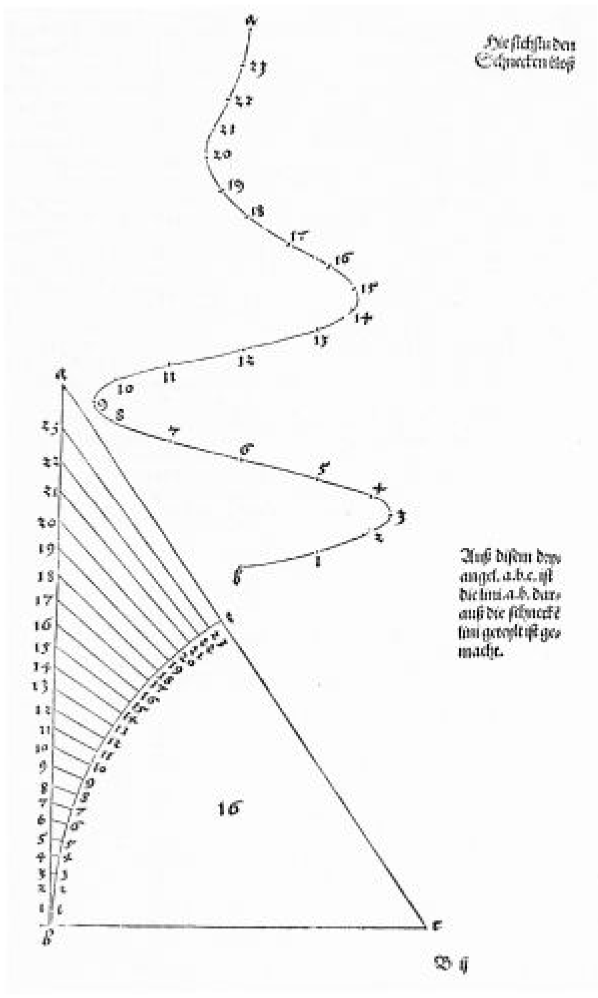

Figure 8.2. Division of a line segment. (Dürer 1983 figure I.16.)

he conceives of will be displayed in a visual space. How do the laws of perception intervene in the representation of the perceived objects?

In Dürer's geometry, objects are often represented by two of its projections, a ground plan and an elevation. As Wolfgang Lefèvre has convincingly shown in section II of his chapter, orthographic plans like ground plans and elevations, and especially their combinations, were hardly in use before the sixteenth century.[2] The context in which the latter technique was developed appears to be that of painters, like Piero della Francesca, who were interested in central perspective. Albrecht Dürer gives in his *Underweysung der Messung* powerful testimony of an excellent command of these methods, which are considered today to be the first occurrences of descriptive geometry. Dürer himself is silent on his method and his sources. We only know from the introduction of his *Proportionslehre*[3] that he considers this technique to be well known to stonemasons. The vocabulary[4] used by Dürer mirrors material procedures, like extracting a curve out of the ground plan, and can be traced back to the medieval building crafts. As I have argued in my book,[5] Dürer's achievement is outstanding for its application of a practical method to abstract mathematical objects like skew curves. While Dürer's text can hardly have been inspiring—he was hardly read—his drawings were. They have been interpreted as starting point for a branch of geometry that developed only towards the end of the eighteenth century in France, namely descriptive geometry.

And yet, there is a strange element, hardly ever noticed, in one of these drawings, which contributed to Dürer's fame. As Lefèvre rightly says, architectural recording like drawing plans and deriving elevations from them, needs to suspend vision in favor of construction. However, vision seems to play some role in Dürer's construction of the helix (figure 8.1).

Dürer's drawing I.15, which Lefèvre has also commented in his chapter (see his figure 7.15), represents the plan and elevation of a skew curve, a cylindrical helix on which a conical one is seated. In the elevation, the intervals of the subdivision of the axis of the cone are not equal, while those of the cylinder are. This is not obvious in Dürer's figure I.15, as it was engraved by the "Formschneider" based on drawings by Dürer himself, but the ensuing figure I.16 leaves no doubt that the intervals grow as the axis rises to a greater height (figure 8.2). Dürer even explains the rule that served to divide the axis, namely the one given by his figures I.8 and I.16. In this last figure, Dürer adapts the method of dividing a line segment, described in his figure I.8, to the construction of the helix. He divides an arc *be* into equal parts and projects this division on the vertical tangent to the arc *ba* or on a parallel to this tangent. The intervals thus obtained on the axis are proportional to a function of $tg(\alpha/n)$ and are no longer equal. They increase at greater heights. The helix becomes steeper and steeper.

The accompanying text is ambiguous in the sense that two different constructions collide. First Dürer divides the vertical axis of the helix into 24 equal parts. This

2 See also Sakarovitch 1998, chap. 1, for a depiction of the combined plan and elevation technique.
3 Dürer 1996.
4 See for instance Peiffer 1995, 55 and Lefèvre in this volume.
5 Peiffer 1995.

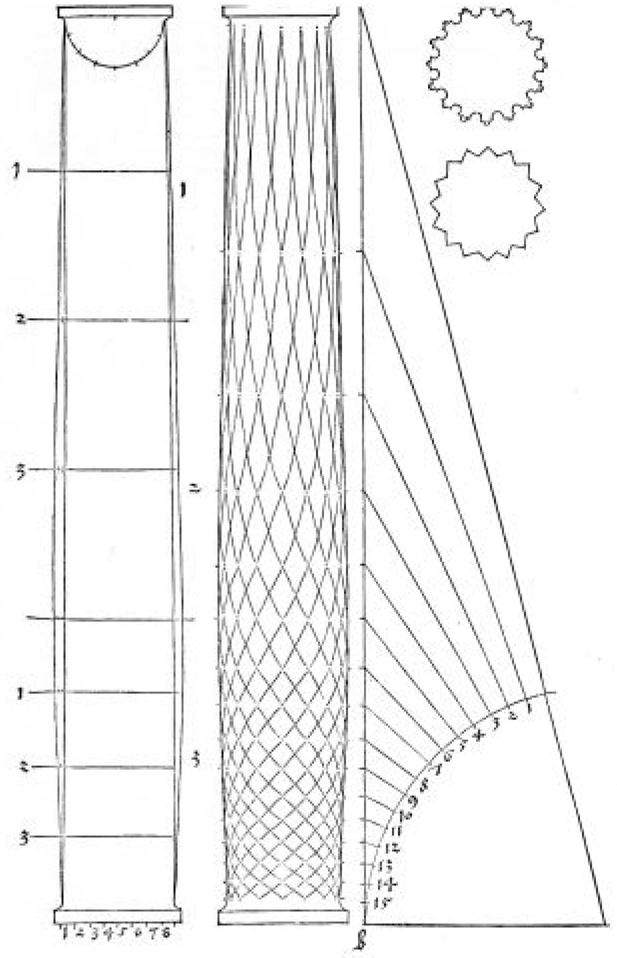

Figure 8.3. Construction of a convex column. (Dürer 1983 figure III.7.)

instruction is immediately followed by a contradictory one: "However I want here to have the intervals grow in the order explained above"[6] (i.e. in his figure I.8). First, Dürer explains how to construct a skew curve by extracting its elevation from its plan. Second, if the curve is applied to the construction of a spiral staircase in the spire of a tower, then the steps must be reduced in length in orderly progression the higher they rise in the tower, i.e. the helix is conical, and the steps will grow progressively steeper such that the axis is divided as in figure I.16 above. In the case of the construction of a mathematical object, the intervals dividing the axis of its elevation are of equal length. In an architectural context, however, when building a circular staircase in the spire of a tower, the subdivisions have to be altered in order to conform with the experience of steps becoming smaller and steeper when climbing up the tower. Dürer offers a construction that allows the desired effect to be obtained.

Book III of the *Underweysung*, which is dedicated entirely to architectural studies, offers some other applications of the same dividing technique. In the opening section, after some definitions of volumes, cones and pyramids, Dürer designs primarily columns, monuments and towers. Elements of decoration, like sun-dials and lettering, close the book. Dürer explains how to generate a column or a pointed solid, a pyramid or a cone, from a given ground plan. For instance, if the ground plan is a circle, he produces a column by raising this circle up to the desired height[7] and a cone by raising it towards a point.[8] More generally, the "kunstreychen bauleut," i.e. the artful builders in Strauss' translation,[9] know how to give the right volume, "in rechter maß beleyben," to a ground plan drawn with simple lines. They also know, from the given sections, how to decorate the columns and their different elements.[10] When constructing convex columns, Dürer applies the procedure described above (figures 8.1 and 8.2) to decorate the shaft with spiraling lines (figure 8.3). He wastes not a single word on that application, which apparently is straightforward to him, except that he insists on using the triangle *abc* in order to expand the intervals as they ascend, and to contract them as they descend.

The construction of a round twisted column, which follows, yields an even nicer example (figure 8.4). Here Dürer starts with a description of the dimensions and proportions of the column he wants to construct, i.e. effectively, the description of a profile. He uses the diameter above its plinth as a module. The proportions he gives are those of a ionic column in Vitruvius' *Architecture*. Then he goes on with the ground plan "from which to extract the column while twisting it."[11] As the figure clearly shows, this plan is composed of three circles with different diameters touching each

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6 "Ich will aber hie die felt ubersich in einer ordnung erlengen / wie voren angetzeigt" (Dürer 1983, fol. B^r).

7 "Ich nym zum ersten ein cirkelrund felt … und far eben mit ubersich so hoch ich will / so wirdt ein runde seulen darauß" (Dürer 1983, fol. G^r). From this quotation, the role played by plans and elevations in the design of columns seems straightforward: the elevation fixes the different levels to which the forms contained in the ground plan will rise.

8 "Ich far aber auß allen forgemelten gründen ubersich / so hoch ich wil in ein spitz / so werden kegel darauß" (Dürer 1983, fol. G^v).

9 Dürer 1977, 191.

10 I am paraphrasing fol. $G_{ij}{}^v$ of Dürer's *Underweysung*.

11 "leg ein grund darauß du dise seulen winden must" (Dürer 1983, fol. H^v).

other. Their peripheries are numbered with sixty subdivisions, beginning with point *a*, the center of the large circle. In the smallest circle, point *c* has the number 6. Then, continuing with the middle-sized circle, along half his periphery, Dürer counts up to 18 and starts with 19 on the large outer circle. Its whole circumference is subdivided into points 19 to 42, then the division continues on the second half of the middle-sized circle until the point *c* is reached with the number 54. The second half of the small circle is divided into points 55 to 60, which coincides with the center *a*. These points in the ground plan show how the axis of the column must be twisted.

Next, the axis of the column in profile is divided into 60 irregular intervals. The procedure used by Dürer is by now familiar. He projects an equally divided arc on the vertical axis. Thus the intervals increase as they ascend. They all fit along a compass opened to a constant angle, or, in other words, in the triangles defined by the branches of the compass and the intervals, the latter are the sides opposing the equal angles. According to the fourth postulate of Euclid's *Optics*,[12] these intervals would appear equal to an eye located at point *f*. Thus, let's presume that Dürer's diagram refers to a visual space in which the laws of Euclidean optics apply, i.e. that the visual rays proceed in straight lines from the eye, and that the collection of such rays constitutes a cone, of which the vertex is at the eye and the base at the surface of the objects seen. The apparent size of a visible object is determined by the visual angle that encloses the extremities of the object and the vertex of which is located in the observer's eye. In such a visual space, the division process used by Dürer induces that the growing vertical intervals of that subdivision are perceived as equal by an eye *f*.

In other words, in his diagram, Dürer takes into account the fact that a magnitude seen from underneath appears smaller than it is. To correct this perception, he increases the intervals as they ascend in accordance with the order of Euclidean optics. I will come back to the presence of an optical theory in these sophisticated practitioner's drawings of architectural and mathematical objects. But for the moment let me stress the highly innovative state of the construction as it is further described by Dürer. We have seen that he conceives of the ground plan, with its three circles divided into the numbers 1 through 60, as a horizontal projection of a helix, which is to represent the twisted axis of a column. That's why Dürer divides the elevation into 60 horizontal planes, numbered from 1 at the bottom up to 60. Each horizontal section of the column is represented by a horizontal line segment whose length is equal to the diameter of the column, the axis of the column being an axis of symmetry. In order to twist the axis, Dürer applies the technique described in his figure I.15 (figure 8.1), and applies to each horizontal line segment the displacement, taken from the ground plan, which the corresponding point of the axis undergoes when twisted. He then imagines that each point of the twisted axis of the column can be the center of a sphere whose diameter is precisely that of the column. If all their circumferences are connected by a line, the contour of the column appears. The surface is thus defined (in our modern terms) as the envelope of a family of spheres with constant radius and

12 Let it be assumed that things seen under a larger angle appear larger, those under a smaller angle appear smaller, and those under equal angles appear equal. Quoted from Lindberg 1976, 12.

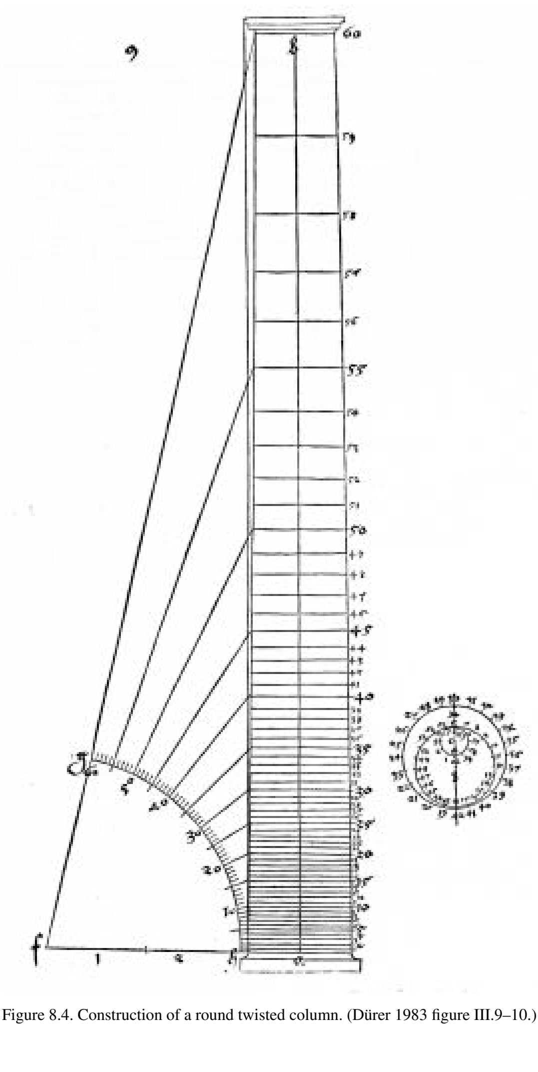
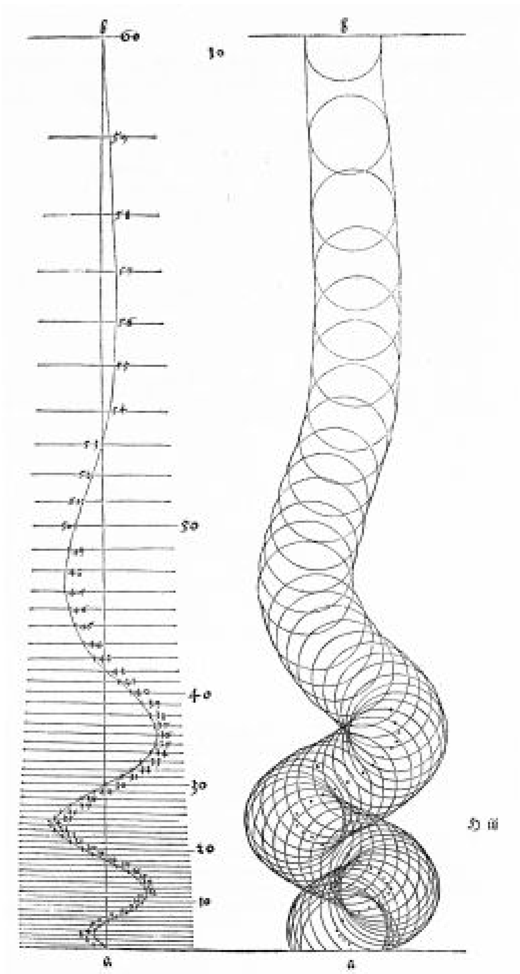


Figure 8.4. Construction of a round twisted column. (Dürer 1983 figure III.9–10.)



(Figure 8.4 continued)

with their centers on a curve. Dürer's drawing certainly inspired the founder of descriptive geometry, Gaspard Monge,[13] when in his lessons held centuries later at the newly founded *Ecole de l'an III* in revolutionary Paris, he taught on the so-called "surfaces de canaux."

In my opinion, it is in this part of his *Underweysung* that Dürer's excellent command of methods later called descriptive is most obvious. His last sentence of this section shows a good understanding of the nature of sections:

> Whereas in a round column the horizontal lines represent only round surfaces and the lines are placed evenly one on top of the other, in the case of twisted columns these lines no longer lie one flat on top of the next; instead, they are displaced, or moved to and fro above or below the next horizontal line, or to one side of it. They are transformed into oblique or curved shapes. This is shown in its simplest form in the following diagrams.[14]

But let's come back to the optical theory possibly underlying the drawings we have commented upon. While Dürer clearly describes how to divide the vertical lines in the drawings discussed above, he is silent on a possible connection to vision. He writes in a context where Euclidian optics[15] is known to some extent, and we know that he himself is well-read on this subject. When in Venice, Dürer bought, in 1507, a copy of Euclid's *Elementa*, edited and commented upon by Bartolomeo Zamberti[16]. This book contains a Latin translation of Theon's recension of Euclid's *Optica*. In Dürer's estate in London[17] we find a manuscript in Dürer's hand containing an extract, in German, of the axioms and propositions of that optics. Except for theorem 16, which is incomplete, the translation is correct. Hans Rupprich has shown that theorem 17 is not in Dürer's, but in Willibald Pirckheimer's hand. That was for him an opportunity to illustrate the kind of collaboration that the painter had established with his humanist friend: while Pirckheimer translated Euclid's text into German, Dürer wrote it down.

Moreover, Dürer bought in 1523 "ten books useful to painters" from the Senate of Nuremberg, among which a copy of a manuscript of Witelo's "De perspectiva," which had belonged to Regiomontanus' library and was inherited by the astronomer Bernhard Walther. The manuscript, today in Basel,[18] doesn't show any testimony of a

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13 Gaspard Monge, *Application de l'analyse à la génération des surfaces courbes* (1807), fol. XIV. See Bruno Belhoste and René Taton, Leçons de Monge in Dhombres 1992, 267–459, 301 for the "surface de canaux."

14 See Dürer 1977, 213. In Dürer's original German version: "Aber so in der geraden seulen die zwerchlini all rund ebnen bedeuten und gerad auf einander stend / so beleyben doch die selbenn linien in der windung der krumen seulenn nicht mer blat auf einander / sunder schieben / hencken / und keren sich hin und her / ubersich undersich und nach der seyten / unnd werden schlemet ablang rundlecht linien darauß. Dise hab jch nach dem schlechtesten nachfolget aufgeryssen" (Dürer 1983, fol. H_{ij}^{r}).

15 On the spread of Euclidean optics in medieval and Renaissance Italy, see for instance Federici Vescovini 1983, and Cecchini 1998a.

16 *Euclidis megarensis philosophi platonici mathematicarum disciplinarum janitoris : habent […] elementorum libros XIII […] Bartolomeo Zamberto Veneto interprete*, Venice 1505. Dürer's copy with his monogram is kept today at the Herzog August Bibliothek Wolfenbüttel. It bears the following inscription: "daz puch hab ich zw Venedich vm ein Dugatn kawft im 1507 jor. Albrecht Dürer" (This book I bought it in Venice for one ducat in the year 1507).

17 London, British Library, Add. Ms. cod. 5228, fol. 202 and fol. 211–219; 5229, fol. 77.

18 See Steinmann 1979, who identified Dürer's copy.

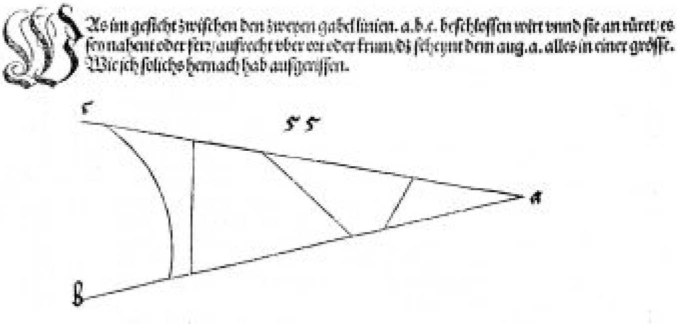

Figure 8.5. Diagram by Albrecht Dürer to the angle axiom of Euclidean optics. The axiom in Dürer's terms: "Whatever is seen [in the field] enclosed by the two forked lines ab and ac and touches them, be it near or far, vertical, oblique, or curved, it appears to the eye in the same size." (Dürer 1983 figure IV.55; translation from Dürer 1977, 373, modified by the author.)

reading by Dürer. Last but not least, Dürer includes the famous axiom of angles in Book IV of his *Underweysung* (figure 8.5). Like his Italian counterparts and sources, Dürer begins his exposé on the rules of perspective with some elements on vision that are clearly Euclidean.

Thus there can be no doubt that Dürer was aware of the existence of the Euclidean laws of optics where the apparent magnitudes of the perceived objects depend on the angle at which they are seen. And yet he doesn't explicitly relate this postulate to the technique detailed above. How are we to explain this silence? One possible explanation is that he learned the technique in a workshop environment and that he applies it tacitly. His knowledge of Euclid probably has its roots in quite a different context, namely in his encounter with Italian perspectivists. This knowledge was then later developed with the active help of Pirckheimer. A last example, taken from Book III of Dürer's *Underweysung*, seems to point to a practical workshop context. This time, the technique is applied to proportioning the letters of inscriptions located at a great height on columns, towers or walls. Dürer finishes his instructions by stating:

> This division can be applied to all sorts of things and not only to letters, especially to decorating towers by paintings located at different levels, in such a way that images located higher seem equal to those which are located lower.[19]

This points to a well-known practice of painters and architects. On the other hand, this example, as the diagram (figure 8.6) already shows, clearly establishes the link

19 My translation, see Dürer 1977 for a better one, if accurate. In Dürer's original words: "soliche teylung hat nit allein stat in den pustaben / sonder in allen anderen dingen / und in sonders so man einen hohen thuren in allen gaden mit bildwercken ziren will / also daß die oberen bild gleych den underen scheynen kan durch disen weg geschehen" (Dürer 1983, fol. K^{v}).

we were missing between vision and the observer's eye, "dein gesicht," *c*. The vertex of the triangle *abc*, used to subdivide *ab*, is identified with an eye, and the arc, which is divided in equal arcs, is an angle of vision. Moreover, Dürer is aware that the technique described here in a practical context is identical to that which he has applied to the construction of an abstract mathematical object: the helix, to which he refers explicitly.[20]

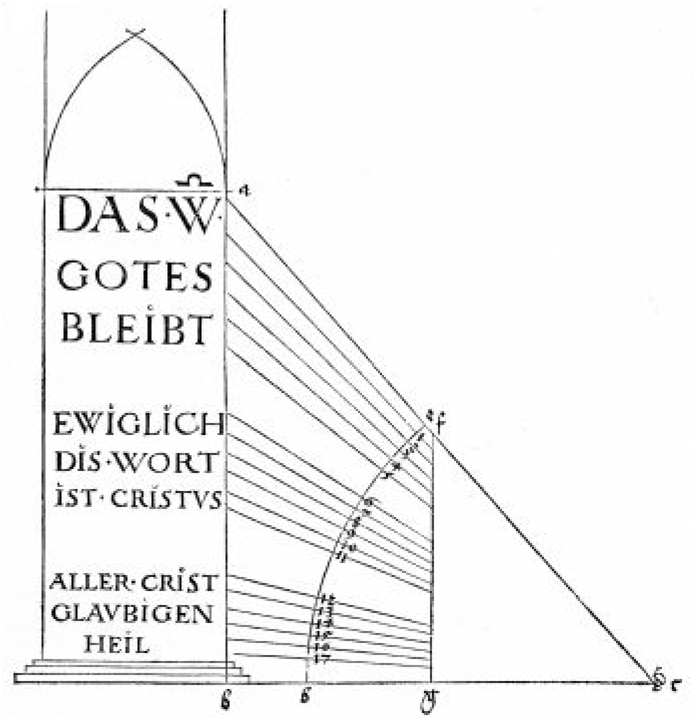

Figure 8.6. Proportioning the letters of inscriptions. (Dürer 1983 figure III.[28].)

20 "welicher an ein thuren schreyben will das man die oberst zeyl der bustaben als wol gesech zu lesen als die underst / der mach sie oben gröser dann unden / durch ein solichen weg / stell dein gesicht so weyt von dem thurn / und in der höch wie du wild / diß sey ein punct *c* und nym für dich den weg des dryangels *abc* der 16. figur des lini büchleins/ ..." (Dürer 1983, fol. K^{v}).

Of course, the technique of correcting optical errors—equal magnitudes appear smaller to the observer's eye when situated higher—has a long history,[21] going all the way back to Greek classical art. Plato[22] already mentions artists who lend their sculptures proportions that appear harmonious without really being so. Recall the story reported by Pliny, or by Iohannes Tzetzes (in the twelfth century), of a contest between Pheidias and Alcamenes for a Athena to be placed on top of a column. While Alcamenes' statue was beautifully proportioned, Pheidias offered a statue with distorted members and he was barely saved from lapidation. The situation reversed completely when the statues were put on top of the column. Vitruvius also gives advice to correct "falsa iudicia oculorum." The procedure used to do this is the one based on Euclidean optics, known in the Western world since its Latin translation "De visu" and circulated in manuscript form in the second half of the twelfth century.[23] It was probably part of the tradition of practical optics[24] as established by Simi and Camerota.

All of the examples we have discussed in Dürer point to an architectural context, even the helix, which Dürer thinks of as at a staircase in the spire of a circular tower. Thus, the division technique, the aim of which is to make things located at different heights appear equal, may well be a workshop technique distinct from, even if linked with, the Euclidean theory of vision.[25] But on the other hand, his examples don't fit the conditions of effective practice. They seem to be a kind of school exercise as shown by the location of the eye at the bottom, and not at man's eye level. The procedure may have been taught in practical abacus schools.

To sum up, an optical theory underlies Dürer's architectural drawings, and even his diagram of a helix, when this mathematical object is used in an architectural context. The stonemason's practice of representing things by means of a ground plan and a vertical section, an *Auszug*, enables Dürer to visualize the object in three dimensions and to display it in a visual space. In the construction process itself, he takes into account the laws according to which vision proceeds. His vertical *Auszug* is an elevation only if we look at it in a space ordered by Euclid's postulate.

Dürer's diagrams are thus located at the intersection of practice and theory. This interpretation is confirmed by at least one Italian follower of Dürer: Daniele Barbaro. In *La pratica della perspettiva* (1568), he comments on Dürers "artificio," "instrumento,"[26] and on

> the rule and form of the quadrant ..., which serves to fix the proportions of the letters or figures which are located high up in some column or wall. With the help of it, painters and architects know to divide the heights in proportional parts[27]

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21 See Frangenberg 1993 and Cecchini 1998b.
22 Plato *Sophistes,* 235–236.
23 See for instance Cecchini 1998a.
24 See Simi 1996 and Camerota 1998.
25 In a paper entitled "L'optique euclidienne dans les pratiques artisanales de la Renaissance," to appear in *Oriens-Occidens. Sciences, mathématiques et philosophie de l'Antiquité à l'Âge classique*, I try to find traces of this procedure closely linked to the practical tradition of "measuring by eye."
26 See Barbaro 1568, 23.

and establishes a double link with the painters' and architects' knowledge on one hand, and with Euclid's *Optics* on the other, whose angle axiom he quotes.[28]

The technique described by Dürer, developed considerably in the baroque world, where it became important in creating artificial effects such as accelerating perspectives, but that's beyond the scope of the present study.

2. FROM EUCLIDEAN OPTICS TO PERSPECTIVE: THE CASE OF SIXTEENTH-CENTURY *KUNSTBÜCHER*. WITH PARTICULAR EMPHASIS ON HANS LENCKER'S *PERSPECTIVA* (1571)

In this section, the emphasis will be on the use Dürer's immediate followers in Nuremberg made of orthographic plans. As it is well known, Dürer's instructions were not immediately understood by the practical men to whom his book was addressed. In their eyes, these instructions needed some mediations, simplifications and further explanations to be useful and applicable to the crafts. Thus during the sixteenth century a host of booklets, the so-called *Kunstbücher*, was published mainly, but not exclusively, in Nuremberg, the aim of which was explicitly pedagogical. For instance, Augustin Hirschvogel,[29] painter on glass in a Nuremberg workshop, author of a *Geometria* (1543) and later "Mathematicus" of the city of Vienna, intends to offer beginners a better understanding of the basic principles and the practical applications of the art of measuring, "die edle und nützliche kunst des messens (Perspectiva in Latein genant)," unlike previous books, which are often obscure and hide the most necessary and noble aspects.[30] To a certain extent these books may thus grant us a glimpse of what was transmitted to apprentices in sixteenth-century Nuremberg, especially in the domain of constructive geometry, plan and elevation techniques, as well as in perspective.

The first point I want to make is that the understanding of Dürer's notion of "Messung" seems to have undergone some change. It is now used as a German translation of the Latin "perspectiva." Count Johann II of Simmern in his *Underweysung der Kunst des Messens* (1531) had already transcribed the Latin *Perspectiva* in German as "kunst des Augenmeß" (art of measuring by eye).[31] The notion of representation, namely in the form of perspective painting, is thus added to Dürer's constructive geometry. To be more precise, the visual appearance of things displayed in a three-

27 "Dalle dette cose si comprende la regula, & la forma del quadranto di Alberto Durero, col quale egli proportiona le lettere, overo le figure, che sono nell'altezza di qualche colonna o parete. Dalche sono avertiti i Pittori, overo gli Architetti a partire le altezze in parti proportionate" (Barbaro 1568, 9).

28 "Le cose, che si vedeno sotto anguli eguali pareno eguali" (Barbaro 1568, 10, § G).

29 On Hirschvogel, see Kühne 2002.

30 See the dedicatory letter of Hirschvogel 1543, fol. a2^r: "Nach dem bißher / durch unsere vorfordern / ein langezeit dise edle und Nützliche kunst des messens (Perspectiva in Latein genant) in Deutscher sprach verborgen gehalten / und den gemeinen man / zulernen schwerlich zu bekommen / Auch den meren Thail in Griekischer und Lateinischer sprach verfast / Was derselben bücher in Truck sein kommen / etwas tunckel / und das notigist und fürnembst / gewönlich dahinten behalten worden / Das dann manchen kunstbegirigen / der solchs gelesen / den buchstaben verstanden / aber zu keinen handtbrauch gelangt hat / abschewlich / und verdrießlich ist gewesen."

31 Johann II von Simmern, in the title of his 1546 *Perspective*, addressed his instructions to "all those who want to use the art of measuring by eye (called Perspectiva in Latin)."

dimensional space and depending on the visual angle are replaced by perspective representations of those same things, the foreshortening of which depends on their distance from the eye. The means to obtain those images remain in part the same, namely orthographic plans, which can be conceived of as belonging to architecture,[32] as Walter Ryff states in the perspective book of his *Der fürnembsten / notwendigsten / der gantzen Architectur angehörigen Mathematischen und Mechanischen künst / eygentlicher bericht* (1547), where he translates passages by Alberti, Serlio, and Vitruvius into German. Ryff (or Rivius), a physician who spent some years in Nuremberg and published a German translation of Vitruvius' architecture, is a quite prolific writer on perspective. He is conscious that teaching perspective by the written word is a difficult task namely because "of the solids on the plane raised out of the ground plan."[33] And yet without this technique of drawing geometric plans ("Geometrischer grund"), nobody will be able to master the art of perspective, which Ryff identifies with Vitruvius' *Scenographia*.

By including perspective in their understanding of "Messung," these men—like Dürer—see it as part of a practical geometry in which the observer's eye plays a role. But instead of constructing three-dimensional objects, they represent them on a picture plane, as Dürer already did in the very last section of his *Underweysung der Messung*. But while Dürer clearly referred the picture plane to the intersection of the visual pyramid, this Albertian model is completely absent from the *Kunstbücher*. These seem related rather to that older tradition of practical (applied) optics.Going back to the Arabs, it was taught in the medieval abacus schools, and its methods were used by astronomers, geographers, and architects in order to deduce from the appearances of distant or inaccessible magnitudes, their real measures. According to the Arab philosopher al-Farabi (tenth century),[34] who lists this practical optics in his classification of the mathematical disciplines, it also includes optical corrections of the sort studied in my first section. So, at least in Nuremberg, although the Albertian rules are available in Ryff's writings, very rough perspective workshop methods develop as part of the art of measuring by eye. Note that for Francesco di Giorgio Martini, the painter's perspective also belonged to that tradition of "misurazione con la vista."

"Messung," as understood by the authors of the sixteenth-century Nuremberg *Kunstbücher*, thus aims to construct perspectival representations of more or less complex solids, most of them polyhedral. Some applications to architectural settings are discussed in most of these treatises. As far as the methods are concerned, they essentially consist in drawing plans and their perspectives in already foreshortened planes. Hirschvogel, for instance, teaches in his *Geometria* "how to obtain the ground plans of regular and irregular solids, to throw them into perspective, and to give an eleva-

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32 Ryff 1547, in the lengthy summary of Book III, speaks of: "der Architectur angehörigen Kunst von der Geometrischen Messung."

33 See Ryff 1547, fol. cc_{iij}^{r}: "Obgleich wol die trefflich kunst der Perspectiva vast schwer und müsam / schrifftlich zuhandlen und tractieren / fürnemlichen aber der Corper halben / auff der ebne aus dem grund auffgezogen."

34 See Jolivet 1997, 260.

tion."[35] He starts from a ground plan, called "Stainmetzenfierung," or the stonemasons' square (figure 8.7). Next he constructs a foreshortened square, called "Geometria der perspectiva." This same square, foreshortened once for all solids, is used through the whole book as a kind of template. The bulk of the book is dedicated to explaining in detail how to divide the square or "geometria," i.e. how to draw in the geometrical square the ground plan of the solid , which is to be represented in perspective. This

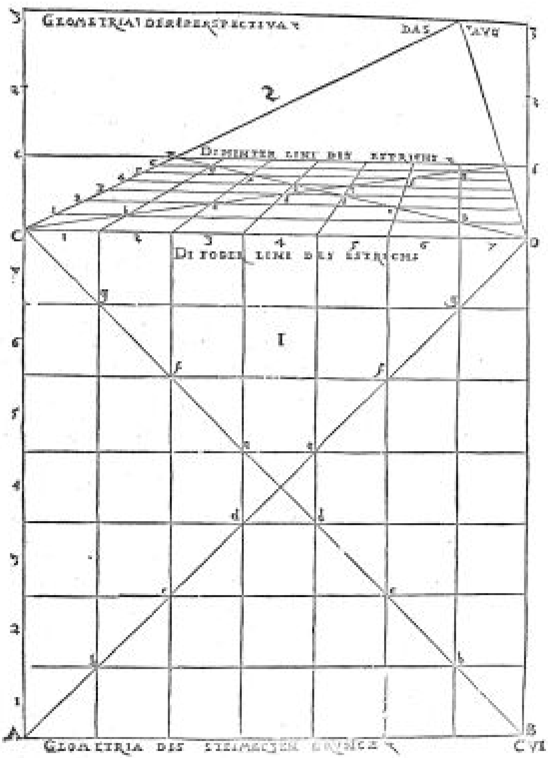

Figure 8.7. Stonemason's plan and *Perspectiva*. (Hirschvogel 1543 figure 2.)

35 That's what he announces in the title of his *aigentliche und grundtliche anweysung / in die Geometria / sonderlich aber / wie alle Regulierte / und Unregulierte Corpora / in den grundt gelegt / und in das Perspecktiff gebracht / auch mit jren Linien auffzogen sollen werden.*

appears to be the most important step in the construction process, at least the one, which deserves ample explanation and description. To throw this ground plan into perspective, Hirschvogel uses the method known in the literature as the diagonal method (which I prefer to call the template method). To keep from losing his way in transferring the division of the ground-plan to the foreshortened one, he makes use of colours and small symbols like + and ↓, thus allowing a glimpse into the workshop projection techniques. Finally he uses an elevation of the object to complete the perspective view of it.

Heinrich Lautensack, a goldsmith and painter from Frankfurt, who follows Hirschvogel's instructions (without always completely understanding them) in his *Underweysung deß rechten gebrauchs deß Circkelß und Richtscheyts, auch der Perspectiva …* (1563), comments on the basic use of the ground plan in those perspective drawings:

> Those plans which are drawn in this way are called plans in the Geometria or stonemasons' plans, because here you may see how all things are in the ground plan. If you are to throw them into perspective you thus know where to put things while foreshortening.[36]

These so-called stonemasons' plans are also central in the practical method described by the goldsmith Hans Lencker in his *Perspectiva*, which was to appear seven years later and which earned him a position at the Saxon court in Dresden.

Lencker's *Perspectiva* (1571) is addressed to craftsmen who build in wood or stone, like carpenters and stonemasons. Before they began working so, Lencker entreated them to visualize the planned construction in small models[37] obtained with the help of perspective, i.e. plane representations. His book, although full of mistakes,[38] is interesting in so far as it grants a glimpse into the workshop methods of the time, the "Praxen" as Lencker puts it, and the use of plans and instruments in implementing these practical methods. Perspective, understood as diminution and drawing orthogonals to a central [vanishing] point, consists in constructing first a ground plan, the *perfetto* of the Italian artists, then deriving from it by means of compasses, strings and more compound instruments a foreshortened plan, the *degradato*, and finally obtaining the perspective of the solid by using an elevation (figure 8.8). The latter part of the process is described as easy going. The bulk of the book is dedicated to the techniques of drawing elevations and especially ground plans, and to foreshortening them ("gründe verjüngen").

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36 Here is the original German text from Lautensack 1618, fol. $D_{iv}{}^{v}$: "Diese gründt die man also auffreist / werden die gründt in der Geometria oder Steinmetzengrund genennet / denn da sihet man wie alle ding im grundt kommen / so man es dann in die Perspectiff wil bringen / das man wisse wo ein jegliches ding sol im verjüngen hingebracht werden."

37 See Lencker 1571, fol. V^{v}: "So kan nun ein solches muster / oder was es sonst ist (doch nicht das es Corperlich sein müsse / sondern nur mit höhe / leng und breite / im sinn fürgenommen) auff einer vierung *a* … auff zweierley gantz unterschiedliche form unnd gestalt gerichtet / unnd auß rechtem grund der Geometria / in die Perspectief fürgerissen werden."

38 See for instance the analysis of Elkins 1994, 96–101.

After having introduced in a first section the different instruments he needs, including templates of already foreshortened squares,[39] he describes in a second section how to prepare the plans, i.e. ground plan, "geometrischer Grund," called R throughout the book, and elevation, labelled P. His definition of both plans shows that

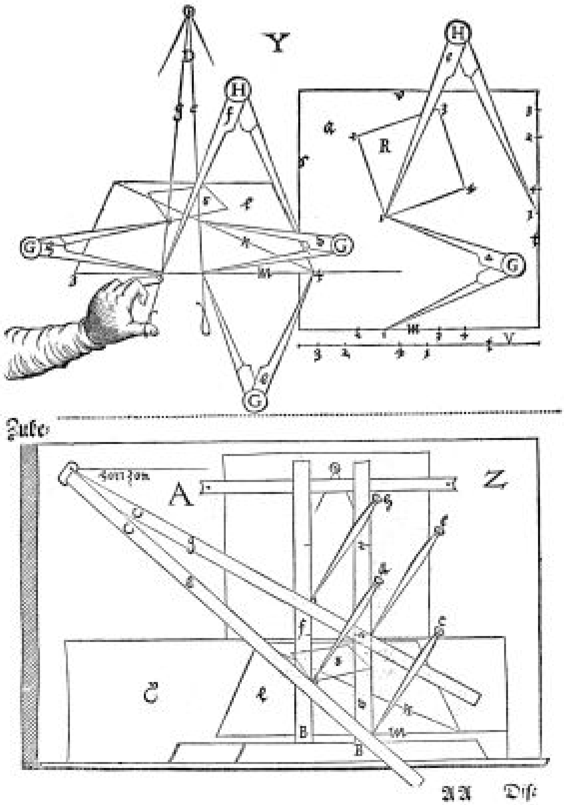

Figure 8.8. Foreshortening with the help of compasses and strings. (Lencker 1571 figure Y/Z.)

39 I study this aspect of Lencker's practical method in the Proceedings of the ESF Exploratory Workshop "Artists, work, and the challenge of perspective" in Rome (19–21 September 2002), to appear with the Ecole française de Rome.

we are far from a familiar projection technique. Lencker is appealing to the imagination and the experience of the apprentice. The ground plan located under the ground line, i.e. the intersection of the picture plane and the ground plane, is introduced as follows:

> all these plans you have to imagine them as the place or space straight from above which each thing (as a building, *corpora* or other, may it be polyhedral or circular, standing lying or oblique, largest at the bottom, in the middle or at the top, elevated at its two corners, at one or in the middle, or however you may imagine) would cover perpendiculariter on the horizontal plane, table or square, if it were a solid.[40]

He similarly defines an elevation as "the space or place which would be covered by some solid or building, by the rising points of its height and width (not perpendiculariter) but *à latre* (sic) on the sides like on a vertical wall."[41]
Two of Lencker's remarks deserve to be put forward in the context of this chapter. To bring the notions of plan and elevation closer to his reader, Lencker uses an analogy with the work of carpenters,[42] who first prepare floors and ceilings according to their length and width from their geometrical plans R. Even if they build columns, walls and vertical elements when laid down, all their thoughts are oriented towards how these elements will raise out of the ground plans (ligend gründ) according to plans P. Thus, it appears that, among craftsmen, carpenters are the ones who know best how to realize plans and to build in wood according to those plans. In Lencker's eyes painters are less informed. But once they have learned to draw the plans, it will be easy to obtain a perspective representation. The second remark points to architects, who already know how to draw plans. Lencker's perspective method will be convenient and particularly handy for them.[43] It is the method[44] called the circumscribed rectangle method by Elkins, the diagonal method by Kirsti Andersen and which I want to call the template method.

Taking a square and an already foreshortened square as a picture plane, this method allows the image of an arbitrary point of the geometrical square to be found

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40 See Lencker 1571, fol. VII: "Und alle dise gründe unterthalb der Erdlinie / mustu dir eigentlich also für und einbilden / als den platz oder rhaum / welchen ein jedes ding (als Gebew / Corpora oder anders / es sey ecket oder rund / es stehe / lige / leine / es sey unten / mitten oder oben / am breitesten / es sey mit beiden orten / mit einem / oder nur mitten erhoben / oder wie es sonst erdacht werden mag) gerad von oben herab auff einem Estrich / tisch / oder vierung *a* Perpendiculariter bedecken würde / wans Cörperlich were / das ist sein rechter Geometrischer grund."

41 See Lencker 1571, fol. VII$^{r/v}$: "Den grund aber oberhalb der linie *m* mustu verstehen gleich wie disen / als den rhaum oder platz / welchen ein jedes Corpus oder Gebew mit den auffsteigenden puncten seiner höhe unnd breite (doch nicht Perpendiculariter) sondern *à latre*, nach der seiten / als an einer auffrechten wand / bedecken würde…"

42 See Lencker 1571, fol. VII^{v}: "Und haben dise beide gründe R und P gar ein ebens gleichnuß / mit dem werck der Zimmerleut / welche erstlich alle deck unnd böden nach der leng unnd breite auff jre Geometrische gründ richten / nach art des grundes R. Demnach ob sie wol alle auffrechte Gebew / als Seulen / Wend / unnd Gibel / auch niderligend zu werck ziehen / so stehen doch gleichwol alle jre gedancken dahin / wie sich hernach im auffrichten / solche Seulen / Wend / und Gibel / mit Düren und Fenstern / auff die ligenden gründ schicken werden / nach art des grundes P."

43 See Lencker 1571, fol. VIII^{v}: "Bistu ein Architectus des Maßwercks und der Gebew / sampt der selben gründe / verstendig / so wird dir sonderlich und vor allen diser weg und gebrauch der Perspectief bequem / leicht / und sehr dienstlich dazu sein."

44 On this method, see Elkins 1994, 96–101; Ivins 1973; Peiffer 1995, 111–112; Schuritz 1919, 43–44; and Staigmüller 1891, 46.

in the foreshortened one. Piero della Francesca, Dürer, and Hirschvogel, for instance, used the diagonal to find the image. Lencker makes very clever and virtuosic use of compasses and strings. Once he has found the image point, he raises it to a height given by the plan P (an elevation).

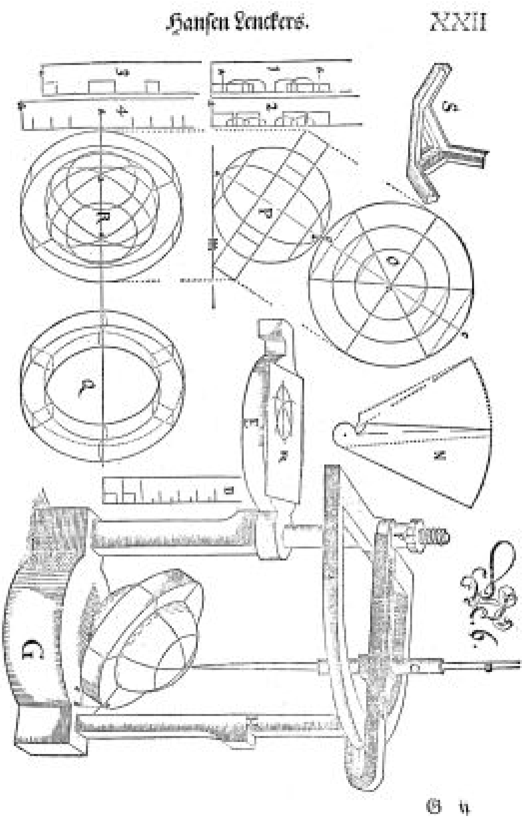

Figure 8.9. An instrument to draw plans and elevations. (Lencker 1571 fol. XXII, figure 6.)

As already mentioned, making the plans and elevations is considered by Lencker to be the core of the business. Seven of the ten figures of his *Perspectiva* are dedicated to obtaining various planes P and R. Lencker seems to have an optimal command of the technique. He combines the plans drawn in different ways such as to obtain different objects, or the same object in a different position. His examples include the sphere, polyhedra, a spiral staircase and an architectural setting, which closes and crowns the whole book. Thus he considers (see his figure 6) a sphere enclosed by a ring whose section is a square, and teaches how to draw it in two different positions: the ring stands vertically, the ring is oblique. His juggling with the different plans to obtain the desired result is quite impressive. For those who spare no expense, Lencker even conceives an instrument (G on his figure) to draw the ground plan and elevation of a given solid (figure 8.9). The description of this instrument is quite straightforward. In the half circle A made in brass, you fix half a diameter B pivoting around the screw F. In the socket (Hülslein) C moving along B, you fix a dry-point (or a lead pencil) D, which can be lifted and dropped. To obtain the ground plan R, you drop this point D onto the solid G, half of the ground plan of which you want to draw, turn the platform E into position H and transfer the coordinates of the point of the solid to the plan R. The heights of the different points of the solid are measured with the help of point D. Lencker prefers to work without this instrument, because the solid you want to draw must already be given, which is far from always being the case, especially if you are an architect and want to show a model of what you are going to realize. We should also add that the method described by Lencker is quite accurate for drawing the complex polyhedral representations, which were Nuremberg's specialty and even its trademark in the sixteenth century. Just think of Wentzel Jamnitzer's beautiful plates!

Johannes Faulhaber, 40 years later and in a quite different professional and theoretical context, includes Lencker's instrument in his *Newe Geometrische und Perspectivische Inventiones* (1610). Faulhaber,[45] a schoolmaster, "Rechenmeister" and "Modist" in the city of Ulm, was a prolific writer and earned his living in teaching mathematics. In his *Inventiones*, he is interested in military applications of perspective. He describes some instruments useful for easily drawing what he calls "planimetrisches Grundlegen" of fortifications, bastions, cities, and camps. To do so he places himself in a tradition of practical perspective, which he traces back to Albrecht Dürer. He follows this line through Gualterus Riff (sic), Heinrich Lautensack, Wentzel Jamnitzer, Daniele Barbaro, and Hans Lencker among others. For him, Dürer is at the beginning of a perspectival tradition based on the use of instruments. He was copied by Daniele Barbaro, and he inspired the so-called *Perspectivtisch* in which the lute was replaced by a simple plan, and which was included by Paul Pfinzing in his *Ein schöner kurtzer Extract der Geometriae unnd Perspectivae* (1599), where he claims to have seen it in Jamnitzer's house. Faulhaber improves Lencker's instrument which, according to him, is based on the same theoretical foundation as Dürer's. The figure

45 On Faulhaber, see Schneider 1993.

is inverted and from the point of vision the sight falls on the object perpendicularly; this is not the case in Dürer's lute woodcut, where the point of vision is fixed on the wall (figure 8.10).[46]

In my view there is another big difference between the two figures by Dürer and Lencker that is interesting for my purpose. Dürer offers a material model for the

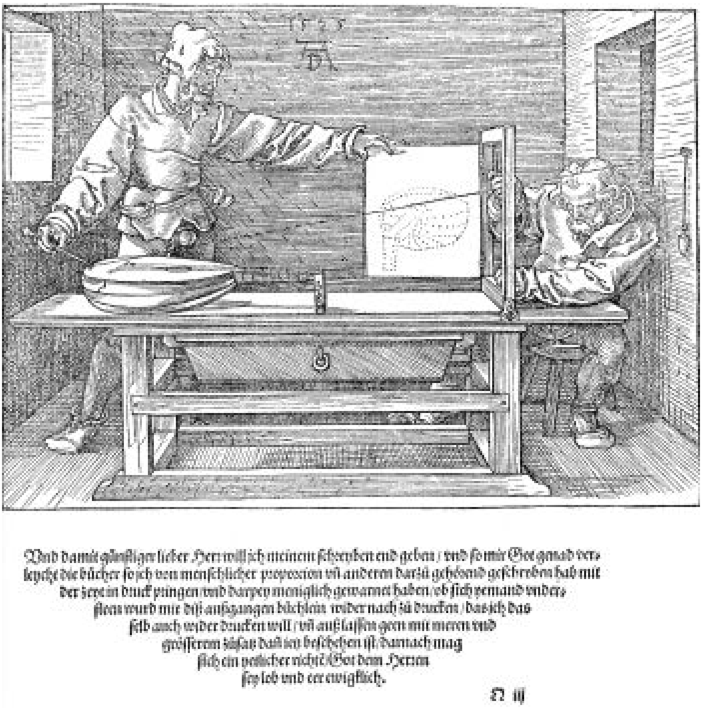

Figure 8.10. The painter with a lute. (Dürer 1983 figure IV.[64].)

46 See Faulhaber 1610, 35: "Weil er dardurch / so wol mit dem Perspectivtisch (doch auff die Maaß und Weiß / wie Albrecht Dürer / durch das Exempel / mit der Lauten angedeutet) als mit deß Lenckers Instrument / so alle Puncten fleissig observiret werden / zum fertigen Handgriff kommen kan / dann beyde Instrument / mit jhrem Gebrauch / auß einem Fundament zu demonstriren : Allein wird in deß Lenckers die Figur umbgekehrt / der gestallt dz der Augpunct *perpendiculariter* herab fällt. Mir ist der Perspectiv Tisch mit der Seyten umb etwas angenemer / dieweil der Augpunct stetigs an einem Orth bleibt / und nit verändert wird."

visual pyramid, the vertex of which is at the point of vision, which is not a human eye in Dürer's representation, but a nail in the wall. The sight is materialized by a string, cut by a transparent plane, the picture plane. The painter is doing nothing more than manipulating strings and passively recording measures. While Dürer's famous woodcut embodies a theory, the Albertian definition of perspective, Lencker's instrument and the way he represents it is first and foremost a technical device to draw plans. Thus, in the practical workshop tradition of such craftsmen as the Nuremberg goldsmiths, carpenters, and architects, the descriptive and perspective methods developed or described by Dürer are transformed into material devices, simple machines like strings and templates, and into instruments designed to apply those techniques rather mechanically.

3. THE GEOMETRY OF PROJECTION INVESTIGATED BY MATHEMATICIANS IN THE SEVENTEENTH CENTURY

Orthographic projections, like those which the Nuremberg craftsmen learned to realize through some of the booklets we looked at in the preceding section, were used systematically in architectural treatises of the late sixteenth century, especially in Italy (Palladio, for instance). Plans and elevations, or horizontal and vertical orthographic projections, were combined, as in Piero della Francesca's *De prospectiva pingendi*, in Dürer's *Underweysung der Messung*, and also much later, for instance in Salomon de Caus' *La perspective avec la raison des ombres et miroirs*,[47] in order to represent three-dimensional objects on a plane, either in perspective, i.e. as they are perceived, or by methods that were later subsumed under descriptive geometry. Thus two distinct mathematical techniques are involved, which led, when taken over by mathematicians, to two distinct domains of geometry, namely projective geometry founded in the seventeenth century by Desargues and Pascal, and descriptive geometry introduced systematically by Monge towards the end of the eighteenth century. In this last section, we will have a rather quick look[48] at the way mathematicians up to the seventeenth century handle orthographic projections and their combinations. Then we will examine some of the known reactions of practical men, namely painters and stonemasons, to the writings of professional mathematicians.

As Martin Kemp and J. V. Field have each emphasized, in the second half of the sixteenth century in Italy, painters' perspective was taken over by humanists like Daniele Barbaro and by professional mathematicians like Egnazio Danti, Giovanni Battista Benedetti, and Guidobaldo del Monte. In their hands, perspective underwent a significant change as different and more fundamental questions were asked, and could slowly develop into a mathematical technique.

Daniele Barbaro, a Venetian patrician with a good humanist education, "a notable patron"[49] who had Palladio and Veronese work for him, wrote a *Pratica della pers-*

47 London 1612. On Salomon de Caus, see Bessot 1991.
48 For this section we can rely on a rather important literature. See in particular Andersen 1991, Field 1997, Flocon and Taton 1963, Kemp 1990, Le Goff 1994.
49 See Kemp 1990, 76.

pettiva published in Venice (1568). J. V. Field sees it as a "spin-off from Barbaro's work on Vitruvius."[50] Indeed, Barbaro prepared an Italian translation of Vitruvius' *De architectura* (Venice 1556) and published the original Latin text ten years later. His interest in the work of Vitruvius, which he amply annotated, is quite probably, according to Field, at least part of the reason for his interest in perspective. Piero della Francesca and Albrecht Dürer are the main sources for the methods displayed in his *Pratica della perspettiva*. He uses combined orthographic projections—plan and elevation—to give a perspective representation of objects, which are in most cases geometrical solids. He also offers a perspective machine inspired by Dürer's. Barbaro seems to take pleasure in the speculative opportunities offered by the study of mathematics. From the start, in his dedicatory letter, he claims that his mathematical studies "opened the route to high and subtle speculations"[51] and regrets the absence of demonstrations in the work of contemporary painters who "let themselves be led by simple practice."[52] In the first chapter of his treatise, introducing the principles and foundations of perspective, he offers a twofold approach to perspective: a practical one (as the title announces) and a more theoretical one referring to Euclid and Apollonius. His intention is to combine the experiments of art with the decrees of science. Being aware of the dependence of perspective on geometry and natural philosophy or physics, he treats perspective as a "*scienza subalterna*," subordinate to both sciences: from geometry, it receives the straight line, from physics the process of vision. The visual rays of perspective are simple lines without width if considered mathematically, but belong to the tangible world if they are to be visible. Even though Barbaro's treatise offers hardly any geometrical demonstration, it assigns the painters' technique a place as a science subalternate to physics and geometry in the classificatory tree of science.[53] The status of that technique is thus changed significantly.

Egnazio Danti,[54] a professional mathematician who translated Euclid's *Optics* into Italian (1573), considers perspective as an art depending on a science (optics) subordinated to geometry. To him, this is to say that it is impossible to proceed in perspective according to the rigor of the Euclidean method. Nevertheless, Danti attempts to provide geometrical proofs of the practical rules used by artists. Born in a Perugian family of artists, employed at the papal court in Rome, he is the author of an extensive commentary on the perspective rules of the architect Giacomo Barozzi da Vignola. He edited for publishing the latter's *Le due regole* (Rome 1583), making substantial additions printed in a different typeface and providing additional illustrations. Vignola's aim was to demonstrate that the two principal methods, i.e. the combined plan and elevation technique and the distance point technique, give the same result even if they look somewhat different in use. Showing the equivalence of two methods is a completely new objective absent from earlier treatises. Practitioners cre-

50 See Field 1997, 133.
51 Barbaro 1568 states: "ci è stata aperta la strada ad altissime e sottilissime speculationi" in the dedication to "Al molto magnifico et eccellente M. Matheo Macigni," which opens the book.
52 See Barbaro 1568, Proemio: "I Pittori dei nostri tempi ... si lasciano condurre da una semplice pratica."
53 On perspective as a mixed science, see Peiffer 2002a.
54 On Danti, see Dubourg–Glatigny 1999, Field 1997, 145–150 and Kemp 1990, 78–83.

ated new methods and applied them to the representation of single forms, rather than asking questions about the compatibility of existing methods.[55] Danti further develops this approach to perspective by focusing on geometrical proof and by offering different kinds of proofs for one proposition. Euclidean style proofs, which are sometimes beyond the understanding of most practitioners, are often followed by an instrument that makes the result tangible.

Mathematicians like Federico Commandino, Giovanni Battista Benedetti, and Guidobaldo del Monte moved further in this direction, as Kemp has shown convincingly. With their treatises, as he puts it, "we see the birth of projective geometry as a discipline in its own right, related to but increasingly separate from the painters' science in its means and ends."[56] The university-trained Commandino, well remembered for his editions and translations of ancient Greek mathematics, recognized that the projection methods used in Ptolemy's *Planisphere*, of which he gave an edition in 1558, are related to perspective,[57] of which he provides a mathematical treatment in Latin. As most commentators have underlined, Commandino gives here a piece of mathematics addressed to mathematicians. Giovanni Battista Benedetti too made "first-class original contributions to the high tradition of mathematics arising out of the problems conceived within the practical one."[58] Best remembered for its mathematical style and approach, however, are the *Perspectivae libri sex* (Pesaro 1600)[59] written by Guidobaldo del Monte, a nobleman trained by Commandino. His six books on perspective were to become "the main source of reference for anyone seriously interested in the underlying geometry of perspectival projection."[60] Right from the start, Guidobaldo states his conviction that perspective is a mathematical science: "I would like to make it quite clear that the proper and particular object of the perspectival science doesn't differ from the object of geometry on which it depends."[61] The mathematical approach to perspective, which was definitively his, appears even in the structure of the work. In his first book, he proves in a purely Euclidean style a corpus of propositions, which are to be applied to problem-solving in the following books.

Yet in the history of mathematics it is Girard Desargues who receives much attention as the founder of a new mathematical discipline, now called projective geometry, the methods of which easily can be connected from a mathematical point of view to those of perspective. His most important work, a short treatise, which contains concepts and methods later subsumed under the term of projective geometry, appeared in

55 See Elkins 1994, 89.
56 See Kemp 1990, 85.
57 For a clear exposé of Commandino's summary of perspective, see Field 1997, 150–161.
58 This is the conviction of J. V. Field 1997, 171. See also Field 1985, Kemp 1990, 86–89.
59 Guidobaldo del Monte has been studied in recent years especially by Rocco Sinisgalli 1984, who translated his *Six books on perspective* into Italian, and Christian Guipaud, who presented an as yet unpublished French translation. Guipaud 1991 focuses on the important result proved by Guidobaldo, namely that any set of parallels in the scene to be portrayed will appear in perspective as a set of lines converging to a point.
60 See Kemp 1990, 91.
61 "Hoc namque in primis praecognitum esse cupio, proprium, ac peculiare obiectum scientiae perspectivae nequaquam a subiecto geometriae, cui subalternatur, diversum esse." (Del Monte 1600, I 3; Sinisgalli 1984, 240).

1639 in 50 copies and was entitled: *Brouillon project d'une atteinte aux evenemens des rencontres du cone avec un plan.*[62] In this treatise you can find a projective theory of the conic sections considered as perspective images of a circle. Some properties of the circle can be transferred to the other conics—properties concerning intersection or contact for instance, but not measure—if you introduce new elements like points at infinity. That is exactly what Desargues did. The perspective images of parallel lines, as you know, intersect at a point of convergence, the vanishing point of the picture plane. If the parallel lines are considered as a pencil of intersecting lines (having their intersection point at infinity), then they are projected into a bundle of intersecting lines.

As early as 1636 Desargues had already published a leaflet of scarcely twelve pages on perspective: *Exemple de l'une des manieres universelles du S.G.D.L. touchant la pratique de la perspective sans emploier aucun tiers point, de distance ny d'autre nature, qui soit hors du champ de l'ouvrage*.[63] It shows a worked example, without proof, of Desargues' perspective method. The core of the method is the introduction of scales, which give the progressive diminution in the width and height of lengths seen at greater distances from the picture plane, the position of the eye remaining unchanged. Desargues speaks of "échelle des éloignements," which is a perspectival scale for reducing orthogonal lengths in the picture and "échelle des mesures," a scale of measures, which is geometrical and allows the measures of the depicted transversals to be read on the ground line according to how far away they are located. The treatise is explicitly addressed to practitioners, even though a twentieth-century scholar has called it "a parody of a practical treatise."[64] It is of course interesting to know the family, social and training background of such an author. Archival research done on the occasion of Desargues' 400th birthday shows that Desargues stemmed from a very wealthy family in Lyon and was perhaps better acquainted with the works of the learned mathematical tradition than his published work reflects. He also knew a great many of the practical traditions, but there is not much evidence that he was a working engineer or architect. The only architectural realizations that can be attributed to him date from the period after 1644 and were motivated by the necessity to prove the superiority of his methods, which had come under attack.

In the light of current historical reconstructions, taking for granted that the projective concepts introduced by Desargues, such as the point at infinity and involution, were rooted in his perspectival work, it is of course challenging to inquire about the connections between Desargues' work on perspective (1636) and his treatise on projective geometry (1639). This was investigated by Kirsti Andersen and Jan Hogendijk:[65]

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62 *Rough draft for an essay on the results of taking plane sections of a cone* in the translation of Field and Gray 1987.
63 Example of one of S.G.D.L.'s general methods concerning drawing in perspective without using any third point, a distance point or any other kind, which lies outside the picture field.
64 See Field 1997, 195.
65 In a special issue of *Centaurus* devoted to Desargues, namely *Centaurus* 34(1991). For the following quotation, see Andersen 1991, 45.

> The mere fact that perspective and projective geometry are related has been taken as an evidence that the insights Desargues gained while working on perspective were essential for his new approach to geometry.

Jan Hogendijk has shown that there is a strong historical relationship between the *Brouillon project* and the *Conics* of Apollonius. Thanks to his introduction of points and lines at infinity, Desargues was able to derive most of the Apollonian theory of diameters and ordinates in a much easier way than Apollonius.[66] According to Hogendijk, Desargues took not only most of his initial motivation from the *Conics* but also more raw material than has hitherto been realized. Kirsti Andersen studies more specifically the connection in Desargues' writings between points at infinity and the vanishing points of perspective and obtains a more nuanced picture than the usual one. She discards vanishing points as a source of inspiration for Desargues' introduction of points at infinity.

Stereotomy and the Development of Descriptive Geometry

According to Joël Sakarovitch, who wrote an extensive history *Epures d'architecture* with the significant subtitle "From stonecutting to descriptive geometry. 16th to 18th centuries," stereotomy is to descriptive geometry as perspective is to projective geometry,[67] but he doesn't tell what precisely in his view perspective is to projective geometry. The writings referred to in the previous subsection show that from a historical perspective connections between the two are not straightforward but rather problematic. From the abstract mathematical point of view, it is true that the geometrical configuration underlying perspective, a visual cone cut by a plane, has an obvious link to conic sections interpreted as perspective images of a circle at the basis of the cone. It is also true that a vanishing point is a point at infinity, which one can add to a line. But from a historical perspective, things may be more complex, and it is not obvious that Desargues took his inspiration from the existence of vanishing points. Today, historical interpretations of the relations between painters' perspective and projective geometry are wide-ranging and divergent. Thus, at one end, the historian of art James Elkins claims that the connection is "virtually nonexistent." To him, "mathematics and perspective have developed in parallel, mutually isolated streams since the mid-sixteenth century."[68] At the other end, one can find the Husserlian thesis of Hubert Damisch,[69] interpreting perspective as an experimental ground for geometry before it took a new start in the seventeenth century and again adopted its

66 Hogendijk 1991, 2
67 In Sakarovitch 1994, 347.
68 See Elkins 1994, 3.
69 "Tout se passe en effet comme si, avant de prendre au xvii^e siècle un nouveau départ, la géométrie avait dû se donner, se constituer, comme elle l'aura fait à ses débuts, dans la Grèce antique, ce que le philosophe Edmund Husserl a décrit, dans son opuscule sur *L'Origine de la géométrie*, comme un sol préalable d'expérience: quitte pour ce faire à s'écarter pour un long temps des voies qui étaient les siennes et à parler un autre langage que strictement conceptuel, mais qui n'était pas non plus purement technique: celui des peintres et autres «perspecteurs», décorateurs, architectes ou tailleurs de pierre." This quotation is taken from Damisch's preface to Le Goff 1998, 5, but Damisch develops this thesis in Damisch 1987, which has been translated into English.

usual conceptual language. In between, one can find number of interpretations, most of them tying perspective to certain mathematical developments, some (as Kemp, Field and Gray, Andersen, Hogendijk) painting a more nuanced picture of these ties.

What Joël Sakarovitch shows in his *Epures d'architecture* is that methods belonging to what we call descriptive geometry were developed out of problems that stonecutters had to solve. The treatises of stereotomy, from Philibert de l'Orme's *Le premier tome de l'architecture* (1567) to Amédée-François Frézier's *La théorie et la pratique de la coupe des pierres* (1737–39), are seen as direct sources for Gaspard Monge's descriptive geometry. Monge himself was active for two years in the workshop for drawing and stonecutting at the *Ecole du génie* at Mézières, and knew quite well the problems and methods of that craft. His aim in *Géométrie descriptive* (1795) was to describe the mathematical principles underlying the complex graphical techniques, i.e. the method of projections, to build a unified theory that could easily be taught, and to apply it to such crafts as stonecutting and carpentry.

In the sixteenth century, various methods were traditionally applied to stonecutting, among which one, which used horizontal and vertical projections of the archstone to be cut out on the faces of the rough block of stone. Thus, geometrization in the form of orthographic projections—not necessarily laid down in drawings, but inscribed directly on the stone—was a necessary step in preparing stones before utilizing them in the planned building. It was at the origin of a large variety of graphical techniques, which were transmitted orally before they were collected and spread through the first treatises. The famous French architect Philibert de l'Orme included a treatise on stereotomy as Books III and IV of his *Le premier tome de l'architecture*. It is a kind of a catalogue of practical stonecutters' methods put together without establishing links between them, without any effort to single out common principles. From a geometrical point of view, most problems to be solved by stonecutters concern intersections of two different surfaces. The stone-dresser ("appareilleur")—the one who prepared the drawings allowing the stonecutter to obtain neatly the various surfaces bordering archstones—had to handle with sufficient precision complex surfaces such as cones, cylinders, ellipsoids, and spheres, which require a good command of methods for representing space. De l'Orme makes extensive use of the plan and elevation technique in the stereotomy books, while he refrains elsewhere in the treatise from using combined orthographic views. In describing the methods of stonecutters—which wasn't an easy task—, in putting into a written form procedures that were usually transmitted orally, in struggling to make them understood, in spreading them outside the realm of the craft, de l'Orme made a significant first move to single out the underlying geometrical theory.

While de l'Orme had described various isolated practical cases more or less familiar to his fellow architects, Girard Desargues, in his rough draft on stonecutting in architecture published in 1640, pretends to give a general method, as the title of his book claims: *Brouillon project d'une manière universelle du S.G.D.L. touchant la pratique du trait à preuves pour la coupe des pierres en l'architecture*. Like in his rough draft on perspective, he works out a single example. This example was not

usual in the architectural practice of the time. Abstractly speaking, it involves a plane cutting a cylinder, i.e. a cylindrical vault penetrates a non-vertical wall and angles downward (a situation, which implies that the axis of the cylinder is not horizontal). The geometrical construction of Desargues is elegant, but the text is dense and obscure (as usual with Desargues). It is impossible to enter into technical detail here,[70] but it is interesting to report on the reactions Desargues' writing provoked among practitioners.

Practitioners' Responses to a Mathematician's Work

The practitioners violently rejected Desargues' geometrical approach to stereotomy. This led to a conflict between Desargues and Jacques Curabelle,[71] known as the best "appareilleur" (dresser) of his time. A series of pamphlets, leaflets and posters were printed in 1644 by the two protagonists, the aim of which was to show the falseness or the superiority of the stonecutters' traditional methods or of Desargues' proposal. To settle the question, Curabelle offered to organize a competition between two stonecutters, one following Curabelle's preparatory drawings, the other applying Desargues' procedures. The outcome of the competition was to decide, in his mind, which of the two methods was the more exact and efficient. For him, only a concrete piece of work could test the precision of the two competing models. Desargues didn't agree with this vision at all. What was important to him was the accuracy of the geometrical arguments. He asked that the two methods be submitted to a board of mathematicians able to evaluate and judge the theoretical foundations of both constructions. The working drawing was in his opinion sufficient to prove the exactitude of the construction; for Desargues there was no need to turn to a concrete realization in stone. On the contrary, the power of geometry is strong enough to legitimate the construction. As Desargues formulates it with some brutality: "geometers … do not go to the school of stonemasons, on the contrary, masons … go to the school and lessons of geometers, and thus the geometers are masters and the masons apprentices."[72] This intense polemic between Desargues and Curabelle is the expression of power struggles between mathematicians and practitioners for the control over the crafts.

While Curabelle had the whole profession behind him, Desargues came more and more under the attack of mathematicians and perspectivists such as Jean Dubreuil or Jean François Niceron. But not everybody turned against Desargues. In particular, the competent engraver Abraham Bosse spent much energy in explaining, defending and transmitting Desargues' ideas.[73] He is said to have followed courses by Desargues on geometry applied to drawing. A convinced defender of Desargues' methods, he wrote a treatise on stonecutting as well as a famous work on Desargues' perspective.

70 See Sakarovitch 1994 and Sakarovitch 1998, 149–179.
71 See Le Moël 1994 for biographical detail. See Desargues 1864, II 219–426 on the polemic.
72 Quoted by Sakarovitch 1998, 181, who gives a beautiful analysis of the polemic.
73 On Bosse and his relations to Desargues, see Heinich 1983, Kemp 1990, 120–123, Bottineau–Fuchs 1994, and Andersen 1991, 65.

Although not a full member of the *Académie royale de peinture et de sculpture,* founded in 1648 and located in Paris, whose president was the painter Charles Le Brun, Bosse was allowed to teach perspective at the Academy. He gave his first lesson on 9 May 1648, but became increasingly involved in a harsh dispute on the role of perspective in painting, which lead finally to his exclusion on 11 May 1661. Such an exclusion seems to be unique in the history of the institution and one might wonder what the motives behind this severe treatment were. They are probably to be found in Bosse's notion of representation and in his allegiance to Desargues' methods.[74] Bosse aims to represent things as they are (according to the geometrical laws of projection) and not as they are perceived subjectively: "You ought be careful not to draw the relief or the natural as seen by the eye, but you should be able to reconstruct the proper ground plan of a painting putting various objects together."[75] This is exactly what Desargues' methods tend to provide. However, the notion of representation, founded on strict geometrical laws, is violently rejected by the academicians who at that time were seeking to discard such rigid laws.

For Abraham Bosse, the two practices of representing a solid by its ground plan or by a perspective view are identical.[76] Bosse only repeats what Desargues had already formulated in his 1640 rough draft on stonecutting: there is no

> difference between the way to draw, reduce or represent a thing in perspective and the way to draw, reduce or represent it by a ground plan, ground and perspective plans are thus two species of a same genre, and they can be described and proved together, with the same words.[77]

By identifying orthographic projections and perspective, Desargues shows that he had understood clearly the link between cylindrical and central or conical projections. This is also obvious in his rough draft on conic sections, where he introduces cone and cylinder by a single definition of a roll ("rouleau"). A line is said to move so that one of its points is fixed at finite or infinite distance, and that another turns around a circle. When the fixed point does not lie in the plane of the circle, the figure obtained is called a roll. Thus in writings partially addressed to practical men, Desargues rose to a level of abstractness that allowed him to treat horizontal and vertical orthographic projections (with the center of projection at infinity) in the same way as perspective, i.e. central projection. Bosse made himself the champion of Desargues' abstract ideas. One may doubt that such ideas were useful to the artists at the Academy. But this does not explain the vehemence of the debates and the virulence of the attacks against Bosse.[78] Painters and artists, whose activities had gained

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74 This is at least the thesis of Nathalie Heinich's analysis of the bitter polemic at the Royal Academy of Painting and Sculpture. See Heinich 1993, 147–152.

75 Quoted by Heinich 1993, 148: "Il faut bien se garder de dessiner le relief ou naturel comme l'œil le voit, mais bien de […] pouvoir remettre en son véritable géométral un tableau composé de divers objets perspectifs."

76 "ces deux pratiques de faire le trait de la représentation d'un corps en géométral, et en perspective, […] ne sont qu'une même chose" (quoted by Heinich 1993, 148).

77 "[il n'y a] différence aucune entre la manière de figurer, réduire, ou représenter une quelconque chose en perspective & la manière de figurer, réduire, ou représenter en géométral, aussi le géométral & le perspectif ne sont-ils que deux espèces d'un mesme genre, & qui peuvent estre énoncées ou démontrées ensemble, en mesmes paroles." (Quoted by Saint-Aubin 1994, 364).

recognition as liberal arts and who were organized in an academic structure with royal patronage, were struggling for their autonomy by eliminating the control of geometry.

CONCLUSION

In this chapter, we have analyzed some of the complex drawings in Albrecht Dürer's *Underweysung der Messung* (1525), which historical scholarship since the beginning of the twentieth century places in the same line as Gaspard Monge's constructions. In these drawings, Dürer takes into account the laws of vision according to which these mathematical and namely architectural objects must be seen. These laws are those of Euclidean optics, a theory of appearances, and not the rules of perspective, although Dürer did include a short presentation of perspective constructions in his book. His immediate followers in the Nuremberg workshops drop the reference to Euclidean appearances. They invent various material devices and instruments in order to make Dürer's perspective methods more efficient. In the books describing these devices, they call the whole domain *Messung*, a term which translates in their eyes to the Latin *perspectiva*. Thus, *Messung* includes methods, like central projections, which do not preserve measure. Even if the Nuremberg craftsmen are not conscious of this fact, their main preoccupation, quite explicit in the treatises of Hirschvogel and Lencker, is the problem of obtaining the horizontal orthographic projection, the *Geometria* or ground plan, of the objects they want to portray. Having constructed the *Geometria*, or stonemasons' square as they call it, they of course know the real measures of the object. While the fabrique or the representation of the concrete object is at the center of their concern, once professional mathematicians, like Danti, Commandino or del Monte move in, the projective ideas underlying this kind of construction increasingly are studied for their own sake. In the seventeenth century Desargues clearly understands that measure is not preserved in projective transformations like those of perspective. Other properties are, if the Euclidean space is completed by some ideal elements. In his hands, a new geometrical discipline takes form, which is no longer concerned with measure and can no longer be called *Messung*. That's what Desargues is suggesting when he writes: "The words perspective, appearance, representation and portrait are the names of one and the same thing."[79] The thing, which remains unnamed by Desargues, is projective geometry.

78 See Desargues 1864, II 49–113, for some aspects of the polemic.
79 See Desargues 1636, 1: "Les mots perspective, apparence, représentation, & pourtrait, y sont chacun le nom d'une méme chose."

PART V
PRACTICE MEETS THEORY

INTRODUCTION TO PART V

In the period when the decisive steps toward classical mechanics were made (1500–1700), technical drawings played an important role in mediating between practical and theoretical knowledge. This much-neglected function of early modern engineering drawings is the topic of the last section of this volume.

Reflections and inquiries of a more theoretical nature were not at all beyond the horizon of early modern engineering. Design processes are themselves reflection processes of a specific kind and cannot be cordoned off clearly from reflections on issues connected with the working of machines that properly could be called theoretical. As already discussed at several places in this volume, particularly in the chapters by Marcus Popplow, David McGee, and Pamela Long, technical drawings played an important role in design processes. Naturally, the possibilities and restrictions that such drawings provide for and impose on these designing reflections mark the power and limits of drawings as means for general reflections about machines as well as their working principles. With respect to the use of drawings for such more theoretical purposes, the advantages and shortcomings of specific pictorial languages are of particular significance. It is not by chance that, as in the case of Leonardo's notebooks, drawings that doubtless served such general reflections often switched from a pictorial style of representation to the diagrammatic one that was common in treatises on mechanics. In this way, such reflections could make use of geometrical demonstrations and proofs that served as chief means of reasoning in early modern mechanics. However, these diagrams, whatever their advantages over more pictorial representations, shared some of the principal limits characteristic of engineers' models on paper. Their representational potency was overtaxed if one tried to represent on one and the same diagram both the spatial relations of a device and physical quantities such as its mass or forces acting upon it.

These limits of engineering drawings as means of theoretical reflections on machines, however, must not obscure another significant role these drawings played as mediators between practical and theoretical mechanics. As is now almost generally acknowledged, early modern mechanics developed along with the technological innovations of this period and hardly can be understood without this background. Almost all of the pioneers of preclassic mechanics were either engineers themselves, Tartaglia being probably the most prominent instance of such a theorizing engineer, or were occupied occasionally with engineering issues and tasks, as was the case with Galileo. The new technology provided a wealth of new subtle objects whose investigation advanced the understanding of the patterns and laws of the natural potencies of which these devices made use. Yet to become truly familiar with the advanced technology of the age, even in highly developed regions like the Padua-Venice area, personal experiences with real machinery of some sort must be complemented by knowledge gained through representations of machines, either three-dimensional models or technical drawings, and knowledge acquired elsewhere through the study of tracts. It even might be possible that, for investigations of machines in a theoretical

perspective, drawings and other models were of even greater significance than real machines.

The main topic of Michael Mahoney's chapter is an exceptionally instructive instance of the power and limits of technical drawings for mediating between theoretical and practical mechanics. His example concerns diagrams by Christian Huygens that were at the interface between practical and theoretical mechanics in a twofold way. First, some of them served the communication between Huygens and Thuret, the Parisian clock-maker who translated Huygens' concept of an *Horologium Oscillatorium* into a working design, thereby proving how differently they could be read by a theoretician and by a practitioner. Second, some of them are highly intricate compounds of different layers of diagrams that represent entities from completely different worlds—from the practical world of machines, from the multidimensional world of physics, and from the ethereal world of mathematics. These diagrams demonstrate to which extremity technical drawings in combination with geometrical diagrams could be pushed when used in theoretical investigations and, at the same time, how impracticable this means of representation became for this employment.

9

DRAWING MECHANICS

MICHAEL S. MAHONEY

1. SETTING THE QUESTION

As the preceding chapters show, a variety of practitioners in the Renaissance drew machines for a variety of apparent reasons: to advertise their craft, to impress their patrons, to communicate with one another, to gain social and intellectual standing for their practice, to analyze existing machines and design new ones, and perhaps to explore the underlying principles by which machines worked, both in particular and in general. I say "perhaps," because this last point is least clear, both in extent and in nature. We lack a basis for judging. We have no corroborating evidence of anything resembling a theory or science of machines before the mid–sixteenth century, and what appeared then reached back to classical Antiquity through the newly recovered and translated writings of Aristotle, Archimedes, Hero, and Pappus, which came with their own illustrations of basic machine configurations.

The absence of a textual tradition to which the drawings themselves are linked, or to which we can link them, makes it difficult to know what to look for in them.[1] How does one know that one is looking at a visual representation of a mechanical principle? It will not do to invoke what is known from the science of mechanics that emerged over the course of the seventeenth century. That sort of "ante hoc, ergo gratia huius" identification of a valued feature of modern science or engineering begs the historical question of precisely what relationship, if any, the drawings bear to the emergence and development of that body of knowledge.[2] What did practitioners learn about the workings of machines from drawing them, and how did it inform the later theory?

That is actually several questions, which do not necessarily converge on the same result. What people learn depends on what they want to know, and why. What questions were the practitioners of the fifteenth and sixteenth centuries asking about the

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1 Cf. Marcus Popplow's observation in his contribution to this volume (above, p. 42): "The analysis of early modern machine drawings is often confronted with such problems of interpretation. To narrow down the possibilities of interpretation, descriptive texts, text fragments on the drawing and textual documents preserved with the drawings again and again prove most useful. Where such additional material is missing—which is often the case due to the frequent separation of pictorial and textual material practised in a number of European archives some decades ago—interpretation is often confronted with considerable difficulties."

2 Popplow has pointed to the dangers of such retroactive identification: "Die wenigen Blicke, die von Seiten der Wissenschaftsgeschichte auf die Maschinenbücher geworfen wurden, suchten in erster Linie zu beurteilen, inwiefern ihre Diskussion mathematischer und mechanischer Prinzipien bereits auf die wissenschaftliche Revolution des 17. Jahrhunderts verweist. Wiederum ist damit die Tendenz zu erkennen, die Maschinenbücher also noch defiziente 'Vorläufererscheinung' späterer, wissenschaftlich exacterer Abhandlungen zu betrachten. Wie im folgenden deutlich werden wird, entsprechen die dieser Betwertung zugrundeliegenden Maßstäbe jedoch nicht den Intentionen hinter der Abfassung der Werke." (Popplow 1998a, 69) As Edgerton's example shows, it is not only the historians of science who have measured these works by later standards.

machines they were drawing? What sorts of answers were they seeking? What constituted an explanation for how a machine worked, and what could one do with that explanation? How were the questions related to one another, and where did the answers lead? What then (and only then) did these questions and answers have to do with the science of mechanics as shaped by Galileo, Huygens, and Newton?

What follows addresses this historical inquiry and the contents of the foregoing chapters only by contrast. It revisits an argument made almost 25 years ago in response to a claim by Samuel Edgerton about the long-term theoretical importance of Renaissance innovations in the depiction of machines.[3] In essence, he maintained that ever more realistic pictures of machines led to the science of mechanics. Three salient passages from the mechanical literature reveal the difficulties with that thesis. The first is found in a letter written to Galileo Galilei in 1611 by Lodovico Cigoli, who observed that "a mathematician, however great, without the help of a good drawing, is not only half a mathematician, but also a man without eyes."[4] The second comes from Leonhard Euler, who in the preface of his *Mechanics, or the Science of Motion Set Forth Analytically* lamented that the geometrical style of Newton's *Principia* hid more than it revealed about the mathematical structures underlying his propositions.[5] Finally, capping a century's development of the subject, Joseph-Louis Lagrange warned readers of his *Analytic Mechanics* that

> No drawings are to be found in this work. The methods I set out there require neither constructions nor geometric or mechanical arguments, but only algebraic operations subject to a regular and uniform process. Those who love analysis will take pleasure in seeing mechanics become a new branch of it and will be grateful to me for having thus extended its domain.[6]

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3 Mahoney 1985. Cf. Edgerton 1980.

4 Quoted by Santillana 1955, 22, in a rather free translation. Cigoli made the remark in a letter to Galileo dated 11 August 1611 in a perplexed effort to understand why Christoph Clavius continued to resist Galileo's telescopic evidence of the moon's rough, earth-like surface (Galilei 1890–1909, XI 168): "Ora ci ò pensato et ripensato, nè ci trovo oltro ripiegho in sua difesa, se no che un matematico, sia grande quanto si vole, trovandosi senza disegnio, sia non solo un mezzo matematico, ma ancho uno huomo senza ochi." It is not clear just what Galileo's drawings of the moon's surface have to do with mathematics.

5 Euler 1736, Preface, [iv]: "Newton's *Mathematical Principles of Natural Philosophy,* by which the science of motion has gained its greatest increases, is written in a style not much unlike [the synthetic geometrical style of the ancients]. But what obtains for all writings that are composed without analysis holds most of all for mechanics: even if the reader be convinced of the truth of the things set forth, nevertheless he cannot attain a sufficiently clear and distinct knowledge of them; so that, if the same questions be the slightest bit changed, he may hardly be able to resolve them on his own, unless he himself look to analysis and evolve the same propositions by the analytic method." (Non multum dissimili quoque modo conscripta sunt Newtoni Principa Mathematica Philosophiae, quibus haec motus scientia maxima est adepta incrementa. Sed quod omnibus scriptis, quae sine analysi sunt composita, id potissimum Mechanicis obtingit, ut Lector, etiamsi de veritate eorum, quae proferuntur, convincatur, tamen non satis claram et distinctam eorum cognitionem assequatur, ita ut easdem quaestiones, si tantillum immutentur, proprio marte vix resolvere valeat, nisi ipse in analysin quirat, easdemque propositiones analytica methodo evolvat.)

6 Lagrange 1788, Avertissement: "On ne trouvera point de Figures dans cet Ouvrage. Les méthodes que j'y expose ne demandent ni constructions, ni raisonnemens géométriques ou mécaniques, mais seulement des opérations algébriques, assujetties à une marche régulier et uniforme. Ceux qui aiment l'Analyse, verront avec plaisir la Mécanique en devenir une nouvelle branche, et me sauront gré d'avoir étendu ainsi le domaine."

Over the short range, looking from Renaissance treatises on machines to the work of Galileo might support the notion that the science of mechanics emerged from ever more accurate modes of visual representation. However, looking beyond Galileo reveals that the long-range development of mechanics as a mathematical discipline directed attention away from the directly perceived world of three spatial dimensions and toward a multidimensional world of mass, time, velocity, force, and their various combinations. Practitioners ultimately found that the difficulties of encompassing those objects of differing dimensionality in a single, workable diagram reinforced a transition already underway in mathematics from a geometrical to an algebraic mode of expression and analysis.[7] One could not draw an abstract machine; one could not even make a diagram of it. But one could write its equation(s). Since the dynamics could find no place in the diagram, the diagram disappeared from the dynamics.[8] But that did not happen right away nor directly, and I want here to take a closer look at the process. In particular, I want to consider the role of drawings and diagrams in mediating between the real world of working devices and the abstract world of mathematical structures.

Edgerton insisted on the capacity of the new techniques to depict machines as they really appeared, rendering their structure in life-like detail. However, the science of mechanics was created as the science not of real, but of abstract machines. It took the form of the science of motion under constraint; as Newton put it in distinguishing between practical and rational mechanics,

> ... rational mechanics [is] the science, accurately set forth and demonstrated, of the motions that result from any forces whatever, and of the forces that are required for any sort of motions.[9]

The clock became a model for the universe not because the planets are driven by weights and gears, but because the same laws explain the clock and the solar system as instances of bodies moving under constraint with a regular, fixed (and measurable) period. Indeed, as Newton himself pointed out, his mathematical system allowed a multitude of possible physical worlds, depending on the nature of the particular forces driving them and on the nature of the initial conditions. Which of these corresponded to our world was an empirical, not a theoretical question.[10]

What made it an empirical question was that the entities and relationships of the mathematical system corresponded to measurable objects and behavior in the physi-

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7 See Mahoney 1998, I, 702–55.

8 John J. Roche notes that Lagrange did not go unchallenged over the next century, pointing in particular to Louis Poinsot's complaint in 1834 that the "solutions are more lost in the analytical symbolism than the solution itself is hidden in the proposed question (Roche 1993, 225)."

9 Newton 1687, Praefatio ad lectorem, [I]. John Wallis had already spoken in similar terms at the beginning of Chapter 1 of Wallis 1670. In contrast to traditional definitions of mechanics that emphasized its artisanal origins or material focus, he insisted that, "We speak of *mechanics* in none of these senses. Rather we understand it as the part of geometry that treats of *motion* and investigates by geometrical arguments and apodictically by what force any motion is carried out (p. 2)." ("Nos neutro dictorum sensu *Mechanicen* dicimus. Sed eam Geometriae partem intelligimus, quae *Motum* tractat: atque Geometricis rationibus & ἀποδεικτικῶς inquirit, Quâ vi quique motus peragatur.")

10 See Newton 1687, the corollaries to Proposition 4 of Book I and the hypotheses (phaenomena in the 3rd edition) and opening propositions of Book III.

cal world. The fit was meant to be exact, and it grew ever more precise as theory and instrumentation developed in tandem. As will become clear below, that mapping of real to abstract, and the converse, emerged from drawings and diagrams in which representations of the two worlds were joined at an interface. But the drawings acted as more than a conceptual bridge. The principle that mathematical understanding be instantiated in specifiable ways in physical devices—in short, that knowledge work—entailed the need for a cognitive and social bridge between mathematical theory and technical practice and between the theoretician and the practitioner, and drawings played that role too. These issues emerge with particular clarity in the work of Christiaan Huygens, but they are rooted in earlier developments.

2. PUTTING MACHINES ON THE PHILOSOPHICAL AGENDA

Whatever body of principles might have been created by the designers and builders of machines, the science of mechanics as the mathematical theory of abstract machines was the work of seventeenth-century thinkers who considered themselves mathematicians and philosophers. Hence, before machines could become the subject of a science, they had to come to the attention of the people who made science at the time. That is, a body of artisanal practice had to attract the interest of the theory class. It has not always done so. Machines evidently did not impress medieval thinkers. The fourteenth-century philosophers who first compared the heavens to the recently invented mechanical clock lived in a society teeming with mills: windmills, watermills, floating mills, mills on streams and mills under bridges, grist-mills, sawmills, fulling-mills, smithing mills. And mechanically a clock is simply a version of a mill. Yet, no one before the late fifteenth century thought mills were worth writing about, much less suggested that the heavens might work along the same principles. Mills were of fundamental importance to medieval society, but as part of the mundane, workaday world they did not attract the interest of the natural philosophers.[11]

That had to change before a science of mechanics could even get onto the philosophical agenda. "It seems to me," says Salviati (speaking for Galileo) at the very beginning of the *Two New Sciences* (published in 1638 but essentially complete by 1609),

> that the everyday practice of the famous Arsenal of Venice offers to speculative minds a large field for philosophizing, and in particular in that part which is called "mechanics."[12]

That the everyday practice of mechanics should be the subject of philosophy is perhaps the most revolutionary statement in Galileo's famous work. Clearly, something had to have raised the intellectual standing of mechanics for Galileo to feel that the

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11 On the place of the mill in medieval society, see Holt 1988, which significantly revises the long standard interpretation of Marc Bloch in his classic (Bloch 1935), at least for England. Boyer 1982 calls attention to the urban presence of mills and makes it all the more curious that medieval scholars could talk of the *machina mundi* without mentioning them.

12 Galilei 1638, 1: "Largo campo di filosofare à gl'intelletti specolativi parmi che porga la frequente pratica del famoso Arsenale di Voi Sig. Veneziani, & in particolare in quella parte che Mecanica si domanda." Cf., most recently, Renn and Valleriani 2001.

philosophical audience to whom he was addressing the *Two New Sciences* would continue reading past those first lines.[13] One may take the statement as an exhortation, but Galileo had to have grounds for believing it would receive a hearing and indeed enlist support.

The early machine literature appears to have been the start of that process. As several of the preceding chapters show, the "theaters of machines" were a form of advertising, through which engineers (in the original sense of machine-builders) sought to attract patronage and to enhance their social status. The pictures portrayed not so much actual working machines as mechanisms and the ingenious ways in which they could be combined to carry out a task. Depicting the machines in operation, often on mundane tasks in mundane settings, the engineer-authors of these books offered catalogues of their wares. Yet, in some cases, they pretended to more. The machines were also intended to serve a means of elevating their designers' status as intellectual workers. Claiming to be not mere mechanics but mathematicians, they purported to be setting out the principles, indeed even the mathematical principles, on which the machines were based.[14]

Yet it is hard to find much mathematics in these treatises. Contrary to Edgerton's claim of the Renaissance artist as quantifier, Popplow points out that it is precisely the dimensions that are missing.[15] In many, perhaps most, cases, these are not measured drawings; in some cases, the elements of the machines are not drawn to the same scale. Meant to show how machines worked and what kinds of machines might be built, the drawings were not intended to make it possible for someone actually to build a machine from its depiction, unless that someone already knew how to build machines of that sort. The need to persuade potential patrons of the desirability and feasibility of a device had to be balanced against the need to conceal essential details that would inform potential competitors.[16] The message seems clear: "These are the machines I can build. If you want one, I shall be glad to build it for you, bringing to bear the knowledge of dimensions, materials, and detailed structure that I have omitted from the pictures." If that is the case, then the drawings, however realistically crafted, were not headed toward a science of mechanics.

Dimensions include scale, and the absence of scale enables pictorial representation to mix the realistic with the fantastic without a clear boundary between the two. It is apparent in one of the pictures in Domenico Fontana's *Della trasportatione dell' obelisco vaticano* of 1590 (figure 9.1). Fontana's twin towers for lifting, lowering, and

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13 The use of the vernacular in the first two days should not mislead us about Galileo's intended audience. The same Salviati who protested when Simplicio began to dispute in Latin in the *Two World Systems* slips unapologetically into the language of the schools in the third and fourth days of the *Two New Sciences* when setting out Galileo's new *scientia de motu*, a subject long part of the university curriculum. That part, at least, would not need translation to be understood by philosophers elsewhere in Europe.

14 In addition to the chapters in this volume by David McGee and Mary J. Henninger-Voss, see [Henninger-]Voss 1995 and Cuomo 1998.

15 Popplow 1998a, 72ff.

16 Popplow 1998a, 72: "Der Königsweg der Darstellung lag darin, auf der einen Seite die Umsetzbarkeit der vorgestellten Entwürfe sowie die dabei angewandten wissenschaftlichen Prinzipien überzeugend zu vermitteln und gleichzeitig auf der anderen Seite tatsächlich entscheidende Konstruktionsprinzipien zu verschweigen. Die Tendenz der Autoren der Maschinenbücher, 'unrealistische' Entwürfe zu präsentieren, scheint aus dieser Sicht durchaus vernünftig."

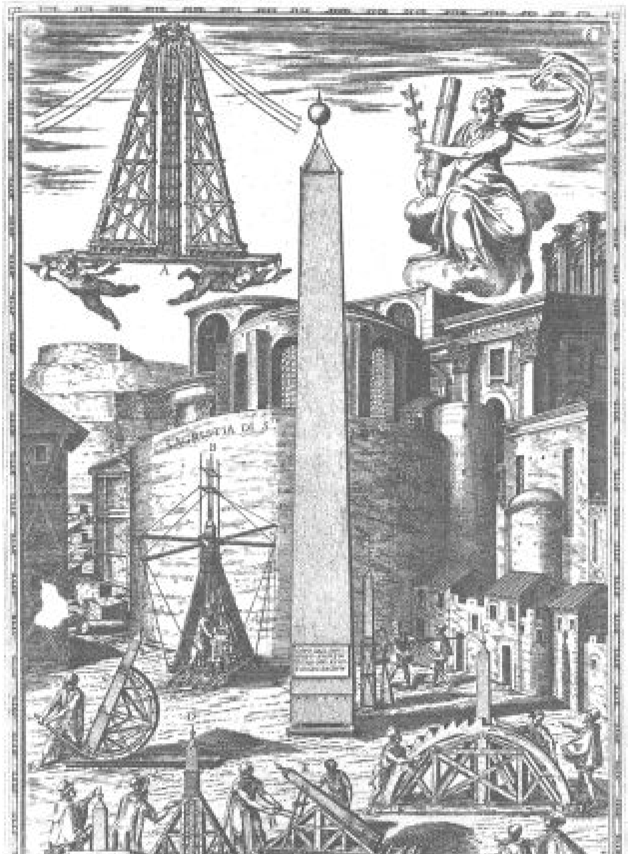

Figure 9.1. Schemes for moving the Vatican Obelisk. (Fontana 1590, fol. 8r.)

raising the obelisk float above a collection of "eight designs, or models, that we consider among the best that were proposed" for carrying out the task.[17] What is striking is that none of these others is even remotely feasible. All but possibly one fail on the issue of scale. They are essentially human-sized devices that either cannot be made larger or wouldn't make any sense if they were, because the humans necessary to work them don't come in such large sizes. In some cases, if they were scaled up as drawn, they would collapse under their own weight, much less that of the obelisk. One can draw them, as one has done here, and one can make models of them, models that might even work. But, scaled up to the dimensions of the obelisk, none of them would constitute a working or workable machine.[18]

It is noteworthy that Galileo begins the *Two New Sciences* with precisely this problem, aiming to provide a theoretical account of why machines do not scale up. It's not merely a matter of geometry, though it may be demonstrated geometrically: at a certain point, the internal stresses and strains of a material device cancel its mechanical advantage.

> ... so that ultimately there is necessarily ascribed not only to all machines and artificial structures, but to natural ones as well, a limit beyond which neither art nor nature can transgress; transgress, I say, while maintaining the same proportions with the identical material.[19]

Several important things are going on in this statement. First, the task of a philosophical account, or a theory, of machines is to set limits *in principle*.[20] There are certain things that machines cannot do, for both mathematical and physical reasons. Second, machines in the hands of the natural philosopher have become part of nature, and nature in turn has been made subject to the limits of machines. The same laws govern both.

In *Diagrams and Dynamics* I pointed to the difficulty that Galileo had in adapting his geometrical techniques from statics to kinematics. Taking his cue in statics first from Archimedes, he transformed his figures while keeping them in equilibrium. For example, in demonstrating the law of the lever, he took a realistically appearing beam suspended at its midpoint and then cut it at various points, adding support at the midpoint of each segment (figure 9.2). The (geo)metrics of the situation coincided with the object under consideration. In shifting then to the so-called "Jordanian" tradition of medieval statics to treat the bent-arm balance, he could continue to work with a

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17 Fontana 1590, fol. 7^{v}.

18 A similar juxtaposition of real and fantastic occurs in Agricola 1556, in which the inventory of machines used in mining begins with the wholly practicable windlass, moves through ever more complicated combinations, and concludes with a water-driven crane that strains credulity once one takes into account the forces involved in reversing its direction in the times necessary to place loads at the desired level. At a certain point Agricola seems to be no longer reporting actual machines but rather imagining potential machines. Another example is Errard 1584. Few of the devices are drawn to scale, nor would they work if they were actually built, in some cases again because humans would not be able to drive them.

19 Galilei 1638, 3: "... si che ultimamente non solo di tutte le machine, e fabbriche artifiziali, mà delle naturali ancora sia un termine necesariamente ascritto, oltre al quale nè l'arte, nè la natura possa trapassare: trapassare dico con osservar sempre l'istesse proporzioni con l'identità della materia."

20 That is what conservation laws do, most notably in thermodynamics. In computer science, theory similarly sets limits on computability, decidability, and complexity.

picture of the apparatus by superimposing directly onto it the geometry of virtual displacements, relying on the similarity of arcs traversed in the same time. Moving from there to the inclined plane by way of the circle, he again could overlay the statical configuration on a picture of the object. But, when it came to tracing the motion of a body accelerating down the plane, the spatial configuration at first misled him into thinking that the triangle that was the picture of the plane could also serve as the triangle made up of the instantaneous velocities of the body. Only by separating the graph of motion from the physical trajectory could he get the mathematics to work.

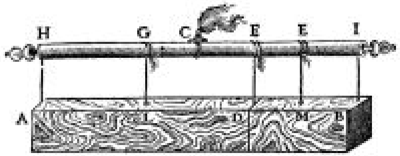

Figure 9.2. The law of the lever derived from a beam in equilibrium. (Galilei 1638, 110.)

It worked only for kinematics. As is well known, a mathematical account of the dynamics of motion escaped him completely, as it did Descartes. Picking up where they had left off, Christiaan Huygens devised a way of moving back and forth between the physical and mathematical realms and thus to get some of the dynamics into the diagram.

Before looking at specifics, it is perhaps worth emphasizing their context, if only to make clear what the second part of this chapter has to do with the first. Huygens' design of a pendulum clock in 1657 marked the beginning of a line of research that continued until his death and that in many respects formed the central theme of his scientific career.[21] In seeking to make his new clock accurate enough to serve for the determination of longitude and durable enough to continue working aboard a ship at sea, Huygens undertook a series of investigations in mechanics that led to fundamental results in the dynamics of moving and rigid bodies. In almost every case, those results led in turn to practical mechanisms that improved either the accuracy or the reliability of working timekeepers: the pendulum clock itself, the cycloidal pendulum, the conical pendulum, the sliding weight for adjusting the period, the balance-spring regulator, the tricord pendulum, the "perfect marine balance." In the end, the complete solution eluded him, in part because it was a matter of metallurgy rather than mechanics. However, subsequent efforts picked up where he had left off, culminating in the success of John Harrison a half-century later.[22]

Huygens thus embodied the union of head and hand that is characteristic of the new science of early modern Europe. His work on the clock and on the determination of longitude at sea are a prime example of what happened when machines did attract the attention of philosophers. In his hands, the clock constituted an interface between the mathematical and the physical world, between theory and practice, and indeed between the scientist and the artisan. Huygens not only derived and proved his results

21 See Mahoney 1980.
22 Andrewes 1996.

in theory, he also designed mechanisms to implement them in practice. He made his own sketches and, in some cases, built working models. But for the finished product he had to turn to master clockmakers and establish productive relations with them. In this he was less successful, in large part because of his inability or unwillingness to recognize the knowledge they brought to the collaboration along with their skill.

Three aspects of Huygens' work warrant closer attention: his use of diagrams in his mechanical investigations, his use of sketches in designing mechanisms, and his relations with his clockmakers. On close examination, his drawings reveal a subtle overlaying of three levels of pictorial representation and establish a visual interface between the physical and the mathematical. That same interface can be found in Newton's *Principia*, and its disappearance in later treatises marks the transition to an algebraic mode of analysis. Huygens' sketches range from the roughest outline to detailed plans, as they show him moving back and forth between theoretical inquiry and practical design. His famous dispute with his Parisian clockmaker, Isaac Thuret, shows what Huygens deemed to count as intellectual property and who could lay claim to possessing it.

3. THE PENDULUM AND THE CYCLOID

In December 1659 Huygens undertook to determine the period of a simple pendulum[23] or, as he put it, "What ratio does the time of a minimal oscillation of a pendulum have to the time of perpendicular fall from the height of the pendulum?" He

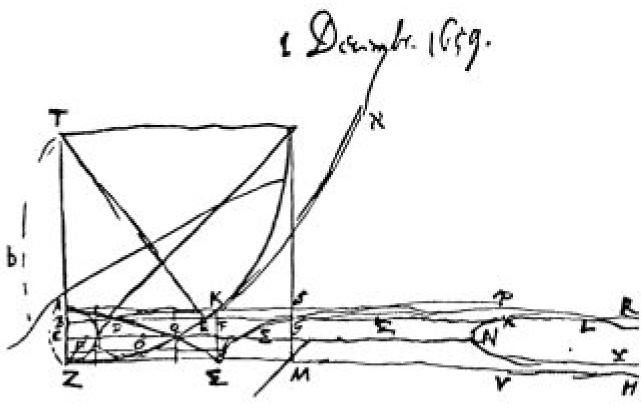

Figure 9.3. Huygens' original diagram. (Huygens 1888–1950, XVI 392.)

23 For a detailed account of what follows, see Mahoney 2000, 17–39.

began with a drawing of the pendulum with the bob displaced through an angle KTZ, which he described as "very small," but which he drew large to leave room for reasoning (figure 9.3). On that drawing of the physical configuration he then overlaid a semi-parabola ADΣ representing the increasing velocity of the pendulum as it swung down toward the center point. Next to the graph of motion, he then drew another curve ΣΣG representing the times inversely proportional to the speeds at each moment of the bob's fall. If the bob were falling freely, the area under that curve would represent the time over AZ. But the bob's motion is constrained along the arc KEZ of the circle, so that at, say, point E, one would have (from Galileo):[24]

$$\frac{\text{time at E}}{\text{time at B}} = \frac{\text{length at E}}{\text{length at B}} \times \frac{\text{speed at B}}{\text{speed at E}} = \frac{\text{TE}}{\text{BE}} \times \frac{\text{BF}}{\text{BD}} = \frac{\text{BG}}{\text{BE}} \times \frac{\text{BF}}{\text{BD}}$$

Huygens then constructed the curve of time over the arc by the relation $\frac{\text{BX}}{\text{BF}} = \frac{\text{BG} \times \text{BF}}{\text{BE} \times \text{BD}}$, sketching it roughly as RLXNYH. Note that the curves of speeds and times introduce non-spatial parameters into a drawing that began as a spatial configuration.

The mathematics of these "mixed" curves led Huygens then to the introduction of two important elements into the diagram. First, to simplify the expression, he drew on a mathematical result of unknown provenance: if BE were the ordinate to a parabola congruent to the one to which BD is an ordinate, then the product of BE and BD would be a constant times the ordinate BI to a semicircle of radius AZ drawn on the common base of the two parabolas. Huygens knew from earlier work on centrifugal force that a circle and a parabola with the right common parameters coincided in the immediate neighborhood of their point of mutual tangency, so he took the circular arc of the bob's trajectory to be a parabolic arc congruent to the graph of its speed and the intersecting curves as congruent, thus fixing crucial parameters, and drew in the semicircle. Second, from another source Huygens knew that the same circle also served the purpose of finding the area under the adjusted curve of times, and so he again shifted his gaze in the diagram and arrived at the result that the area, and hence the time of motion over the arc, varies as π times the product of the radius of the circle and the length of the pendulum. But, in comparing the time over the arc to the time of fall through the length of the pendulum, the radius of the circle cancels out, making the swing of the pendulum a function of the length of the pendulum only.

For the time, the derivation so far was already a mathematical *tour de force* but Huygens was only getting started. The result was only approximate over a minimal swing of the pendulum. He knew that empirically, because others had shown that, contrary to Galileo's claim, the period of a simple pendulum increases with increasing amplitude. But he now knew it mathematically, because he had explicitly made an approximation in deriving his result: he had taken BE as the ordinate to a parabola

24 Relying on the well known mean-speed theorem, Huygens takes the speed at B as the constant speed reached at Z, since the time over the interval AZ at that speed will be twice the time of uniform acceleration over the interval.

rather a circle. He now asked for what trajectory of the bob would BE in fact lie on that parabola? To determine the answer, he had to unpack his drawing to see how the mechanics of the body's motion would generate a parabola to match that of its speed of free fall. He appears to have found the answer by separating BE as ordinate to the physical trajectory from "another BE" as ordinate to the desired parabola, which was the mirror image of a mathematical (mechanical) curve. As he put it:

> ... I saw that, if we want a curve such that the times of descent through any of its arcs terminating at Z are equal, it is necessary that its nature be such that, if as the normal ET of the curve is to the applicate EB, so by construction a given straight line, say, BG is to another EB, point E falls on a parabola with vertex Z.

To determine that "other BE," he seems to have asked how the original BE entered into the expression $BE \times BD$. It came from the ratio $\frac{TE}{BE}$, which expresses the ratio of the "length" of point E on the circle to that of point B on the centerline (figure 9.4). Huygens had simply stated the relation without explanation and without drawing the lines in the diagram from which it derives. To see it, one draws the tangent at E, intersecting the centerline at, say, W. By similar triangles, $\frac{\text{length E}}{\text{length B}} = \frac{EW}{BW} = \frac{TE}{BE}$. Now, if one looks at the lines that Huygens did not draw but clearly had in mind in forming

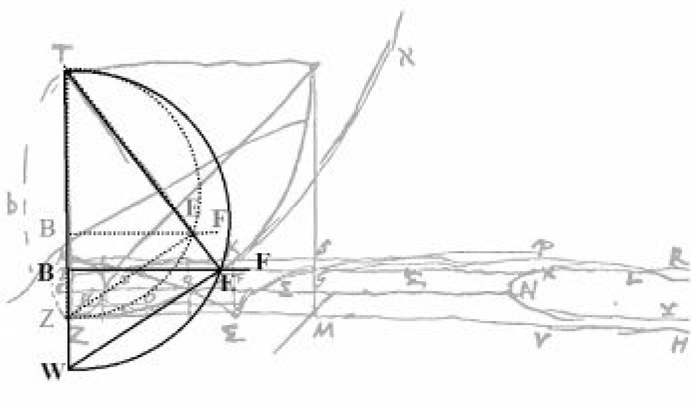

Figure 9.4. Unpacking the relation of physical trajectory and mechanical graph.

the ratio $\frac{\text{TE}}{\text{BE}}$, one more or less readily sees that $\frac{\text{TE}}{\text{BE}} = \frac{\text{TW}}{\text{EW}} = \frac{\text{EW}}{\text{BW}}$. If one takes TW as a "given straight line" (Huygens' BG), then the first proportion maps the ratio $\frac{\text{TE}}{\text{BE}}$ into the form $\frac{k}{\text{EW}}$ (where EW is his "other EB"), and the second pair makes $\text{EW}^2 = \text{TW} \times \text{BW}$. That is, if BE were extended to F such that BF = EW, then F would lie on the parabola $\text{BF}^2 = \text{TW} \times \text{BW}$, with vertex at W and with latus rectum TW. Now, if W coincided with Z, then $\text{BF}^2 = \text{TZ} \times \text{BZ}$. The osculating circle of that parabola is precisely the circle on diameter TZ.

At this juncture, one needs a shift of focus. The original circle, centered at T, is no longer of interest as the trajectory of the pendulum, but rather as a base for that trajectory. One is looking for another curve, with vertex at Z, the properties of whose normal and applicate can be mapped onto the relations just discussed in the circle of *diameter* TZ. That is, if BE extended intersects the curve at H, one wants $\frac{\text{normal at H}}{\text{BH}} = \frac{\text{TE}}{\text{BE}}$. That will be the case if the normal to the curve is parallel to TE, and hence the tangent at H is parallel to ZE. But this last relation, as Huygens says, is precisely, "the known method of drawing the tangent" to a cycloid.

Cycloid? Where did the cycloid come from? Well, it was on Huygens' mind; he had been involved recently in a debate over the curve and so was well aware of the property of its tangent. But, I want to maintain, the curve was also before his eyes. It, or rather its *Gestalt*, had crept into his diagram when he drew that semicircle for auxiliary purposes. With the parabola streaming off from the top and the trajectory of the pendulum swinging up from the bottom, the semicircle now looked like generating circle of a cycloid in the then standard diagram of the curve. Huygens needed no more than a hint; note how the semicircle in the original diagram has become a generating circle in the diagram Huygens drew to show that the cycloid is indeed the curve in question (figure 9.5). Once he had the hint, the details quickly followed.

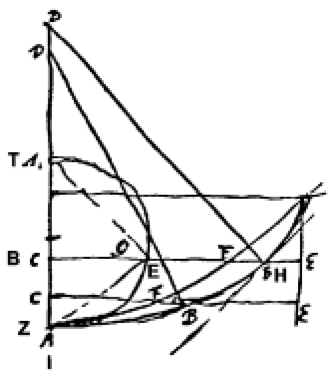

Figure 9.5. The revised diagram, with lettering added from figure IX.4. (Huygens 1888–1950, XVI 400.)

It will not have escaped notice, but I want to emphasize the composite nature of Huygens' main drawing here. It contains three spaces: the physical space of the pendulum, the mechanical space of the graphs of speeds and times, and the mathematical space of the auxiliary curves needed to carry out the quadrature of the curve of times. Huygens combined them without conflating them. That is how he was able so readily to modify the trajectory so as to make an approximate solution exact. He knew not only at what step he had made the approximation, but also in what space he had made it and how it was reflected in the other spaces. He needed a curve in physical space, the properties of whose normal and ordinate could be mapped by way of a mathematical curve so as to generate another mathematical curve congruent to a graph of velocity against distance.

4. PHYSICAL AND MATHEMATICAL SPACE

Huygens conjoined physical and mathematical space in another configuration in his work on the compound pendulum.[25] The problem itself, it should be noted, arose out of a quite practical concern, namely, that in the physical world pendulums have neither weightless cords nor point masses as bobs. Rather one is dealing with swinging objects whose weight is distributed over three dimensions. The task is to find a point in the pendulum at which it acts as if it were an ideal simple pendulum, its center of oscillation.[26]

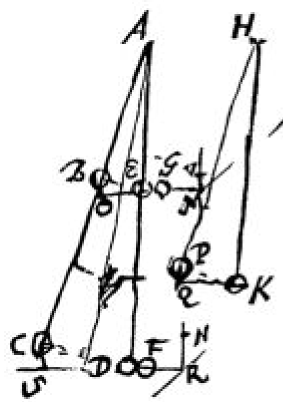

Figure 9.6. The two-bob pendulum. (Huygens 1888–1950, XVI 415.)

To determine that point, Huygens began with two bobs B and C joined by a common (weightless) rod AC and drew a simple pendulum HP swinging through the same angle in the same time (figure 9.6). Under the constraints of the pendulum, the speeds of B and C will be directly proportional to the speed of P at corresponding points of their swings. The speed of P can be measured by the square root of the height QP through which it falls to K, and that height is proportional to the heights BO and CS through which B and C fall toward E and D, respectively. But the speeds of B and C are constrained by their rigid connection and hence do not correspond directly to the heights through which they individually fall. To get a measure of their speeds, Huygens imagines them impacting with equal bodies G and F, respectively, and imagines G and F then directed upward by reflection off planes inclined at

25 For details, see Mahoney 1980, 234–270.
26 The worknotes dated 1661 (Huygens 1888–1950, XVI 415–34) became Part IV, *De centro oscillationis*, of Huygens 1673.

45^{o}. Each will climb to a height proportional to the square of its velocity, which can be expressed as a function of the height CS and of the ratio of the distance of the bob from A to the unknown length.[27]

At this point, Huygens invokes the principle that the center of gravity of G and F will rise to the same height as that of the compound pendulum at the beginning of its swing. Let me come back to the source of that principle in a moment and focus here on its application. It requires that Huygens move away from the geometrical configuration, which lacks the resources for determining the unknown length HK. That is, he cannot construct it directly by manipulation of the lines of the diagram, because the weights have no quantitative representation. Rather, he turns to algebra, translating the elements of the drawing into an algebraic equation in which the unknown is the length of the simple pendulum and the knowns the bobs and their distances from A: if HK = x, AB = b, AD = d, and B and D denote the weights of the respective bobs, then the centers of gravity before and after will be $\frac{\mathrm{B}b + \mathrm{D}d}{(\mathrm{B} + \mathrm{D})d}$ and $\frac{\mathrm{B}b^2 + \mathrm{D}d^2}{(\mathrm{B} + \mathrm{D})xd}$, respectively. That is, $x = \frac{\mathrm{B}b^2 + \mathrm{D}d^2}{\mathrm{B}b + \mathrm{D}d}$, which again cannot be exactly located on the diagram. The same thing happens when Huygens then turns to extend this result, by generalization of an n-body pendulum, to the oscillation of a uniform rod.[28] In the continuous case, he imagines the rod as consisting of contiguous small bodies, swinging down under the constraints of a rigid body and then freed to rise individually to the heights corresponding to their acquired speeds (figure 9.7). But, rather than measuring their heights vertically, Huygens draws them horizontally, thus forming a parabola. By reasoning *mutatis mutandis* from the case of two bodies, he shows

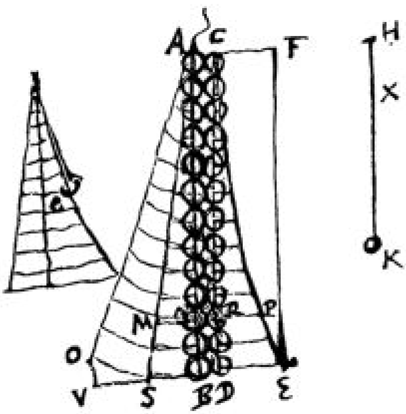

Figure 9.7. A rod resolved into contiguous elements. (Huygens 1888–1950, XVI 421.)

27 Huygens first used this technique in his derivation of the laws of elastic collision in *De motu ex percussione* in 1659. There, however, the bodies fall vertically from initial positions to acquire the speeds at which they collide, and then the speeds after collision are converted upward to resting positions. The basic principle is the same: the center of gravity of the system neither rises nor falls in the process. These drawings confirm what one suspects was the physical setup behind the diagrams in the earlier work, namely experiments on impact using pendulums.

28 The indeterminacy of n adds another reason for moving to algebra, where it can be treated operationally as a magnitude. There is no way to represent pictorially an indeterminate number of bodies.

that the equality of the heights of the centers of gravity before and after corresponds to the equality of areas of the triangle on the left (where BS = OV) and of the parabola on the right. The solution of the center of oscillation now comes down to the quadrature of the parabola. However, since the parameters of the parabola include the weight of the rod, that solution must again be couched in algebraic terms.[29]

Note that the triangle is simply an overlay on the physical picture of the pendulum, while the parabola is a mathematical configuration, graphing the height attained against the velocity as a function of the distance from the point of suspension. The centerline forms an interface between the two realms. The inclined planes that initially rendered that interface mechanically intelligible by directing the motion of the weights upward following collision disappear after the first construction. Thereafter, the physical configuration is pictured on the one side, the mathematical structure of the mechanics is pictured on the other. Transition from the one to the other takes place at the centerline by a transformation corresponding mathematically to Galileo's law relating velocity to height in free fall.

One finds similar configurations in Newton's *Principia*, for example in Proposition 41 of Book I:

> Assuming any sort of centripetal force, and *granting the quadrature of curvilinear figures*, required are both the trajectories in which the bodies move and the times of motions in the trajectories found.[30]

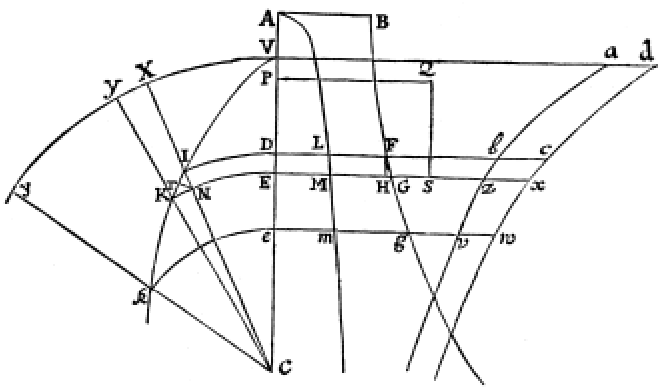

Figure 9.8. Newton's diagram. (Newton 1687, I.8, prop. xli probl. xxviii.)

29 Indeed, Huygens had to retain the algebra even in the finished geometrical form of [Huygens 1673].
30 For an extended discussion of Newton's mathematical methods, see Mahoney 1993, 183–205.

On the left is a picture of the orbit VIK of the body revolving about the center of force at C, together with a circle VXY superimposed as a measure of time; the angles in the drawing correspond to measurements that can be made by an observer (figure 9.8). On the right are a variety of curves, which represent the measures of various dynamic parameters such as force and velocity. They are mathematical structures with which one calculates, at least in principle, since in this diagram they are general curves drawn arbitrarily to demonstrate the structure of the problem, rather than any specific law of force. The lines connecting the two sets of curves at the centerline AC map areas under the mathematical curves on the right to sectors of the circle and orbit on the left, thereby determining the position of the planet on its orbit at any given time. The solution of the inverse problem of forces thus becomes a question of quadrature, of finding the areas under the curves on the right for particular laws of force.

For quite independent reasons, the reduced problem of quadrature took a new form with the development of the calculus. Geometry gave way to algebra as the language of analysis, and the construction of curves was supplanted by the manipulation of symbols. Pierre Varignon's adaptation of Newton's configuration shows the result (figure 9.9).[31] The left side remains the same, but complex of curves on the right is reduced to a single curve representing a general expression of the central force determining the orbit. Except for the two ordinates to the curve, there are no auxiliary constructions in the diagram to render graphically the transformations by which integration of the curve fixes the angle and radius of the corresponding position on the orbit. The curve itself thus plays no operational role in the argument, which

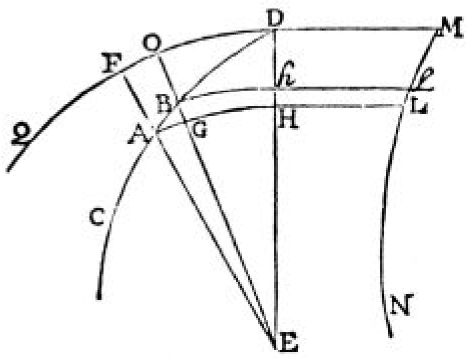

Figure 9.9. Varignon's diagram. (Varignon 1710.)

31 Varignon 1710; at 542. Varignon's solution of the problem of inverse central forces (given the force, to find the curve) rested on his earlier analysis of the direct problem (given the curve, to find the force) in Varignon 1700, which was based on a similar adaptation of Newton's diagram.

instead takes place entirely off the diagram in the Leibnizian notation of the infinitesimal calculus.[32] Indeed, the orbit is expressed in differential form, reflecting the generality of the problem and its solution.

I come out here where I came out in *Diagrams and Dynamics*, albeit along a different route. Here I have placed more emphasis on the importance (and the challenges) of graphical modes of thought in the early development of the science of mechanics, even though they were later abandoned. It is important to see how drawings functioned for Huygens, if only to discern what was involved if one really set out to use pictorial means to analyze the workings of machines and to quantify the underlying principles. Those principles were not to be found in the pictures, precisely because the pictures erased the boundary between the real and the fantastic; there has been no shortage of elegant pictures of perpetual motion machines.[33] What enabled Huygens to equate the areas on the right and the left of his diagram of the compound pendulum was a principle for which Torricelli is often given credit, but which surely predates him in the form of what I have referred to as a "maxim of engineering experience."[34] It comes down to this: bodies do not rise of their own accord, or, as the author of the treatise attributed to Juanelo Turriano puts it for a particular case, "water cannot go upward on its own ... because of its heaviness and weight."[35] In the case of interest to Huygens, a swinging pendulum winds down, or at best it keeps swinging at the same rate. It certainly does not swing more widely. Huygens makes this a quantitative principle by applying Galileo's law of free fall to the center of gravity of a system of bodies, moving at first under constraint and then freed of constraint. In doing so, he translates experience of the physical world into measurable behavior expressible in mathematical terms.

5. THEORY AND PRACTICE, KNOWLEDGE AND KNOW-HOW

What is particularly striking, and perhaps unusual, about Huygens' work on the clock is the close interplay between theory and practice. As noted above, the ultimate task was the reliable determination of longitude at sea, which is a matter of keeping time accurately. His abiding goal was to design a device accurate to within seconds a day and durable enough to withstand the rigors of service aboard ship under all conditions. In his own mind, the relation between theory and practice was seamless. A famous dispute surrounding his invention of a spring-regulated clock suggests other-

32 Indeed, Varignon's statement of the problem, while echoing Newton's proposition, reflected the shift of mathematical focus: "Problême: Les quadratures étant supposées, & la loy quelconque des Forces centrales f etant donnée à volonté en y & en constantes; Trouver en générale la nature de la courbe que ces forces doivent faire décrire au mobile pendent des tems ou des élemens de tems dt donnés aussi à volonté en y & en constantes multipliées par dx ou par dz variables ou non." (Varignon 1710, 536.)

33 David McGee makes a similar point (albeit eschewing the term "fantasy") in his contribution to this volume when he speaks of Taccola's style of drawing enabling him "to work in the absence of real physics." Indeed, he adds in a note, "the whole point of the drawing style is to *eliminate* the need for physical considerations on the part of the artist."

34 Mahoney 1998, 706–8.

35 Ps.–Juanelo Turriano, *Los veintiun libros de los ingenios y de las maquinas*, "el agua no puede ir de suyo para arriba ... por causa de su gravedad y peso." For an example of such maxims filtered through a reading of the *Mechanical Problems* attributed to Aristotle, see Keller 1976, 75–103.

wise and raises a question of interest in this context, namely, of what kinds of knowledge drawings contain and of how they serve as means of communication.

Shortly after publishing his *Horologium Oscillatorium* (Paris, 1673), Huygens uncovered the property of the cycloid that accounted for its tautochronicity: the accelerative force on a body sliding down the inverted curve is proportional to its displacement from the vertex at the bottom, the point of equilibrium. He quickly generalized the property into a principle he called *incitation parfaite décroissante:* in any situation in which the force acting on a body is proportional to its displacement from equilibrium, the body will oscillate with a period independent of its amplitude. By a series of experiments he then confirmed that the regular vibrations of springs rested on that principle and immediately sought to take advantage of the result.[36]

One of Huygens' worknotes shows that on 20 January 1675 he devised a mechanism for regulating a clock by means of a spiral spring.[37] Or rather I should say he sketched such a mechanism, for he did not build the mechanism himself, or even a model of it. Rather, he later related that on the 21st he sought out his clockmaker, Isaac Thuret, but did not find him until the morning of the 22nd, when he had Thuret construct a model of the mechanism while Huygens waited. Evidently, the model was completed by 3 p.m., and Huygens took it with him. The following day, Thuret built a model for himself and then on the 24th and 25th undertook to apply it to a watch. Soon thereafter, unbeknownst to Huygens, he displayed the watch to Colbert, presenting it in a such a manner as to suggest that it was his invention. Dismayed that Thuret would so violate a pledge of secrecy, Huygens vehemently rejected the claim, accused Thuret of violating his trust, and ended their longstanding collaboration.[38]

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36 Worknotes and sketches in Huygens 1888–1950, XVIII 489–98.

37 Huygens 1888–1950, VII 408ff; on Huygens' invention of the spiral balance, see Leopold 1980, 221–233 and Leopold 1982.

38 Huygens appears to have had grounds for being angry. According to his version of the facts, which Thuret's supporters did not contest, Huygens had pledged Thuret to secrecy before explaining the mechanism he wished to have built. Over the following week, Huygens did more work on the design as he planned both its announcement to the scientific community and its presentation to Minister Colbert for the purpose of securing a *privilège* restricting its manufacture to those licensed by Huygens. On 30 January, he wrote Henry Oldenburg, secretary of the Royal Society, announcing a new mechanism for regulating watches accurately enough to determine longitude and encoding its basic principle in an anagram. The next day he visited Colbert. On 1 February, he learned that Thuret had already attended the Minister on the 24th to show him the second model and that people were now speaking of Thuret as the inventor. Thuret had said nothing of this to Huygens, even though the two had collaborated during the week. Thuret had made hints to Huygens that he desired a share in the credit. But Huygens rejected the idea, even as he pointed out that in enjoying Huygens' license to produce watches with the mechanism Thuret stood to reap the greater monetary gain. On learning of what he took as a betrayal of trust, Huygens cut off all relations with Thuret, excluding him from a license. As the leading clockmaker of Paris, Thuret enjoyed the protection of some powerful patrons, not least Madame Colbert and her son-in law, Charles Honoré d'Albert de Luynes, Duc de Chevreuse. Their intercession led to Thuret's written acknowledgment of Huygens' sole claim to the invention and expression of regret that he might have acted in any way to suggest otherwise. In return he received authorization to produce the new watches. But, then, so too did all clockmakers in Paris, as Huygens decided not to ask the Parlement de Paris to register his *privilège*. Although the two men eventually reconciled, and indeed Huygens recognized the superiority of Thuret's craftsmanship, they never resumed their active collaboration, which had constituted a powerful creative force in timekeeping. For details of the dispute, which dominated Huygens' attention for six months, see Huygens 1888–1950, VII 405–498.

Given the ensuing dispute, a question arises: How did Huygens express or record his invention of the balance spring? That question seems to depend on another, to wit, when did he make the sketches accompanied by "Eureka 20 Jan. 1675"? Was it on that day, or was it some two weeks later, when after learning of Thuret's preemptive visit to Colbert he felt the need to defend his ownership of the invention by means of a day-by-day account of what had transpired in the meantime? That the invention occurred on that day seems clear from the evidence. By Huygens' account, not contradicted at the time by any of several people in a position to do so, he had the idea on the 20th, spoke of it to Pierre Perrault on the morning of the 21st and described it to Isaac Thuret around midday on the 22nd. Rather, the question is how he described it to Thuret. Did he make one or more drawings, and, if so, are they the drawings bearing the date? As John H. Leopold has observed, a look at the manuscript itself suggests the answer.[39]

Surrounding the "eureka" are two drawings and some notes. One drawing shows only a coil spring attached to a dumbbell balance, seen in top view. The other is a side view of the dumbbell mounted on an escapement, with the spring in a cylindrical housing mounted underneath a mounting plate (figure 9.10).

At the top is a descriptive heading, "Watch balance regulated by a spring." To the left underneath the coil and balance are two notes:

> le ressort doit se tenir en l'air dans le tambour et estre rivé au costè et a l'arbre (the spring should be held in the air in the drum and be riveted at the side and at the arbor)
>
> le balancier en forme d'anneau comme aux montres ordinaires (the balance in the shape of a ring, as in ordinary watches)

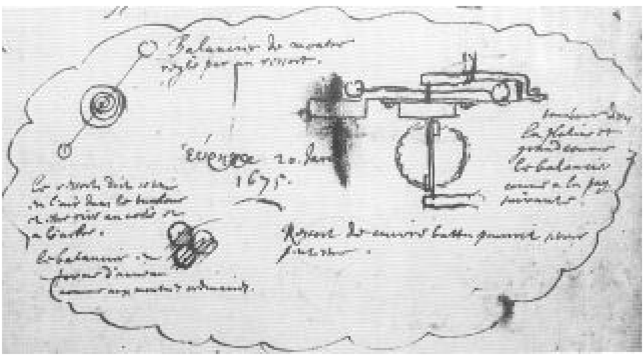

Figure 9.10. Huygens' "Eureka" sketch. (Leiden, University Library, Huygens Collection ms. E, fol. 35; reproduced in Leopold 1980 229.)

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39 Leopold 1980, 228.

The note to the right of the drawing of the escapement reads:

> le tambour dessus la platine et grand comme le balancier, comme a la pag. suivante (the drum above the plate and as large as the balance, as on the following page (figure 9.11))

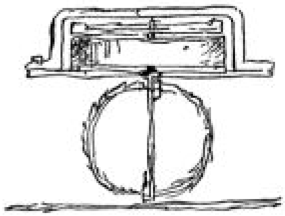

Figure 9.11. The improved escapement. (Leiden, Museum Boerhaave, Huygens Collection ms. E, fol. 36; reproduced in Huygens 1888–1950, VII 409.)

Finally, beneath the drawing, Huygens noted, "ressort de cuivre battu pourroit servir peutestre (spring of beaten copper could serve perhaps)."

Clearly, some of the notes describe not the drawings but rather changes to be wrought on the designs. Leopold suggests that we may have here the record of what transpired when Huygens visited Thuret. Huygens showed Thuret his sketches, described what he had in mind, and then jotted the notes as Thuret made suggestions based on his experience as a clockmaker. Except for the first note, that seems right. Thuret looks at the dumbbell balance and says "Let's use an ordinary balance wheel, the same size as the spring." As to material, he thinks beaten copper might work. He looks at the escapement and suggests moving the spring from under the plate to above it, so as not to interfere with the 'scape wheel. Huygens makes a note, but waits until getting home to make a sketch, perhaps drawing it from the model he has brought home with him.

By contrast, the first note may not reflect the clockmaker's experience but rather have served to reinforce what Huygens was telling Thuret in describing the basic design, perhaps in response to Thuret's uncertainty about it. The spring must be fixed at both ends but move freely in between. Huygens insisted on that as the core of the design in both the *privilège* and the description published in the *Journal des Sçavans*. The idea of using a spring as regulator had been floating around for more than ten years, and several people had tried without success to make it work.[40] Most had worked from the model of the pendulum, replacing its swing with the vibrations of a metal strip or the bouncing of a coil spring, in either case leaving one end free. Quite apart from the difficulties of transmitting those vibrations to the escapement, such a direct quotation of the pendulum led to difficulties in adjusting the period of the spring's oscillations. Huygens had approached the spring as he had the pendulum. The goal was to leave the oscillator swinging freely and to keep it separately adjustable, while communicating its swings to the escapement and feeding back just

40 See, e.g., Huygens' reply of 18.IX.1665 (Huygens 1888–1950, V 486) to a report from Robert Moray (Huygens 1888–1950, V 427) that Robert Hooke had spoken of "applying a spring to the balance of a clock in place of a pendulum." Huygens said he recalled having heard of the idea on a visit to Paris in 1660 but did not think it would work, at least as proposed. Having himself made the idea work in 1675, he never adverted to those earlier suggestions and to what role, if any, they played in his thinking.

enough force to keep the oscillator from running down. In Huygens' design for the spring regulator, the balance constitutes the freely swinging, adjustable weight, held steady in its pivots and communicating its motion to the escapement, either directly by pallets on the arbor or indirectly by means of a pinion. The spring, independent of the escapement and subject only to the coiling force of the arbor, regulates the swing, and its tension may be separately adjusted.

Toward the end of the dispute with Thuret, Huygens reported having reminded Thuret of just this point of the design. In an effort to explain why he had sought some part of the credit for the invention, Thuret claimed to have been thinking of such a mechanism but to have held back from doing anything because he thought that lateral vibrations of the spring would vitiate its regular oscillations. "I responded," said Huygens,

> that what he said of the trouble with these vibrations was something contrived to make it appear that he knew something about the application of the spring, but that this itself showed that he had known nothing about it, because, if he had thought of attaching the spring by its two ends, he would have also easily seen that these vibrations were of no concern, occurring only when one knocked or beat against the clock and even then not undercutting the effect of the spring.[41]

Hence, the first note may account for something Huygens said on several occasions in his dispute with Thuret. "In explaining it to him," reported Huygens of his initial conversation with Thuret, "he said (as yet barely understanding it), 'I find that so beautiful that I still can't believe it is so.'"[42] As Huygens later reminded Thuret, he had said nothing about any investigations of his own prior to Huygens' visit. Yet, if Thuret had been thinking along the lines he later claimed, then he may well have expressed wonder about precisely the way in which the spring was mounted.

And it may well have been wonder born of having wrestled with the problem without seeing a solution. For Thuret evidently understood what Huygens showed him well enough both to make suggestions for its improvement and to build a working model from scratch in a couple of hours. Indeed, when Huygens then departed with the model, Thuret built another, evidently from memory (or did he make his own sketch?), and at once set about to incorporate a working version into a watch (figure 8.12). How much did he have to understand to do that? What did the second model look like, and what changes did Thuret make in adapting it to the watch? In this case, it is not a matter of scaling the model up, but rather of making it smaller and fitting it in with the other parts of the watch. Irrespective of whether Thuret had in fact been exploring the problem independently of Huygens, he might have thought that the know-how involved in that process alone entitled him to share in the privilege for the clock on the grounds that he had helped invent it. Huygens argued that Thuret had simply been following his sketch, which contained the essence of the invention.

41 Huygens 1888–1950, V 486.
42 "En la luy expliquant il dit, ne l'entendant encore qu'a peine, je trouve cela si beau que je me défie tousjours qu'il ne soit ainsi." Huygens 1888–1950, VII 410.

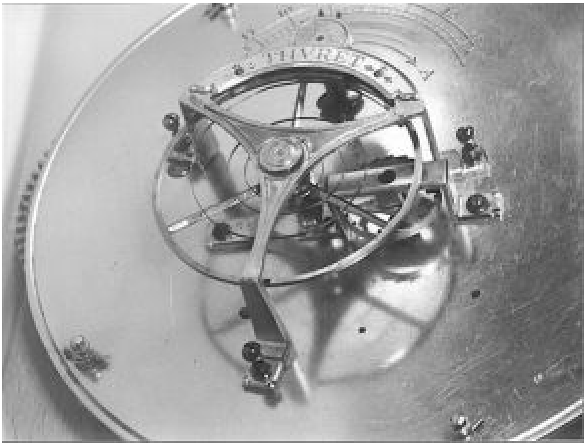

Figure 9.12. The escapement of a Thuret clock, 1675. (From Plomb 1999.)

Huygens' response to another challenge of his priority, this time from a gadfly named Jean de Hautefeuille, is revealing in this regard. Shortly after Huygens announced his invention to the Academy, Hautefeuille contested the claim both intellectually and legally, arguing that he had earlier proposed replacing the pendulum of a clock with a thin strip of steel, the uniform vibrations of which would have the same effect.[43] It would have the further advantage of working irrespective of the position of the clock and hence lend itself to use in watches. Hautefeuille admitted that he had not succeeded in getting his mechanism to work, but he insisted on having established the principle of using a vibrating spring as a regulator. The principle lay in the equal vibrations of a spring, whether straight, helical, or spiral. The particular shape and configuration of the spring was incidental, a matter to be determined by a workman *(ouvrier)*.

Huygens responded by noting that Hautefeuille was far from the first to suggest using a spring as a regulator. It had been proposed repeatedly, but none of the earlier designs had actually worked in practice. The trick lay in transforming the spring's equal vibrations into the uniform advance of an escapement, and more than the spring was involved. Thinking that one could simply replace a pendulum by a metal strip, even to the point of adding a weight at the end of the strip to strengthen the vibrations, reflected a basic misunderstanding of the problem.

43 J. de Hautefeuille, "Factum, touchant Les Pendules de Poche" (a petition to the Parlement de Paris to block the registration of Huygens' *privilège*, Huygens 1888–1950, VII 439–53; cf. his letter to the Académie des Sciences, 7 July 1674, describing his idea, Huygens 1888–1950, VII 458–60.

Huygens' claim to the invention of the balance-spring regulator rested on two different sorts of knowledge. First, he knew in principle that a spring, whatever its form, was a tautochronic oscillator. That meant more than knowing for a fact that springs vibrate at the same rate no matter how much they are stretched or compressed. It meant knowing why that was the case. His letter to the *Journal des Sçavans* announcing the invention spoke of the movement of the clock as being "regulated by a principle of equality, just as that of pendulums is corrected by the cycloid."[44] That piece of knowledge stemmed from his researches of 1673–74. As a result he knew that the relation between a spring and a cycloidal pendulum lay in their both instantiating the principle that the driving force is directly proportional to the distance from rest. They are different manifestations of the same kind of motion.

Second, Huygens had determined the particular arrangement that translated the principle into practice for the spring. As in the case of the pendulum, it was a matter of letting a weight swing freely, driven ever so slightly by the force of the driving weight or mainspring, communicated through the escapement. Hanging the pendulum by a cord attached to the frame, and connecting it to the escapement by means of a crutch, had been the key to using it to regulate the clock while using the clock to keep the pendulum swinging. In the case of the spring regulator, the weight was the balance wheel, pivoting about the arbor as an axis and governed by the winding and unwinding of a spiral spring attached at one end to the arbor and at the other to the plate or support on which the arbor was mounted. While his original drawings showed the arbor with pallets directly in contact with the crown wheel, the version for the *Journal des Sçavans* showed a pinion moving a rack wheel, which turned the pallets connected with the crown wheel. That design quoted the arrangement in his original clock of 1658.

What Hautefeuille thought was a mere "accident," a technicality to be left to artisan, Huygens considered essential. It was the weight of the balance wheel that held back the advance of the escapement. Since the wheel was swinging about its center of gravity, its motion was independent of its position in space. The spring maintained the tautochrony of that motion. Neither component could do the job alone, yet they could be independently adjusted for the force of the mainspring and the period of oscillation. The secret of a "regular, portable" timepiece lay in their combination.

For present purposes, the rights and wrongs of Huygens' dispute with Thuret are the least interesting aspect of the episode. What is more interesting is the intersection of craft knowledge and high science, of what one knew from building mechanisms and what one knew from analyzing their dynamics. Regrettably, only the latter knowledge was self-consciously set down in words; we have only Huygens' account of this affair. For the former, we must reason indirectly from the artifacts, something I must leave to the antique horologists who know enough about clockmaking to do it critically.[45] In the case of Renaissance machines we also lack the high science. But

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44 .*Journal des Sçavans*, 25.II.1675; repr. in Huygens 1888–1950, VII 424–5; at 424: "... leur [sc. les horologes] mouvement est reglé par un principe d'égalité, de même qu'est celuy des pendules corrigé par la Cycloïde."

we do have the drawings and must find ways to reconstruct from them and, with the help of modern craft expertise, from the artifacts, the conceptual framework of the artisans a half-millennium ago.

Also interesting is the direction Huygens took subsequently. The spring balance, it turned out, had its deficiencies. The practical goal of the enterprise was an accurate sea-going clock for determining longitude. While the spring was mechanically more stable than a pendulum, it was sensitive to changes in temperature and humidity to a degree that undermined its accuracy. So Huygens pursued other mechanisms, all of which had in common the underlying mechanical principle of the spring: the force driving them varied as the displacement from equilibrium. They were all what subsequently came to be called simple harmonic oscillators. Sketches for such mechanisms run for pages in his works, most accompanied by mathematical demonstrations of their workings. In some instances, it is not clear how they would be incorporated in a clock. In other cases, the designs were in fact realized. In the two best known instances, his tricord pendulum and his "perfect marine balance," Huygens built a model before turning to a clockmaker to produce a working timepiece.[46] In both cases, the model seems to have been a necessary proof of concept before attempting a full-scale clock.

What does this all have to do with the topic of this volume? Two things, I think. First, it shows that just drawing a picture of a device, however realistic the rendering, does not suffice to explain how it works. What it conveys depends to a large extent on what knowledge the viewer brings to it. As Huygens tells it, at least, Thuret built a model of the spring balance from Huygens' drawing without understanding how it worked. But having built one model and having heard Huygens' explanation, he evidently knew enough about the workings of the device to adapt it to a watch. That is, Thuret knew what he could change and what not; he knew how to scale it to his own needs. But he went beyond that. He claimed a share in the credit for the invention on the grounds that he had transformed Huygens' idea into a working device. If the interpretation above of what occurred between the two men is correct, Huygens' drawing did not suffice for that; it required a clockmaker's knowledge of the field of application. To Huygens' way of thinking, the invention lay in the idea. For all Thuret's expert knowledge, he had not understood at first why it worked.

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45 On this point, see Mahoney 1996, 63–68. The several articles in that volume on the clocks of John Harrison are brilliant examples of such direct "readings" of the artifacts.

46 Huygens seems to have drawn a lesson from his experience with Thuret. In his "Application de Décembre 1683" he wrote: "Le 17 dec. 1683 j'ay portè [*sic*] a Van Ceulen l'horologer le modelle que j'avois fait de ce mouvement de Pendule Cylindrique, pour changer de cette facon les 2 horologes que je luy avois fait faire our la Compagnie des Indes Orientales. J'avois prié mon frere de Zeelhem de venir avec moy: parce que ledit horologer s'imaginoit d'avoir trouvé la mesme chose que moy, apres m'en avoir ouy dire quelque chose en gros. Mais ayant vu le modelle il avoua que ce qu'il avoit modelé n'y ressembloit nullement." (Huygens 1888–1950, XVIII 532) Here a comparison of models seems to have prevented confusion between what Huygens and Van Ceulen had in mind respectively and suggests that a drawing such as the one Huygens had earlier made for Thuret might have been too vague or, literally, sketchy to do so.

6. MATHEMATICAL MODELS

That difference of perspective leads to the second point. By the late seventeenth century, two kinds of people were emerging from the machine literature of the Renaissance, those concerned with machines and those concerned with (mathematical) mechanics. What Thuret, the craftsman, built was a model of a specific mechanism, a coiled-spring balance. From that point on, he was interested in the various ways in which it could be used in watches and clocks. As was the case for his counterparts in England, his clockmaking skills recommended him as an instrument maker for the new scientific institutions. In the following century, clockmakers built the first textile machinery. In short, artisans like Thuret became machine builders, for whom a new genre of machine literature would develop in the nineteenth century.[47]

What Huygens, the mathematician, showed Thuret was a model of a general principle, "perfect incitation." For Huygens the spring balance was one of a series of models that began with the cycloidal pendulum and included the vibrating string, the spring, and the variety of forms of the "perfect marine balance." Once he had discovered the principle, he began to look for the ways in which it was instantiated in physical systems. His drawings of those systems all aimed at bringing out the proportionality between the accelerative force and the displacement from equilibrium. In some cases, it emerged more or less directly, as in the case of chains being lifted from a surface or of cylinders being raised and lowered in containers of mercury. In other cases, it took some sophisticated and ingenious manipulation to relate the geometry of the mechanism to that of the cycloid. Later mechanicians would seek it symbolically by reducing the equation of the system to the form

$$F = m\frac{d^2S}{dt^2} = -kS.$$

To work that way is to build a model of another sort, namely a mathematical model, in which one seeks to map a physical system onto a deductive structure. As Newton showed, using the same mechanics as Huygens but positing another kind of *incitation*, one in which the accelerative force varies as the inverse square of the distance, unites Kepler's empirically derived laws of planetary motion in a mathematical structure, of which Galileo's laws of local motion are limiting cases. In a second set of "Queries" added to his *Opticks* in 1713, he turned his attention to the chemical and electrical properties of bodies and wondered rhetorically whether they might not be explained in terms of small particles of matter attracting and repelling one another by central forces of a different sort from gravity. "And thus Nature will be very conformable to her self," he mused, "and very simple, performing all the great Motions of the heavenly Bodies by the Attraction of Gravity which intercedes those Bodies, and almost all the small ones of their Particles by some other attractive and repelling

47 In England, the great clock and instrument makers could aspire to learned status, as Richard Sorrenson has shown in his dissertation [Sorrenson 1993]; for an example of one such clockmaker, see Sorrenson 1996.

Powers which intercede the particles."[48] His suggestion became the agenda of mathematical physics for the next two centuries, as practitioners sought to apply the increasingly sophisticated resources of analytic mechanics to mechanical models of natural phenomena. Following that agenda into the late twentieth century and to its encounter with complexity leads to a new problem of scaling and to what appears to be a redefinition of the relation of mathematics to nature. But that issue really would take this contribution beyond the question of the pictorial means of early modern engineering.

48 Newton 1730, 396.

APPENDIX

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